Heterogeneous expectations in experimental asset markets

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Abstract

Beliefs play a fundamental role in economic choices and aggregate market outcomes. A substantial theoretical literature argues that the disagreement among investors about the value of assets is responsible for price bubbles in financial markets. So far, no study was able to directly proof this line of theory, neither in secondary data analysis nor in experimental studies. Contrary to prior studies, we use a direct test of this relationship by varying the belief dispersion in Smith et al. (1988) markets. Using a novel experimental treatment, we allocate subjects into markets based on their elicited beliefs of future prices prior to trading. Our results suggest that heterogeneous beliefs are related to more pronounced price bubbles, while there is no effect on trade volume and share concentration. We further find that beliefs are positively related to risk aversion.

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1 Introduction

Financial bubbles, during which the market value of assets vastly exceeds a reasonable assessment of fundamental value, have occurred frequently in modern history.[1] Historical bubbles have often been accompanied by several characteristics. First, bubbles tend to coincide with technological and financial innovations. New technology and financial innovations introduce fundamental uncertainties about the value of investments, increasing the disagreement among investors about future prospects and prices. Second, the new uncertainty creates an unexplored environment of overconfidence, causing investors to overreact to their favorite signals resulting in excessive and speculative trading. During the boom period, optimistic beliefs result in excessive trade volumes and asset price inflation. Eventually the bubble bursts, resulting in financial crises and real economic depression.

Since the late 1970's, several scholars have argued that differences in investors' beliefs may be responsible for mispricing and trade volumes in financial markets.[2] Miller (1977) claims that differences in investor beliefs about the value of an asset can lead to persistent overvaluation in market with constraints to short sales. When investors with lower valuations are unable to express their beliefs in these markets after selling their shares, prices will only reflect beliefs of relatively optimistic investors.[3] According to this *overvaluation hypothesis*, asset prices increase with the dispersion in beliefs. Heterogeneous beliefs have also been used to explain trade volumes (Varian, 1992; Harris and Raviv, 1993; Hong and Stein, 2003) and share concentration (Harrison and Kreps, 1978). Disagreement about an asset's value causes optimistic investors to buy from pessimistic investors, hence increasing the volume of trade. As a consequence of these trades, share ownership will become increasingly concentrated among optimistic investors.

Within empirical literature, there is considerable disagreement about the relationship between beliefs and market behavior. Several empirical studies claim to have found evidence supporting the

[1] For treatises on historical bubbles and crashes, see Kindleberger (2000) and more recently Xiong (2013).

[2] Economic literature makes a distinction between differences in *posterior* beliefs and differences in *prior* beliefs. Whereas the former is explained by different information, different *priors* relates to beliefs about possible states of the world. In this paper, differences in beliefs strictly relates to different priors. For a discussion on the common prior assumption, see Morris (1995).

[3] In this paper, *optimists* refers to market participants with higher beliefs of an asset's future prices relative to *pessimists*, which are market participants with lower beliefs of future prices. The variance in the distribution of optimistic and pessimistic beliefs is termed *belief dispersion*.

conjecture that heterogeneous beliefs result in inflated asset prices (Brown and Brown, 1984; Diether et al., 2002) and elevated volumes of trade (Hong and Stein, 2007). Others (Avramov et al., 2009) argue that these relationships disappears when controlling for specific characteristics in the sample data. Moreover, there is considerable debate about the methodologies employed to measure beliefs and overpricing. Since both belief dispersion and the value of assets are unobservable, empirical literature has to rely on proxies for the variables of interest (Abarbanell et al., 1995; Garfinkel, 2009). These proxies may be biased and correlated with other characteristics that affect valuation, thereby limiting the ability to make causal inferences.

In response to conflicting empirical results and methodological difficulties, several economists have used experiments to study the effect of belief dispersion on market behavior. Within the laboratory setting, asset prices and beliefs can be measured directly, which avoids the need to use proxy variables. The researcher can control the market environment in order to isolate the causal mechanisms that drive market behavior. Moreover, the laboratory provides an environment where subjects can truthfully report on their beliefs (Bloomfield and Anderson, 2010).

Experimentalists commonly study market behavior using the asset market design by Smith et al. (1988). The key finding in most research is that prices start below the asset's fundamental value, exceed fundamental value to produce a persistent bubble, before crashing violently just before maturity. This design has hitherto been used to test a wide variety of factors governing price bubbles.[4] Studying the relation between expectations and market prices, Haruvy et al. (2007) elicited expectations of future period prices from subjects in repeated markets of the design by Smith et al. (1988). While not discussing belief dispersion in detail, Haruvy et al. (2007) notes that beliefs become more homogeneous as subjects become experienced in later markets. Carle (2016) revisited the data by Haruvy et al. (2007) and reports a positive relation between the initial diversity in subject opinion and price levels. In line with the overvaluation hypothesis, the initial dispersion of beliefs is indicative of later market price levels, with disperse beliefs resulting in a more pronounced bubble formation. At the same time, Carle (2016) does not find a relationship for belief dispersion on trade volume, neither at the market level nor for individual periods.

[4] For surveys of studies employing the design by Smith et al. (1988), see Sunder et al. (1992) and more recently Palan (2013); Powell and Shestakova (2016).

The previously mentioned experimental studies relate belief dispersion and market behavior *ex post*, and do not use belief dispersion as a variable of experimental manipulation. This paper aims to provide an alternative approach to answering the suggested relationship between beliefs and market behavior, by experimentally manipulating the dispersion in beliefs prior to the start of the experimental session. By varying the initial dispersion of beliefs, we seek to answer the following research question: *How does the dispersion of beliefs of future prices among market participants influence market behavior in experimental asset markets?* We approach this question by eliciting beliefs of current and future period prices prior to assigning subjects into markets. Contrary to Fellner and Theissen (2014), we manipulate belief dispersion using beliefs that subjects bring into the laboratory. Subjects are ranked based on their initial beliefs and allocated to markets in order to vary the initial belief dispersion between parallel markets. By varying the belief dispersion within our markets, our experimental method provides a direct test of the relationship between belief dispersion and price levels, as predicted by theoretical literature.

This study makes several contributions to the literature on heterogeneous beliefs and market behavior. First, we find that the distribution of beliefs has a high degree of path dependency. Whereas optimistic subjects are likely to remain optimistic, initial pessimists tend to remain pessimistic. Second, our results indicates a positive relation between belief dispersion and bubble formation, in line with theoretical literature (Miller, 1977) and previous experimental analysis by (Carle, 2016). Markets with higher dispersions of initial beliefs tend to produce larger bubbles, although the number of observations is too low to claim evidence. Contrasting the positive relation of belief dispersion on price level, we find no difference for either trade volume or share concentration between markets that are different in belief dispersion. We also relate beliefs to subject characteristics, and find that beliefs are negatively correlated to risk aversion. Finally, by assigning subjects to markets based on their beliefs, this paper adds to recent experimental studies involving the ex ante manipulation of subject groupings in markets of the design by Smith et al. (1988).[5]

Because of the limited size and scope of our experimental setup, our results suffer from a low level of statistical significance. Additional experimental tests are necessary to determine the success

[5] Manipulation of subject grouping has been used to study cognitive ability (Hanaki et al., 2015; Bosch-Rosa et al., 2015; Breaban and Noussair, 2015), experience (Dufwenberg et al., 2005; Deck et al., 2014), willingness to speculate (Janssen et al., 2015), and gender (Eckel and Füllbrunn, 2015; Holt et al., 2015).

of our novel experimental treatment. Moreover, while our data suggests a positive relationship between belief dispersion and price level, the analysis provides little insight into the mechanism underlying this result. Finally, we argue that future research should aim to better understand the intricate relationships between individual beliefs, trading decisions and market prices.

This paper is structured as follows. Section 2 reviews the related literature on bubble formation and heterogeneous beliefs, leading to several hypotheses listed in Section 3. The experimental setup and treatment design are described in Section 4. Section 5 reports on the experimental results and test our hypotheses. Section 6 discusses our findings and makes suggestions for future research.

2 Literature review

"Bulls don't read. Bears read financial history. As markets fall to bits, the bears dust off the Dutch tulip mania of 1637, the Banque Royale of 1719-20, the railway speculation of the 1840s, the great crash of 1929."

- James Buchan, 2001, Frozen Desire

How does the divergence of opinion between bullish and bearish investors influence market prices? Dominant financial literature, such as in the capital asset pricing model, assumes that market participants have homogeneous beliefs about the future performance of financial assets. Investors and potential investors have access to the same information and agree about risk and expected returns. In response to recurring price bubbles, market volatility and excessive trading volumes, an increasing body of literature has considered the role of disagreement among investors. Disagreement can the result of different information, a different interpretation of the same information or different preferences and views of the future.

A substantial theoretical literature suggests that the disagreement among investors about the value of assets lead to inflated asset prices.[6] Miller (1977) argues that in a market with short selling constraints, heterogeneous investors beliefs lead to persistent overvaluation. When the pessimist

[6] Several scholars disagree with this hypothesis. For example, Varian (1985) argues for a negative relation between the divergence in opinion and asset prices. In his model of the financial market, disagreement among investors leads to additional uncertainty. To compensate for this uncertainty, risk averse investors will demand an additional risk premium, leading to higher expected returns and lower asset prices instead. Additionally, Merton (1985) claims investor disagreement leads to market segmentation, which has a negative effect on asset prices.

investors are unable to express their opinion by taking a short position, market prices will only reflect the opinion of the optimist investors. Optimists will express their beliefs through higher bid and ask orders, driving up prices. At the same time, pessimist investors will exit the market by selling their shares to optimist investors. As the heterogeneity of investor beliefs increases, optimist investors drive up prices in accordance with their inflated beliefs, causing ownership to become increasingly concentrated within a smaller group of optimist investors.

In addition to Miller's overvaluation hypothesis, the disagreement about the value of an asset can induce speculative behavior.[7] Morris (1996) builds on Miller (1977) by using a dynamic model of beliefs to explain price bubbles following initial public offerings. In his model, investors have heterogeneous prior beliefs about the stock's fundamental value. As additional information becomes available, investors update their beliefs. During this updating process beliefs fluctuate and may cross each other, inducing investors to anticipate capital gains by reselling the asset at a higher price to a more optimistic investor. The ability to reap these speculative profits will be reflected as a premium, allowing the current price to exceed even the fundamental valuation of the most optimistic investors (Harrison and Kreps, 1978). In addition to inflated price, the updating of investor beliefs is accompanied by higher volumes of trade, as investors rebalance their portfolio in accordance with their beliefs.

The vast body of empirical evidence provides mixed and inconclusive evidence on the relation between belief dispersion and market behavior. Brown and Brown (1984) provides the earliest empirical test of the overvaluation hypothesis by examining the effect of heterogeneous expectations on prices of U.S. farmland. In their model, land prices include a speculative component in addition to fundamental factors. They report evidence for a speculative resale component in land owner's reservation prices that increases with the dispersion in beliefs, in line with theoretical predictions by Harrison and Kreps (1978). Diether et al. (2002), which has become a benchmark for empirical studies of belief dispersion, found evidence for overvaluation in stock markets by studying the dispersion in analysts' earnings forecasts and future stock returns. The authors found that belief dispersion and concentrated share ownership are associated with lower future returns, and hence

[7] Following Keynes (1936), speculation is defined as the willingness to pay more for an asset when able to resell rather than being obligated to hold it forever.

higher asset prices. Chang et al. (2007) studied the effect of heterogeneous beliefs under short selling constraints. Instead of comparing the dispersion in analyst forecasts, Chang et al. (2007) used trade volume and the stock's beta as proxies for belief dispersion. By comparing stock returns under different short selling regimes in the Hong Kong stock market, they report that short selling constraints tend to cause overvaluation. Moreover, the overvaluation effect is more dramatic for individual stocks for which there is a larger dispersion of investor beliefs.

Many other empirical studies (Chen et al., 2002; Park, 2005; Boehme et al., 2006; Hong and Stein, 2007) have replicated Miller's (1977) overvaluation result.[8] Opponents argue that the overvaluation implied by lower future stock returns is only valid for assets with certain characteristics, such as small or illiquid stocks (Manski, 2006). Avramov et al. (2009) claims that financial distress can account for the proposed relation instead. Using credit ratings as a proxy for the dispersion in opinion, they show that the dispersion effect is caused by lower rated stocks during periods of financial distress. When adjusting for credit rating, the negative relation between belief dispersion and returns disappears. Additional conflicting results are provided by Doukas et al. (2006), who argues that the dispersion in analysts' forecasts is a biased measure of belief dispersion. Using a different measure of belief dispersion, they report a positive correlation between prices and future returns, contrary to the overvaluation hypothesis.

In addition to conflicting results, empirical research on the effects of beliefs on prices faces several shortcomings (Garfinkel, 2009). Both market behavior and belief dispersion are unobservable and therefore require measures based on proxies. These proxy measures of belief dispersion and price level may be noisy or biased and suffer from unobserved confounding effects, for which the researcher is unable to control (Fellner and Theissen, 2014). Finally, financial analysts and investors may not face incentives to honestly report on their beliefs. Trueman (1994) reports that analysts time earnings and price forecasts and exhibit herding behavior, whereby they release forecasts similar to those announced by other analysts. By providing a controlled environment, experimental research can solve for these issues. The experimentalist can control the information flows available to

[8] While most empirical tests of the overvaluation hypothesis involve stock markets, some have tested the effects for other asset markets. For example, Carlin et al. (2014) studied the level of disagreement about MBS prepayment speeds among Wall Street mortgage dealers. Contrary to the overvaluation literature, they report a positive relation between the level of disagreement and expected returns, thus implicating a lower price level instead.

subjects, allowing for tests of different prior beliefs (Bloomfield and Anderson, 2010). Second, an environment can be created were subjects are incentivized to truthfully report their beliefs. Third, the laboratory allows the researcher to manipulate the environment to measure the effect of a single variable while controlling for other factors, allowing tests of causality.

The most commonly used direct study of bubble formation is the experimental asset market design by Smith et al. (1988). Within these markets, subjects trade a risky asset that pays a dividend at the end of each period. The typical finding is that prices start below the asset's fundamental value, exceed fundamental value to produce a bubble, before crashing violently just before maturity.[9] Experimental evidence on markets with a known heterogeneous trader composition shows that increased heterogeneity tends to increase bubble formation. Hanaki et al. (2015) found that markets with heterogeneous cognitive abilities among subjects results in significantly larger asset mispricing, which they relate to increased strategic uncertainty. As traders become aware of the presence of market participants with a lower cognition function, they may seek to profit from their less sophisticated peers by engaging in speculative behavior.

Contrasting the abundance of empirical research, experimental evidence on the relation between belief dispersion and asset prices is scarce. We know of only a single experimental test in which belief dispersion is subject to experimental manipulation. Fellner and Theissen (2014) designed an experiment in which subjects can trade an asset that has either a high or low true value. At the start of the experiment, subjects received a noisy private signal about the true value of an asset. By varying the known precision of the signal, subjects were endowed with different information, thereby creating a divergence of opinion within markets. Contrasting Miller's (1977) overvaluation hypothesis, the authors find that asset prices are not higher for markets with a higher divergence of opinion. When discussing this result, Fellner and Theissen (2014, p. 26) notes that market participants may act on sentiment, which is not necessarily consistent with the information of the signal. Since the dispersion in beliefs was not measured but rather imposed on the experimental market, the extend to which the differential information endowment affected

[9] A common interpretation of this result is that bubbles emerge because subjects expect to pass the asset on to an even less rational subject (hence the "greatest fool hypothesis") before the price eventually crashes. However, a small bubble still emerges when investors have no resale option and are forced to hold the asset until the end (Lei et al., 2001). Therefore, the resale hypothesis (Harrison and Kreps, 1978; Morris, 1996) is unable to explain total mispricing on its own.

beliefs between subjects was not tested. The authors did not report on the effectiveness of their treatment in creating truly heterogeneous markets, which might explain the lack of a relationship.

Other experimental evidence is provided by Carle (2016), who revisited the data of Haruvy et al. (2007), who elicited subject beliefs of current and future prices in Smith et al. (1988) markets. Carle (2016) calculate the coefficient of variance of individual beliefs, serving as a measure of belief dispersion within each market. When regressing the observed overpricing on this measure of belief dispersion, he finds that bubbles significantly increases with belief dispersion. In line with the overvaluation hypothesis (Miller, 1977), an increase in belief dispersion is accompanied by higher asset prices. Moreover, Carle (2016, p. 69) notes that the distribution of beliefs exhibits path dependency, where belief dispersion in the first period is indicative of belief dispersion in the remaining market periods. Therefore, the total market overpricing can already be anticipated using the belief dispersion prior to observing prices.

Despite the advantages of belief elicitation within the laboratory, a direct experimental test of the relation between belief dispersion and market behavior is lacking. This research seeks to fill this gap, by directly varying the dispersion in belief among market participants prior to trading in Smith et al. (1988) markets.[10] In addition, we provide additional evidence on the relation between beliefs and trading decisions.

3 Hypotheses

The theoretical framework in Section 2 provides several testable predictions about the effects of heterogeneous beliefs. Our first hypothesis concerns the relationship between belief dispersion and price level, which is the focus of this research. The static model of heterogeneous beliefs by (Miller, 1977) suggests that the relatively optimistic investors determine prices when there are significant costs to short selling. When the heterogeneity of beliefs increases, the expression of more optimistic beliefs results in more pronounced bubbles. In addition, prior research of subject heterogeneity in Smith et al. (1988) markets generally finds that prices increase with the degree of heterogeneity.

[10] Theoretical literature (Miller, 1977; Harrison and Kreps, 1978; Morris, 1996) emphasizes the role of short selling constraints on the positive relation between beliefs dispersion and price levels. Due to our limited experimental resources, we solely focus on the effect of belief dispersion, given complete constraints on short selling.

Therefore, the heterogeneity of beliefs is expected to be positively related to price level.

Hypothesis 1: *Belief dispersion is positively related to bubble formation.*

Our second hypothesis concerns the relationship between belief dispersion and trade volume. Theoretical literature (Varian, 1985; Hong and Stein, 2003) predicts that the disagreement about asset values results in increased volumes of trade among investors. Therefore, the heterogeneity of beliefs is expected to be positively related to trade volume.

Hypothesis 2: *Belief dispersion is positively related to trade volume.*

The third hypothesis concerns the relationship between belief dispersion and the concentration of share holdings. Miller (1977); Harrison and Kreps (1978) claim that the dispersion in beliefs leads to more concentrated share ownership, as relatively pessimistic investors will sell their shares to relatively optimistic investors. Therefore, when the diversity of beliefs increases, share ownership will be increasingly concentrated within the most optimistic investors.

Hypothesis 3: *Belief dispersion is positively related to the concentration of share ownership.*

4 Methodology

The focal point of this study is to consider the effect of belief dispersion on price, by comparing markets with relatively heterogeneous beliefs to markets with more homogeneous beliefs. The hypothesis in the previous section are tested by conducting an experimental asset market in which we directly vary the heterogeneity of beliefs in markets while controlling for other variables.[11] We use the seminal design by Smith et al. (1988), which has been used to study a variety of market structures, institutions and trader characteristics and is known to consistently produce price bubbles. During each session, we construct markets that vary in belief dispersion by selectively grouping subjects based on their initial beliefs of future prices prior to the start of trading. In the following, we describe our experimental procedure and treatment in detail.

[11] Contrary to Fellner and Theissen (2014), all subjects in each session receive the same information and are part of the same training session. Therefore, the dispersion in initial beliefs is due to differences in *ex ante* subject beliefs. In section 5.4 we test whether subject characteristics can be used to explain the variation in initial beliefs.

4.1 General procedure

The data were gathered in one session at the NSM Decision Lab at the Radboud University in Nijmegen, the Netherlands and two sessions at the Rhine-Waal University of Applied Sciences in Kleve, Germany. Sessions were conducted in the period from May 2016 to June 2016. The session at the Radboud University consisted of two parallel markets, each running two market rounds.[12] The sessions at the Rhine-Waal University each consist of three parallel markets, which ran a single market round.[13] This resulted in a total of ten experimental markets, of which eight are first round types. After the trading phase, subjects are asked to complete a short Cognitive Reflection Test (CRT) (Toplak et al., 2011). This tests comprises 7 questions, each having a single correct answer, an intuitive (incorrect) answer and an incorrect answer. Each session lasted approximately two hours, including the first thirty minutes during which the experimenter read the instructions and trained participants in the use of the market software.[14] The computerized market used z-Tree (Fischbacher, 2007) and participants were recruited using ORSEE (Greiner, 2004). A total of 60 students participated in the three experimental sessions, all of whom were inexperienced in asset market experiments, with the majority of students majoring in economics and business administration.[15]

4.2 Market structure

Adopting design 4 of Smith et al. (1988), subjects are endowed with shares and cash. Subjects can trade these shares during a sequence of 15 call market trading periods. Within rounds, a participant's cash balance and inventory is carried over from one period to the next. At the end of each period, each share pays a dividend drawn from the set [0, 8, 28, 60] Gulden, with equal probability and independent from each period. To control for the effect of dividend payments on price development, each experimental market had the same sequence of dividend payments. In

[12] Due to the low number of participants (13) in the Radboud University session, we were able to construct two instead of three parallel markets, each containing six or seven subjects.

[13] Because of time constraints at the Rhine-Waal University sessions, we were unable to run a second market round.

[14] Appendix A.5 contains experimental instructions provided to participants. A table of holding values and histograms of simulated share holding values at each period was provided on a separate sheet.

[15] Appendix A.4 contains descriptive statistics of subject characteristics for each of the three sessions.

Table 1: Parametrization

Category	Variable	Parameter(s)			
Market	Number of rounds	1 or 2			
	Number of periods	15			
	Subjects per market	7 or 8			
	Clearing institution	Call market			
Shares	Dividends per share	0	8	28	60
	Probability of dividend	25%	25%	25%	25%
	Buy back value	50			
	Expected (fundamental) value	410			
Endowments	Shares per subject	5			
	Cash	600			
	Loan	1,500			
	Expected value of endowments	2,650			

Note: This table shows the parametrization of the experimental setting. The experimental currency is *Gulden*, with 200 Gulden = 1 Euro. The number of rounds and subjects per market is different per session. Since the loan has to be repaid at the end of the last period, it is not included in the expected value of the endowments.

addition, after paying the final dividend at the end of the last period, each asset has a buy back value of 50 Gulden. Therefore, the expected (fundamental) value of each asset in any period t equals the expected dividend in each period of 24 Gulden, multiplied by the number of dividends remaining $(16 - t)$, plus the value of the asset buy back. Given the 15 dividend draws per market and the fixed buy back value, the initially expected value of each share is 410 Gulden. At the start of each market, subjects are given five shares and 600 Gulden each, plus an interest free loan of 1,500 Gulden, to be repaid after the share buy back. Our generous initial endowment of shares and cash is considerably larger than endowments granted in previous experiments (Smith et al., 1988; Haruvy et al., 2007), in order to provide optimistic subjects sufficient liquidity to submit bids in excess of fundamental value in early periods, thereby stimulating bubble formation.[16] In addition, we do not allow subjects to short sell, in order to provide optimists the best opportunity to increase prices in excess of fundamental value. Table 1 summarizes the parametrization of our experiment.

Each subject received an equal number of shares and cash, which served two purposes.[17] First, the equal initial endowment prevents differential endowment effects from interfering with subject

[16] For evidence on the effect of endowments and the cash to asset ratio on bubble formation, see Caginalp et al. (2001) and Haruvy and Noussair (2006).

[17] Smith et al. (1988) provides subjects with different endowments of shares and cash in order to stimulate trading based on portfolio rebalancing. Subsequent tests (Smith et al., 1993) found that equal endowments among subjects does not reduce bubble formation.

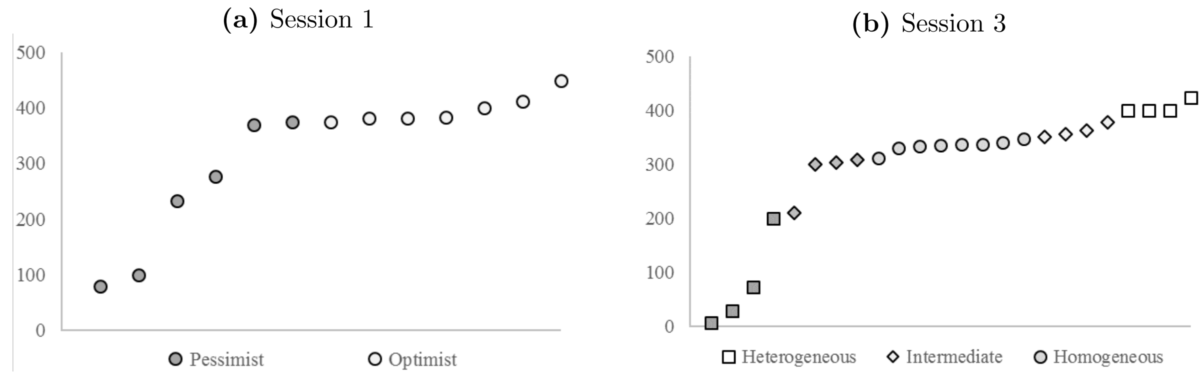
beliefs and trading decisions. Second, the process of grouping subjects based on their beliefs as part of our experimental treatment would result in a different number of shares and cash per market. The distribution of endowments, the dividend payments and buy back value was common knowledge among participants. In addition, participants received a table at the beginning of the experiment containing the expected dividend stream and fundamental value at the beginning of each period. Dividend payments, the buy back value and proceeds from share sales increased cash holdings while purchases of shares decreased them. Throughout the experiment, margin purchases and short sales were unavailable.

The market for the experimental shares operates each period and uses a call market design. In call markets, bids and asks for a period are submitted simultaneously and aggregated into a market demand and supply curve. The *bid* order consists of the maximum proposed purchase price and the maximum quantity of shares the subject seeks to buy. The *ask* order consists of the minimum proposed selling price and the maximum quantity of shares the subject seeks to sell. The clearing price for all transactions of the particular period is determined by the intersect of the demand and supply curve, ensuring the largest number of transactions. We chose a call market design over a continuous double-auction design for two reasons. First, each period in a continuous market has to last for several minutes, which significantly increases the length of each experimental session. Second, asking subjects to predict future market prices may be confusing when there is no single clearing price for each period. While a continuous double auction market provides subjects the possibility to speculate both across and within periods, Cheung and Palan (2012) reports no qualitative difference between market institutions.

4.3 Treatment

Before submitting bid and ask orders in each period, participants are asked to predict market prices of future trading periods. Participants submit their beliefs of the current period t, as well as the next two period prices $t+1$ and $t+2$, into separate fields on their trading interface. We follow Holt et al. (2015), arguing in favor of a limited number of predictions, as this might result in more thoughtful responses and less subject boredom.[18] Subjects are incentivized to submit accurate price expectations by providing small monetary rewards to accurate forecasts. For each forecast that falls within a 10 Gulden interval of the subsequent market price, the subject is awarded 0.10 Euro. Beliefs are elicited during the first 13 periods of trading, resulting in a total of 39 forecasts per subject per market.

Figure 1: Subject grouping into markets



Note: Each point in the graph represents one subject prior to grouping into markets ($t = 0$). On the vertical axis is the average of price expectations for $t = 1$, $t = 2$ and $t = 3$. Marker shape indicates the type of market the subject is grouped into. Square is heterogeneous; Diamond is intermediate; Circle is homogeneous. Darker tones indicate relative pessimism. In session 1, we split our subjects into an optimist and a pessimist market. In sessions 2 and 3, we created two heterogeneous markets and one homogeneous market.

Before the start of the first market round, subjects are grouped into markets based on their initial beliefs. First, we rank all subjects in the session based on the sum of their submitted beliefs of prices for periods $t = 1$, $t = 2$ and $t = 3$. The four participants with the highest price expectations (optimists) are grouped in the market together with the four participants with the lowest price expectations (pessimists). This *heterogeneous market* comprises the most optimistic and most pessimistic subjects within the session, thereby maximizing the dispersion in beliefs. The second (less) heterogeneous market consists of the four participants that rank just below the four most optimist participants, and the four participants ranking above the four participants that are most pessimistic. Third, the *homogeneous market* consists of the remaining eight participants, whose price expectations rank between the participants in the heterogeneous markets. Figure (1) provides a graphical representation of our treatment in session 1 and session 3.

[18] The questionable value of eliciting longer term beliefs is acknowledged by Carle (2016, p. 61), who finds that subjects in the data of Haruvy et al. (2007) base trading decisions primarily on current period beliefs, with long term expectations adding little if any additional predictive power.

Our experimental treatment results in a total of four heterogeneous and four homogeneous first round markets.[19] In addition, the session at the Radboud University resulted in two second round markets, for which the subject grouping of the first round was unchanged. To prevent strategic motivations in submitted beliefs and demand effects, subjects are not informed of the experimental treatment. In addition, the trading environment provides no information about either beliefs or trading decisions made by other subjects.

4.4 Participant payoffs

Participant payments consist of four elements. First, subjects each received a show up fee of 4 Euro. Second, participants received 1 Euro for each 200 Gulden in end-of-round cash. The end-of-round cash balance is the sum of the cash balance after the share buyback and loan repayment after the final period. Third, subjects received 0.10 Euro for each of the 39 submitted forecasts within the price interval. In the session with two market rounds, a random draw determined the round used to calculate the Euro payment for trading and belief accuracy. Finally, subjects received 0.20 Euro for each correct answer in the CRT questionnaire. The average earnings per participant were 20.12 euros. Payments were made in cash and in private at the end of the experiment.

5 Experimental results

Because of the novelty of our experimental treatment, we first reflect on its effectiveness in creating markets that vary in belief dispersion in subsection 5.1.[20] Second, subsection 5.2 analyzes the experimental data by comparing market outcomes across and within treatments.[21] Third, subsection 5.3 considers the relationship between belief dispersion and trade volume and share ownership at the market and period level. Finally, subsection 5.4 tests for alternative explanations for the sug-

[19] Due to a coding error, subjects with similar beliefs were grouped together in the first session, resulting in a homogeneous market with relatively optimistic beliefs and a homogeneous market with relatively pessimistic beliefs instead.

[20] In this section, *homogeneous markets* refer to markets populated by subjects with relatively similar price beliefs at $t = 0$, resulting in a lower dispersion in initial beliefs. On the other hand, *heterogeneous markets* refer to markets populated by subjects with relatively optimistic and subjects with relatively pessimistic beliefs at $t = 0$, resulting in a higher dispersion. In the figures, triangles denote homogeneous markets and squares denote heterogeneous markets.

[21] Since we only conducted second round markets for session 1, our analysis focuses on the eight first round markets.

gested relationship between belief dispersion and bubbles and considers the extent to which subject characteristics determine beliefs.

5.1 Beliefs

For beliefs to have an influence on aggregate market outcomes, we first need to establish a relation between individual beliefs and trading behavior in our experimental data. When subjects trade in line with their beliefs, optimistic subjects will submit higher bids and asks, while pessimistic subjects submit lower bids and asks. Subjects with higher beliefs buy shares from pessimistic subjects and consequently have larger share holdings. Appendix A.1 provides separate panel regressions of share purchases, share holdings, bids and asks of individual subjects for each of the 15 market periods. Based on the positive coefficients of ranked beliefs in each panel regression, we conclude that elicited beliefs are significant predictors of subsequent trading behavior. Subjects with higher relative beliefs submit higher bids and asks, therefore they tend to be net buyers and hold larger share positions than subjects with lower relative beliefs. However, beliefs are only partially able to explain trading behavior.[22] Therefore, our experimental data indicates a noisy relationship, in line with previous findings by Carle (2016).

Next, we reflect on our experimental treatment of creating diverse markets that vary in belief dispersion. First we consider the effect of our experimental treatment on the initial ($t = 1$) belief dispersion. Initial belief dispersion ($BD_{mt=1}$) is measured as the standard deviation of each subject i's forecasts of prices in market m at the beginning of the trading phase:

$$BD_{mt=1} = \sqrt{\sum_{i=1}^{n} \frac{\left(\overline{B}_{mit=1} - \overline{B}_{mt=1}\right)^2}{n-1}} \tag{1}$$

where $\overline{B}_{mit=1}$ is the simple average of price beliefs for periods $t = 1$, $t = 2$ and $t = 3$.[23] A larger measure of $BD_{mt=1}$ indicates that within market m, the average deviation of subject beliefs from the mean belief increases, thus indicating more disperse beliefs of future prices. Since we are interested in the belief dispersion during the entire market round, we want to see whether the initial

[22] When ranked bids and asks would be fully explained by ranked beliefs, regression coefficients are equal to unity.

[23] The results do not change significantly when using a different subset of beliefs. We use the average of elicited beliefs (equation A.3) since this is the measure used to allocate subjects into markets.

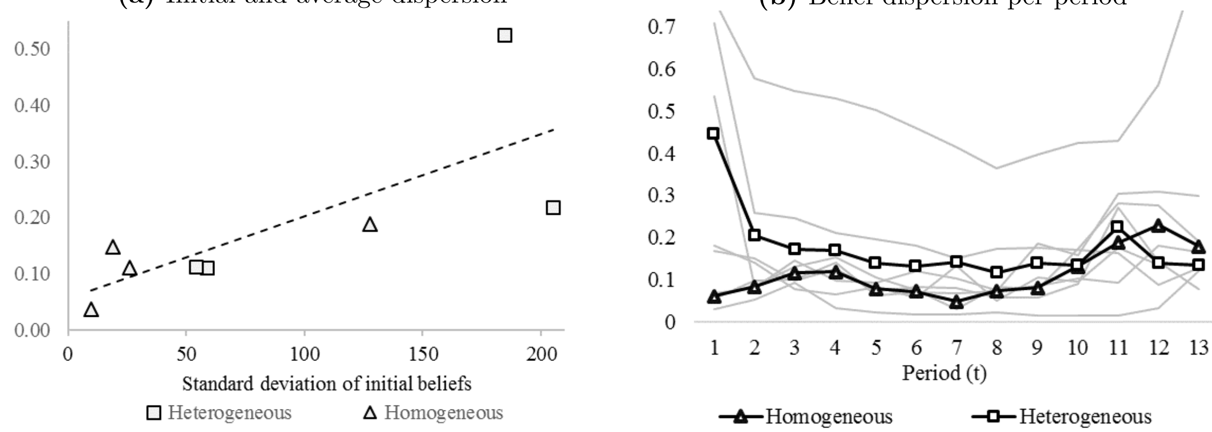
dispersion is maintained during the remaining market periods. Belief dispersion in later periods is measured by the coefficient of variance of average individual subject beliefs.

$$BD_{mt} = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(B_{mit} - \bar{B}_{mt})^2}}{\bar{B}_{mt}} \tag{2}$$

where B_{mit} is the average of a subject i's beliefs elicited at the start of period t. We use the coefficient of variance instead of the standard deviation to correct for the declining fundamental value on the magnitude of the dispersion metric during the later periods.

Figure 2: Belief dispersion per market

(a) Initial and average dispersion **(b)** Belief dispersion per period



Note: The horizontal axis in Figure 2a shows the initial belief dispersion at ($BD_{mt=1}$) and the vertical axis shows average belief dispersion ($\overline{BD}_m$) for all 13 periods in which beliefs are elicited, for each market. The dashed line depicts the relationship between both variables. Figure 2b shows individual market and median belief dispersion (BD_{mt}) for homogeneous and heterogeneous market types in each period t. The black lines with markers are medians of belief dispersion for each market type.

Observation 1: *Belief dispersion in the first period is a strong predictor of the belief dispersion in the remaining periods of the market round.*

Figure 2a shows initial belief dispersion and average belief dispersion for the entire market round. The dispersion in initial beliefs between markets is shown by the distribution on the horizontal axis. Subjects in markets on the right have heterogeneous beliefs, while subjects within markets on the left have homogeneous beliefs. Each market's value on the vertical axis depicts the average belief dispersion for all 13 periods in which beliefs are elicited. The upward sloping dashed trend line indicates that the initial belief dispersion is highly indicative of the average belief dispersion in each market. This is acknowledged by a Pearson correlation test ($\rho = 0.74$), which is significant at the

5% level. In general, we find that the initial belief dispersion is a strong predictor of the average belief dispersion. Extending on this result, Figure 2b shows that the difference in belief dispersion between homogeneous markets and heterogeneous markets extends well beyond the first periods. Up until period 10, the median of heterogeneous markets is higher than the median of homogeneous markets.[24] The stability of belief dispersion within markets hints that initial individual beliefs are good predictors of beliefs for the remainder of the market.

Carle (2016) tested the stability of the belief distribution in de dataset of Haruvy et al. (2007). He computed the average change in the rank of beliefs between periods of subjects within each market. Next, he compared this realization to the theoretical random distribution for the same market, using a Monte Carle simulation. The real markets all have fewer average rank changes than the percentile value of the simulation of 1,000 markets, leading (Carle, 2016, p. 84) to conclude that relative (ranked) beliefs exhibit significant path dependency. In addition, Carle (2016) finds that mean rank changes increase in later market repetitions, as beliefs become increasingly homogeneous. We apply the same method to our dataset, but also calculate rank changes for individual periods.[25] When comparing the observed rank changes to our markets with randomly ranked beliefs, we find that all of our markets fall in the most extreme percentile of the simulated distributions, in line with Carle (2016). Homogeneous markets have a mean rank change per period of 1.17, which is less than the heterogeneous market mean of 1.36, although this result is not significant using a two-sided permutation test ($p = 0.26$).[26] However, we would expect the opposite result, since beliefs in homogeneous markets are less disperse and therefore more likely to cross one another between periods, resulting in a higher rank change instead. Although ranked beliefs in heterogeneous markets appear to be somewhat more volatile than in homogeneous markets, the overall change in beliefs is very low for both market types.

Next, we investigate changes in ranked beliefs at the period and individual subject level. The

[24] The characteristic U-shape in Figure 2b is due to the normalization of the dispersion metric BD_{mt} by the mean belief $\overline{B}_{mt}$ (see equation 2). When prices increase during the bubble phase, the denominator $\overline{B}_{mt}$ rises as well, decreasing the value of our dispersion metric. When prices crash in the final periods, $\overline{B}_{mt}$ increases.

[25] Appendix A.2 describes the ranking procedure and Monte Carlo simulation.

[26] The p-value of the two-sided permutation test is the proportion of possible permutations of bubble metrics between homogeneous and heterogeneous treatments that yield an absolute difference in the mean bubble metric equal to or larger than the difference observed in the data. For information on permutation tests, see Kaiser (2007).

Table 2: Panel analysis of rank change per period

	Rank change (RC_{mt})	
Belief dispersion (BD_{mt})	−0.311**	−0.395**
	(−2.24)	(−2.55)
Relative price (P_{mt}/FV_t)		−0.160*
		(−1.86)
Interaction ($BD_{mt} \cdot P_{mt}/FV_t$)		0.179
		(1.24)
Intercept	1.238***	1.335***
	(20.03)	(16.67)
Regression type	FE	FE
Observations	96	96

Note: Rank changes per period are calculated using equation A.5. Both regressions use standardized independent variables. A Hausman test is used to choose between Fixed-effects (FE) and Random-effects (RE) regression, with a critical of $p \leq 0.10$. Figures in brackets are t-statistics; * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

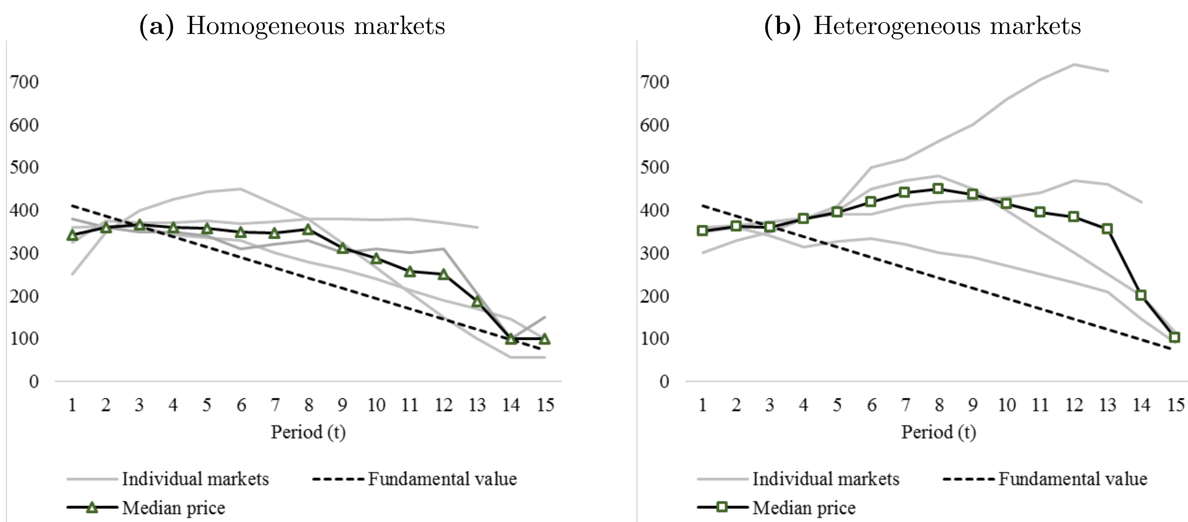
panel regression in Table 2 indicates that belief dispersion (BD_{mt}) is a significant predictor of rank changes for individual periods. The negative coefficient of $\beta_{BD} = -0.311$ shows that the distribution of optimists and pessimists is more stable when the dispersion in beliefs increases. This is in line with our expectations, as more disperse beliefs are less likely to cross one another between periods. In addition, we are interested whether the stability of beliefs varies depending on the degree of overpricing. The second regression includes a measure of relative price, to control for periods in which overpricing was more pronounced. A weakly significant coefficient of $\beta = -0.160$ shows that when prices exceed fundamental value, the distribution of beliefs becomes more stable. Thus, the distribution of beliefs remains relatively stable regardless of price movements away from fundamental value. This result is interesting, for it runs contrary to the theoretical notion by Morris (1996) that investors have more volatile relative beliefs during speculative market phases. Instead, we find that even in markets and periods with the largest overvaluation, the change in relative beliefs remains low. We consider the extent to which initial beliefs can predict beliefs for the remainder of the experimental session. Table A.3 in appendix A.2 provides Pearson correlation coefficients of initial and later period subject beliefs. The correlation coefficient between the rank of initial ($t = 1$) beliefs and the average rank for all 13 periods in which beliefs were elicited of $\rho = 0.39$, significant at the 1% level, indicates that relative beliefs remain relatively stable throughout the experiment. Hence, the extent to which a subject is either an optimist or pessimist can already be anticipated in the first period.

Summarizing, the persistence of the belief distribution in markets and the relative stability of individual beliefs suggest that beliefs exhibit a significant amount of path dependency. Despite often volatile price developments, relative beliefs remain stable, with initial optimists likely to stay optimistic and pessimists likely to stay pessimistic. These results are highly relevant to the significance of our treatment, as they implicate that the initial manipulation in the distribution of beliefs has a lasting effects on our experimental markets.

5.2 Price Bubbles

We now represent the overall patterns in market prices for each experimental treatment. Figure 3a includes the markets with relatively *homogeneous* initial beliefs, while figure 3b represents markets with *heterogeneous* initial beliefs. The line with the markers is the median price of individual markets for each treatment.[27] The dashed line represents the declining fundamental value.

Figure 3: Market prices by initial belief dispersion



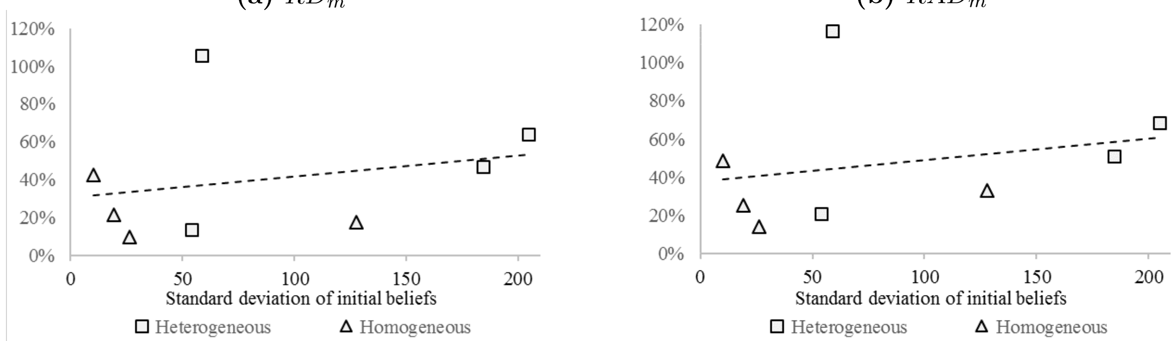
In line with the typical result in Smith et al. (1988) markets, prices start below fundamental value, increase and intersect the fundamental value to reach a peak, before crashing down to fundamental value in the final periods. When comparing the homogeneous and heterogeneous

[27] We depict *median* prices instead of *mean* prices in order to limit the effect of individual markets on treatment market averages. When using mean prices, the difference between treatments becomes more pronounced.

treatments, the figures suggest larger price bubbles for heterogeneous markets. For both treatments, early period prices start just below fundamental value and intersect fundamental value at the third period. However, whereas prices in homogeneous markets generally progress horizontally, prices in heterogeneous markets tend to increase upon crossing fundamental value. Price bubbles seem more persistent in heterogeneous markets, with price peaks occurring later on in the market. In heterogeneous markets the maximum price was 740, whereas the maximum price was 450 in homogeneous markets.

Figure 4: Initial belief dispersion and bubble measures

(a) RD_m **(b)** RAD_m



Note: These graphs depict overvaluation (4a) and absolute mispricing (4b), with each point representing a single market ($t = 0$). On the vertical axis is the initial belief dispersion ($BD_{mt=1}$). The shape of the marker indicates the market treatment. The fitted line plots the relationship between both variables.

The magnitude of bubbles is measured using two of the most widely used metrics within the literature on experimental asset markets.[28] Relative Deviation (RD_m) is calculated as the average of per-period prices deviations from the fundamental value, normalized by fundamental value.[29]

$$RD_m = \frac{1}{15} \sum_{t=1}^{15} \frac{(P_{m,t} - FV_t)}{\overline{FV}_m} \tag{3}$$

A relative deviation of $RD_m = 0.25$ indicates that on average, the price per period is 25% higher than fundamental value. Negative values of RD_m indicate underpricing, whereas positive values indicate overpricing. Relative Absolute Deviation (RAD_m) measures the average absolute level of

[28] Appendix A.3 provides tests of additional bubble measures, which show similar results to RD_m and RAD_m. For an extensive discussion of all bubble measures used in this study, see Stöckl et al. (2010).

[29] Note that $\overline{FV}_m$ is the average fundamental value of periods in which trade occurred, which may vary between markets. Without this correction, markets with no-trade periods would have significantly smaller values.

price deviations relative to the average fundamental value.

$$RAD_m = \frac{1}{15}\sum_{t=1}^{15}\frac{|P_{m,t} - FV_t|}{\overline{FV}_m} \tag{4}$$

A relative absolute deviation of $RAD_m = 0.20$ indicates that on average, the price per period differs 20% from the fundamental value. Larger values of RAD_m indicate that asset prices are further away from fundamental value, which can be the result of both underpricing as well as overpricing. Together, these two measures indicate both the direction (RD_m) and magnitude (RAD_m) of price bubbles. In the following, we use both metrics to discuss the differences across treatments at the market level. As our first hypothesis suggests more pronounced overpricing, we will focus on the former metric.

Observation 2: *Markets with heterogeneous initial beliefs generally produce larger bubbles than markets with homogeneous initial beliefs.*

Table 3: Observed values of bubble metrics

Treatment	Market	Belief dispersion ($BD_{mt=1}$)	Bubble metric (RD_m)	(RAD_m)
Homogeneous	1	26.31	0.10	0.14
	2	127.84	0.18	0.33
	3	19.17	0.22	0.25
	4	9.96	0.43	0.49
	Mean (1-4)	45.82	0.23	0.30
Heterogeneous	5	204.77	0.64	0.69
	6	58.72	1.06	1.17
	7	184.51	0.47	0.51
	8	53.96	0.14	0.21
	Mean (5-8)	125.49	0.58	0.64
Mann-Whitney U rank test		0.08*	0.15	0.15
Fisher-Pitman permutation test		0.17	0.14	0.17

Note: This table reports individual market and averages of observed bubble measures by treatment. Markets 1-4 have homogeneous initial beliefs and markets 5-8 have heterogeneous initial beliefs. The third column denotes the (initial) belief dispersion ($BD_{mt=1}$) at $t = 0$. The bottom rows report p-values of two-sided Mann-Whitney U tests and Fisher-Pitman permutation tests comparing homogeneous (1-4) and heterogeneous markets (5-8). * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Our first hypothesis predicts a positive relationship between belief dispersion and overpricing. Despite the limited number of data points, Figure 4 suggests that bubbles tend to increase with the initial dispersion in beliefs, in line with our first hypothesis. Based on treatment means of RD_m and RAD_m in Table 3, heterogeneous markets produce larger bubbles than homogeneous markets. Since markets generally tend to overprice during most periods, there is little difference between our two bubble measures. While homogeneous markets are overpriced by 23% on average, heterogeneous markets are overpriced by 58%. A two-sided rank tests produces the same p-value of 0.15 for either metric, which is insignificant at conventional levels.[30] We also perform a stratified permutation test, which is sensitive to number instead of rank. The two-sided test yields p-values of 0.14 and 0.17 respectively, with the 4 homogeneous markets in the first strata and the 4 heterogeneous markets in the second strata.

Despite the lack of statistical significance, our data indicates a positive relation between belief dispersion and bubble formation. When the disagreement between optimist and pessimist subjects is more pronounced, price deviations from fundamental value tend to be larger. Our results are in line with previous experimental findings by Carle (2016, p. 69), who used regression analysis to prove that bubbles can already be predicted by the distribution of beliefs in the first period. Both of these results lend credence to the theoretical prediction that an increase in belief dispersion leads to higher prices (Miller, 1977; Harrison and Kreps, 1978; Morris, 1996).

5.3 Trade volume and share concentration

A large body of literature argues that trade volume increases with the heterogeneity of investor beliefs (Varian, 1985; Harris and Raviv, 1993; Hong and Stein, 2007). We evaluate this hypothesis by first comparing the average trade volume of heterogeneous markets with the trade volume of homogeneous markets. Trade volume (V_m) is measured as the mean number of trades per period V_{mt}, normalized by total shares Q_m to account for the varying number of subjects per market.

$$V_m = \frac{1}{Q_m} \sum_{t=1}^{15} (V_{mt}) \tag{5}$$

The fourth column of Table 4 shows trade volumes for individual markets and treatment averages. All values of (V_m) are between 1.15 and 2.23, which means that on average, each share in our markets is traded about two times during the market round. Based on treatments means, heterogeneous markets do not have higher levels of trade than homogeneous markets. While market 3 and market

[30] We report two-sided tests due to the novelty of our experimental treatment and the lack of comparable research. Using one-sided rank and permutation tests would lead us to conclude that heterogeneous markets produce significantly larger bubbles at the 10% level.

4 have the most homogeneous initial beliefs in our data, these also have the highest trade volumes. While the difference is insignificant for both the rank and permutation tests, the average total turnover per share (V_m) is lower for the heterogeneous markets (1.64) than for the homogeneous markets (1.88).

Table 4: Trade volume and share concentration

Treatment	Market	Belief dispersion ($BD_{mt=1}$)	Trade volume (V_m)	Share concentration (SD_m)	Share concentration (SH_m)
Homogeneous	1	26.31	1.80	4.08	19.00
	2	127.84	1.30	3.19	16.40
	3	19.17	2.20	6.19	24.87
	4	9.96	2.23	4.51	21.47
	Mean (1-4)	45.82	1.88	4.49	20.43
Heterogeneous	5	204.77	2.13	4.24	21.73
	6	58.72	1.93	3.55	19.13
	7	184.51	1.15	3.27	18.20
	8	53.96	1.38	3.83	20.47
	Mean (5-8)	125.49	1.64	3.72	19.88
Mann-Whitney U rank test			0.39	0.39	1.00
Fisher-Pitman permutation test			0.46	0.34	0.83

Note: This table reports individual market and averages of observed trade volumes and share concentrations by treatment. Markets 1-4 have homogeneous initial beliefs and markets 5-8 have heterogeneous initial beliefs. The third column denotes the initial belief dispersion ($BD_{mt=1}$). The bottom rows report p-values of two-sided Mann-Whitney U tests and Fisher-Pitman permutation tests comparing homogeneous (1-4) and heterogeneous markets (5-8). * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Observation 3: *Heterogeneous beliefs are not associated with larger trade volumes. This holds for both initial belief dispersion and trade volume at the market level, as well as belief dispersion and trade volume for individual periods.*

Next, we consider whether belief dispersion and trade volume are related on the period level (V_{mt}). The leftmost two columns in Table 5 show panel regressions of trade volume on belief dispersion for each period of all markets. None of the regression coefficients is significant, which suggests no significant relationship relation between belief dispersion and transaction volume. The same result is found when correcting for trade volume in the previous period (V_{mt-1}). These findings are similar to Carle (2016), who used the same metrics of belief dispersion and trade volume when analyzing the dataset by Haruvy et al. (2007). Thus, our results confirm findings that cast doubt on the theoretical prediction that a dispersion in belief among investors encourages trade.

Table 5: Period analysis of trade volume and share concentration

	Trade volume		Share concentration			
	(V_{mt})		(SD_{mt})		(SH_{mt})	
Belief dispersion (BD_{mt})	−0.106	−0.080	−0.012	0.082	−0.007	0.004
	(−0.89)	(−0.41)	(−0.11)	(0.80)	(−0.06)	(0.45)
Previous period (y_{t-1})		−0.137		0.673***		0.989***
		(−1.32)		(10.37)		(41.34)
Interaction ($BD_{mt} \cdot y_{t-1}$)		−0.106		0.117		-0.003
		(−0.82)		(1.54)		(-0.27)
Intercept	0.068	0.045	−0.142	0.018	−0.134	0.075
	(0.46)	(0.45)	(−0.65)	(0.40)	(−0.65)	(10.58)***
Regression type	RE	FE	RE	FE	RE	RE
Observations	104	96	104	96	104	96

Note: This table includes panel regressions of trade volume and share dispersion on belief dispersion and lagged variables. All variables are standardized. A Hausman test is used to choose between Fixed-effects (FE) and Random-effects (RE) regression, with a critical value of $p \leq 0.10$. Figures in brackets are t-statistics; * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

According to theoretical work by Miller (1977); Harrison and Kreps (1978), share ownership becomes increasingly concentrated when the dispersion in beliefs increases, as shares are increasingly held by the relatively optimistic investors. We test this hypothesis by comparing the average share concentration between our market treatments. Similar to Füllbrunn and Neugebauer (2012), the concentration of share ownership is measured using two metrics. Share dispersion (SD_m) is calculated as the average standard deviation of subject share positions:

$$SD_m = \frac{1}{15} \sum_{t=1}^{15} \sqrt{\sum_{i=1}^{n} \frac{(S_{mit} - \overline{S}_{mt})^2}{n-1}} \tag{6}$$

where S_{mit} is the number of shares held by subject i in market m at time t.[31] Share holdings (SH_m), is defined as the average number of shares held by the two subjects with the highest number of shares in their portfolio in each period t in market m. Values for both metrics of share concentration are found in the last two columns of Table 4.

Observation 4: *Heterogeneous beliefs are not associated with increased share concentration. This holds for both initial belief dispersion at the market level, as well as belief dispersion and share concentration for individual periods.*

[31] As was described in Section 4, each subject received five shares at the start of the experiment, so the average number of shares $\overline{S}_{mt} = 5$.

Contrary to our third hypothesis, the average share concentration in homogeneous markets is higher ($SD_m = 4.49$) than in heterogeneous markets ($SD_m = 3.72$). For our second measure (SH_m), we find a similar result, but the variation between individual market is smaller. Based on two-sided Mann-Whitney U tests, we are unable to reject the null hypothesis that share concentration is different between treatments. The stratified permutation tests yield similar results, with p-values far from significant levels for both share dispersion and share holdings.

The rightmost four columns in Table 5 show panel regressions of share concentration in each period, with each second test controlling for lagged share concentration. Because of the low levels of trade volume in our experimental markets, the distribution of shares does not change a lot between periods. Therefore, we would expect the share concentration in $t-1$ to be a strong predictor of the share concentration in t. Indeed, both lagged share dispersion (SD_{mt-1}) and lagged share holdings (SH_{mt-1}) have coefficients close to unity and are highly significant. While some of the coefficients for belief dispersion (BD_{mt}) have the predicted positive sign, none are even weakly significant. The same holds for the interaction variables of belief dispersion and the lagged ($t-1$) dependent variable, both of which are insignificant. Our results do not support the hypothesis that belief dispersion increases with share ownership. Instead, low trade volumes prevented optimistic investors from acquiring large stakes.

5.4 Alternative explanations

Section 5.2 suggests markets with a heterogeneous (initial) belief compositions yield larger bubbles than markets with homogeneous beliefs. This section tests whether subject characteristics have a confound effect on this relationship. Heterogeneous markets are created by grouping specific subjects based on their elicited (ranked) beliefs. By deliberately grouping subjects into markets instead of using randomization, our treatment could implicitly have selected subjects that are prone to speculative behavior (*hidden sorting*). Since subjects are grouped by the relative optimism of their initial beliefs, we test whether initial beliefs are correlated to characteristics that are known to influence bubble formation in experimental literature.[32]

[32] We report the average of risk tolerance measures instead of the individual measures. The significance of regression coefficients tends to decrease when using the individual measures of risk tolerance.

Table 6: Market characteristics

Treatment	Market	Initial beliefs ($\bar{B}_{it=1}$)	($\lvert\bar{B}_{it=1} - FV_{t=1}\rvert$)	Cognition (right)	(intuitive)	Risk tolerance	Gender (female)
Homogeneous	1	398.10	18.00	4.57	0.86	35.86	0.14
	2	239.17	146.83	3.50	2.83	25.33	0.83
	3	358.81	27.19	3.43	2.14	33.00	0.43
	4	333.96	52.04	4.13	1.25	32.50	0.63
	Mean (1-4)	332.51	61.02	3.91	1.77	31.67	0.51
Heterogeneous	5	288.50	166.67	3.75	1.75	32.25	0.63
	6	325.63	60.38	3.13	1.75	41.63	0.50
	7	241.67	164.17	3.13	2.25	30.38	0.63
	8	321.17	64.83	3.50	2.25	33.13	0.63
	Mean (5-8)	294.24	114.01	3.38	2.00	34.34	0.59
Mann-Whitney U rank test		0.25	0.08*	0.19	0.56	0.77	0.64
Fisher-Pitman permutation test		0.37	0.17	0.17	0.66	0.51	0.71

Note: This table reports individual market and averages of subject characteristics measures by treatment. Markets 1-4 have homogeneous initial beliefs and markets 5-8 have heterogeneous initial beliefs. The bottom rows report p-values of two-sided Mann-Whitney U tests and Fisher-Pitman permutation tests comparing homogeneous (1-4) and heterogeneous markets (5-8). * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Table 8 reports OLS regressions of initial subject beliefs ($\overline{B}_{it=1}$), the variable used to allocate subjects to markets, on several subject characteristics. The leftmost regression column shows the effect of subject characteristics on the (linear) measure of initial beliefs. This regression indicates to what extend optimism in beliefs can be attributed to subject characteristics. The second column specifies beliefs as their absolute deviation from the fundamental value, indicating what causes subjects to have beliefs that deviate from fundamental value. The third column uses the normalized rank of initial beliefs within each session.[33] Finally, the independent variable in the fourth column is the absolute deviation from the centered rank of initial beliefs. Since our treatment involves the allocation of relatively pessimistic and relatively optimistic subjects to heterogeneous markets, positive coefficients are indicative of hidden sorting.

5.4.1 Cognition

Available literature (Frederick, 2005; Bosch-Rosa et al., 2015) has found that cognitive sophistication of subjects may explain the occurrence of price bubbles. Bosch-Rosa et al. (2015) tests whether

[33] To correct for the different size of sessions and markets, we divide ranked variables by the number of subjects within their respective group.

the occurrence of bubbles depends on subjects' cognitive sophistication. First, they use several tests to assess subjects' cognitive sophistication. Next, they populate markets with subjects with either low or high cognitive sophistication. They find that the low sophistication markets produce large bubbles, while the high sophistication markets produce no bubbles at all.[34] In Table 6, Cognition (right) indicates the market average of CRT questions answered correctly, and indicates increased cognitive sophistication. Cognition (intuitive) denotes the number of questions answered intuitively, which indicates less cognitive sophistication. Treatment averages for heterogeneous markets indicate a slightly lower cognitive sophistication than homogeneous markets. This is true for both the number of questions answered right, as well as the number of intuitive questions. Rank tests and permutation tests do report a significant difference at conventional p-values, however. Markets 5 and 6 produce the largest price bubbles, but do not have a lower average cognitive sophistications than other markets.

The regression analyses in Table 8 report no relationship between cognitive sophistication and initial subject beliefs. In each of the four regressions, the coefficients for either metric of cognitive sophistication are insignificant. The lack of a (linear) relationship between beliefs and cognitive sophistication implies that our experimental treatment did not implicitly select subjects on cognition, which is confirmed by a two-sided t-test ($p = 0.29$). Markets are not significantly different in average cognitive sophistication, nor do they vary in the distribution of cognitive sophistication. Summarizing, we find no evidence that cognitive differences between subjects are responsible for the larger bubbles in heterogeneous belief markets.

5.4.2 Risk aversion

Our experimental asset pays a stochastic dividend at the end of each period. Because risk averse subjects have lower valuations of the risky asset than risk neutral subjects, prices are lower in markets populated by risk averse subjects (Smith et al., 1993). Additionally, risk aversion may be related to the tendency to engage in speculative trading. In line with this theoretical conjecture,

[34] Hanaki et al. (2015) uses a similar methodology to Bosch-Rosa et al. (2015), but also informed subjects of the dispersion in cognitive sophistication within the market. They find that markets with heterogeneous cognitive sophistication produce larger bubbles, which they attribute to increased speculative tendencies when subjects are aware of the market heterogeneity. In our experimental setup, subjects were unaware of characteristics specific to their market, in order to prevent the perception of heterogeneity from influencing behavior.

Table 7: Correlation matrix initial beliefs and subject characteristics

Variable	1	2	3	4	5	6
1. Initial belief ($\overline{B}_{it=1}$)	1.00					
2. Abs. deviation ($\lvert\overline{B}_{it=1} - \overline{FV}_{t=1}\rvert$)	-0.80***	1.00				
3. CRT (correct answers)	0.11	-0.15	1.00			
4. CRT (intuitive answers)	-0.11	0.23*	-0.70***	1.00		
5. Risk tolerance	0.50***	0.41***	-0.18	0.07	1.00	
6. Gender (female = 1)	-0.22*	-0.15	-0.08	-0.03	-0.47***	1.00

Note: This table includes Pearson correlation coefficients of initial beliefs and characteristics of $N = 60$ individual subjects. Variable 2 is the absolute difference between initial beliefs and fundamental value. Variables 3 to 6 are subject characteristics. The bottom row shows p-values of Student's t-tests between male and female subjects. * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Eckel and Füllbrunn (2015); Breaban and Noussair (2015) find that risk averse subjects tend to produce smaller bubbles in Smith et al. (1988) markets. The second column from the right in Table 6 shows market averages of subject's risk tolerance.[35] The average risk tolerance of subjects within markets ranges from 25.33 for market 2, to 41.63 for market 6. Homogeneous markets' average risk tolerance is 31.67, whereas heterogeneous markets with an average of 34.34 are slightly more risk tolerant. Because of the large variation within experimental treatments, the differences in average market risk aversion are insignificant using both non-parametric rank and permutation tests.

While there are no significant differences in average market risk aversion, we do find a large variation in risk aversion within our entire subject sample. Figure 5b plots individual subjects on risk tolerance and initial beliefs, indicating that risk averse subjects tend to have more pessimistic beliefs. This is acknowledged by the significant coefficient ($\beta = 6.652$) of the first regression in Table 8. By adding risk aversion to the first regression analysis, the explanatory power of our model (R^2) increases from 12% to 32%, indicating that risk aversion has a significant positive relationship with initial beliefs. This means that our heterogeneous markets are therefore populated by subjects with relatively heterogeneous risk attitudes. Consequently, the relationship between belief dispersion and bubbles could thus be partially explained by a dispersion in risk tolerances instead.

[35] We measured subjects' risk aversion by asking about their willingness to take risks in six specific areas. Risk aversion can therefore be understood as the inverse of our risk tolerance measure. We report the average of risk tolerance measures instead of the individual measures. The significance of regression coefficients tends to decrease when using individual measures of risk tolerance.

Table 8: Initial beliefs and subject characteristics

	Initial beliefs		$rank$[Initial beliefs]	
	$\overline{B}_{it=1}$	$\lvert\overline{B}_{it=1} - \overline{FV}_{t=1}\rvert$	$\overline{B}_{it=1}$	$\lvert\overline{B}_{it=1} - \overline{B}_{mt=1}\rvert$
CRT (correct answers)	8.121	−4.264	0.179	0.007
	(0.83)	(−0.46)	(0.69)	(0.05)
CRT (intuitive answers)	−0.750	12.781	−0.203	0.167
	(−0.06)	(1.12)	(−0.64)	(0.94)
Risk tolerance	6.652***	−5.476***	0.140***	−0.012
	(3.98)	(−3.47)	(3.17)	(−0.48)
Gender (female = 1)	14.841	−24.094	0.401	0.078
	(0.48)	(−0.82)	(0.49)	(0.17)
Study (economics = 1)	43.524	−22.113	0.669	−0.310
	(1.47)	(−0.79)	(0.86)	(−0.72)
Location (Radboud University = 1)	45.820	−35.354	0.558	−0.133
	(1.33)	(−1.09)	(0.62)	(−0.26)
Intercept	24.029	293.582***	−5.598**	2.693*
	(0.26)	(3.33)	(−2.28)	(1.97)
Observations	60	60	60	60
Adj. R^2	0.253	0.188	0.154	−0.049

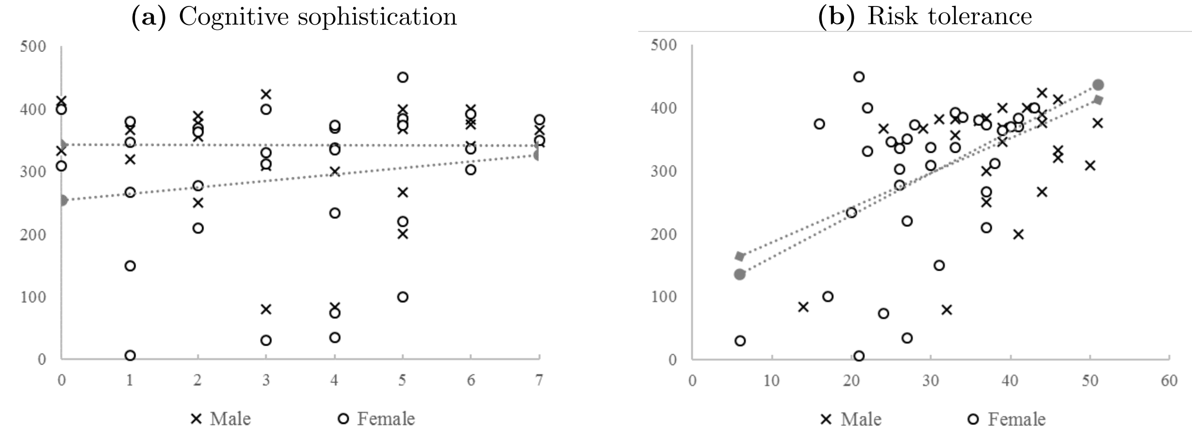
Note: OLS regression of initial beliefs on subject characteristics. We use four independent variables, from left to right: Initial belief ($\overline{B}_{it=1}$); Absolute deviation from fundamental value ($|\overline{B}_{it=1} - \overline{FV}_{t=1}|$); Ranked initial belief ($rank[\overline{B}_{it=1}]$); Ranked absolute deviation from mean rank ($|rank[\overline{B}_{it=1}] - rank[\overline{B}_{mt=1}]|$). Figures in brackets are t-statistics; * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

5.4.3 Gender

A growing number of studies argue that male and female traders behave differently in financial markets. Experimental evidence reveals that woman tend to be more risk averse than men and therefore less willing to take risks (Byrnes et al., 1999). This suggests that woman will tend to avoid engaging in aggressive competition and speculative strategies. The experimental evidence on the relationship between gender and mispricing is mixed. Eckel and Füllbrunn (2015) reports that markets populated by higher numbers of woman produce smaller price bubbles than those populated mainly by men. On the other hand, Holt et al. (2015) finds no differences between male and female markets. We first analyze whether our experimental treatment resulted in hidden sorting on gender. The final column of Table 6 depicts the fraction of female subjects in each market. On average, our subject sample had slightly more female participants than male participants. While differences between treatments are small and insignificant due to the limited size of our dataset, we find a large variation in market composition within treatments. Whereas market 1 contains just a single female subject out of a total six subjects, market 2 contains all but one female subjects. Within this market, our experimental treatment of separating optimists and pessimists resulted in a market populated primarily by female subjects. These findings are in line with Holt et al. (2015, p. 28), who found gender differences in terms of individual measures, but not on bubble amplitude. Indeed, markets with highly unbalanced gender populations produced one of the smallest bubbles in our dataset.

Figure 5: Cognitive sophistication and risk tolerance

(a) Cognitive sophistication **(b)** Risk tolerance



Note: These graphs depict cognitive sophistication (5a) and risk tolerance (5b), with each point representing a male (cross) or female (circle) subject. Cognitive sophistication is measured as the number of right answers on the CRT test. Risk tolerance is calculated as the equally weighted sum of the six specific risk measures in the questionnaire. On the vertical axis are initial beliefs ($B_{it=1}$), the treatment variable in our experimental setup. The fitted lines plot the linear relationship between both variables for each gender.

While evidence on the relationship between bubbles and gender is mixed, experimental literature generally agrees that woman have more pessimistic beliefs than men (Eckel and Füllbrunn, 2015; Holt et al., 2015) A one-sided t-test ($p = 0.04$) confirms that men tend to have more optimistic initial beliefs than woman. However, Table 8 shows this result disappears when controlling for risk aversion.[36] We find that female beliefs are not more pessimistic than men, when controlling for risk aversion. The grouping of circles in Figure 5b shows that female subjects (circles) tend to have a lower risk tolerance, which is confirmed by a one-sided t-test ($p = 0.00$).

Based on gender compositions between markets, we find our experimental treatment did not

[36] The addition of an (insignificant) interaction term of risk tolerance and gender neither changes the result, nor increases the explanatory power (adj. R^2) of our model.

implicitly allocate subjects based on gender. Likewise, we find no (direct) relationship between gender composition and bubble formation within our dataset. In line with previous research, female subjects are significantly more risk averse than male subjects. Summarizing, our results do not indicate a confounding effect of gender on the relationship between belief dispersion and bubble formation.

6 Discussion and Conclusion

Economic and financial literature has long recognized the relevance of beliefs in decision making. When beliefs are disperse, theoretical literature predicts overvaluation (Miller, 1977; Morris, 1996), increased trade volume (Varian, 1985), and more concentrated ownership (Harrison and Kreps, 1978). This paper reports on a novel experimental treatment designed to directly vary belief dispersion in the experimental asset market design of Smith et al. (1988). Our treatment involved the allocation of subjects based on their initial beliefs in order to create markets that vary in belief dispersion. Although subjects within markets witness the same price development, initial optimists are likely to remain optimistic, while initial pessimists are likely to remain pessimistic. Moreover, we find that the experimentally manipulated belief dispersion remains relatively stable within these markets, with beliefs showing a significant degree of path dependency. These findings indicate that our treatment has a lasting effect on the belief distribution throughout the experimental session. Therefore, we argue that this treatment could be used to test the relation between belief dispersion and market outcomes.

In line with the overvaluation hypothesis, we find that markets populated by subjects with heterogeneous beliefs exhibit larger price bubbles than markets with homogeneous beliefs. Thus, bubbles are more pronounced when the dispersion in beliefs increases. At the same time, we find no evidence that belief dispersion is related to either trade volume or share concentration. This research provides an incomplete test of the Miller (1977) hypothesis, as all of our markets have complete constraints on short sales. Previous literature either finds prices under short selling constraints to deflate (Haruvy and Noussair, 2006), while Smith et al. (1993) reports no effect on price levels. It would be interesting to extent our experimental treatment to markets with and without short sale constraints in a 2x2 factorial design.

Recent studies argue that the typical bubble result in Smith et al. (1988) asset markets is influenced by specific subject and market characteristics. We tested whether our experimental treatment explicitly selected subjects based on characteristics that have been identified to influence bubbles in experimental asset markets. Heterogeneous markets are populated by somewhat less cognitively sophisticated and more risk seeking subjects, although differences are statistically insignificant at conventional levels. When considering individual subjects, we find a strong and negative relationship between risk aversion and initial beliefs. While treatment markets are similar in average risk aversion, heterogeneous belief markets also have more heterogeneous risk attitudes. Therefore, the suggested relationship between belief dispersion and price bubbles may be caused by a confounding effect of heterogeneous risk aversion within markets. Indeed, recent research (Breaban and Noussair, 2015) shows that trading strategies are related to risk attitudes, with risk averse agents less likely to base trade on past trends (momentum) and loss averse agents less likely to speculate. These findings hint that within our heterogeneous markets, subjects follow different trading strategies. Moreover, additional research into the relationships between subject characteristics and beliefs is warranted. We argue that future applications of subject selection treatments should include thorough tests of implicit selection on characteristics related to speculative behavior (e.g. Janssen et al. (2015)).

This study faces several important shortcomings. Due to the low number of experimental sessions and subjects, statistical tests tend to produce only weak results. Our initial setup involved the repetition of the first market round using the same cohort of subjects. Because of time constraints, this second round was not conducted during session 2 and session 3. Therefore, we were unable to investigate how the distribution of beliefs develops in subsequent rounds. The lack of available subjects resulted in a rather crude method of ensuring a variety in belief dispersions between markets. Additional research should ensure an adequate number of students, while changing the experiment design into a single market round, to allow the researcher to design markets with a specific belief distribution. The results of this study show that both beliefs and their distribution tend to remain relatively stable within the Smith et al. (1988) design, but provide little insight into the formation of beliefs themselves. Future research should therefore aim to better understand the intricate relationships between subject characteristics, individual belief formation, trading behavior and the aggregate macro environment they create.

Theoretical literature has increasingly incorporated heterogeneous beliefs into asset pricing models, challenging the mainstream assumption of rational expectations in favor of behavioral views. The typical result of these models is that when allowing heterogeneous expectations, price dynamics become unstable and unpredictable (De Long et al., 1990; Brock and Hommes, 1998; Hommes, 2011). Increasingly, laboratory experiments are used to test these new models of beliefs and decision making. One of the most advanced of these research programs is the heterogeneous expectations hypothesis (Heemeijer et al., 2009; Hommes, 2011), which aims to find a more plausible theory of heterogeneous expectations. Using controlled laboratory experiments, Hommes (2011) studies the relationship between individual expectations and aggregate outcomes. They argue for a heuristic switching model, in which agents use a different set of forecasting rules based upon their relative performance in predicting future prices. These forecasting rules include relatively simple heuristics based on adaptive expectations and learning. Allowing for multiple heuristics enables these models to better explain the different outcomes across and within market settings, as is customary in much of the bubble literature. At this point, the extent to which beliefs can be used to explain outcomes at the macro level remains to be seen.

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A.1 Beliefs and trading behavior

This appendix comprises tests of the alignment between individual price expectations and subsequent trading behavior. A number of variables are constructed from the elicited beliefs in the experiment. We separate the elicited price expectations into *short-term* and *long-term* beliefs. The short-term belief is defined as the price expectation of subject i in market m for period t:

$$STB_{mit} = B^{t}_{mit} \tag{A.1}$$

The short term belief is elicited in the same period t as the realization of the forecast. Long term belief elicited is defined as the average of price expectations for period $t+1$ and period $t+2$:

$$LTB_{mit} = \frac{1}{2}\left(B^{t+1}_{mit} + B^{t+2}_{mit}\right) \tag{A.2}$$

We differentiate between short term beliefs (STB_{mit}) for the same period and long term beliefs for the next two periods (LTB_{mit}). Our call market design allows subjects to only trade once per period. Therefore, subjects are unable to speculate on the increasing price of shares within periods. Therefore, long term beliefs contains information about about speculative tendencies, while short term beliefs can show trade intention. Finally, we define average beliefs as the average of both short term and long term beliefs:

$$\overline{B}_{mit} = \frac{1}{3}\left(B^{t}_{mit} + B^{t+1}_{mit} + B^{t+2}_{mit}\right) \tag{A.3}$$

Similar to Carle (2016), we use ranked beliefs and ranked trading metrics in the regression analysis. For example, ranked short term beliefs B^{t}_{mit} in period t are constructed by ranking participants by their expectations of the market price for period t. Participants are ranked from highest to lowest, with the lowest price expectation assigned $rank[STB_{mit}] = 1$, the second lowest $rank[STB_{mit}] = 2$, etc. Due to the varying size of our markets, the subject with the highest submitted price expectations ranges from 6 to 8. To correct for the different size of markets, we divide ranked variables by the number of subjects i within the respective group. Using this ranking higher ranks indicate a higher level of (relative) optimism, while lower ranks indicate less optimism about current period market prices. The ranking of beliefs serves several purposes. First, it eliminates the problem of normalizing beliefs to the decline of fundamental value, as the range of ranks within each market is constant. Second, the procedure is unbiased towards extreme outliers. The use of ranked metrics does come at a cost, as no inferences can be made about the magnitude of regression coefficients.

A.1.1 Net purchases and share holdings

Table A.1: Panel analysis of net purchases and share holdings

	Net purchases (ΔS_{mit})			Share holdings (S_{mit})		
	Short	Long	All	Short	Long	All
$rank[STB_{mit}]$	0.133***		0.051	0.259***		0.286***
	(3.57)		(1.04)	(4.85)		(4.45)
$rank[LTB_{mit}]$		0.226***	0.128***		0.125**	−0.053
		(4.20)	(2.62)		(2.07)	(−0.74)
Intercept	−0.570***	−0.966***	−0.764***	3.890***	4.464***	4.005***
	(−3.17)	(−3.95)	(−3.95)	(9.18)	(10.24)	(8.93)
Regression type	RE	FE	RE	RE	FE	RE
Observations	780	780	780	780	780	780

Note: A Hausman test is used to choose between Fixed-effects (FE) and Random-effects (RE) regression, with a critical of $p \leq 0.10$. Figures in brackets are t-statistics; * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Observation 5: *Long term beliefs are significant predictors of net share purchases, while only short term beliefs are significant predictors of share holdings.*

Table A.1 reports panel regressions of net purchases and share holdings on ranked short and long term beliefs. Net purchases is the change in the number of shares a subject has in his portfolio (ΔS_{mit}) from period $t-1$ to t. The positive coefficients of STB_{mit} and LTB_{mit} indicate that relative beliefs are a significant predictors of net purchases. Coefficients can be interpreted as the expected increase in net purchases when a subject would have held the higher beliefs of the more positive subject within the market instead. The insignificant coefficient in the third column (All) indicates that only long term beliefs are relevant when making purchasing (selling) decisions. We find this result in line with strategic motivations of subjects to profit from price increases, for which at least two additional trading sequences are necessary at elicitation.

Next, share holdings (S_{mit}) measures the total amount of shares held by each subject. When interpreting the size and significance of the coefficients in our panel regressions, we find that beliefs are weak predictors of trading behavior. A coefficient of 0.286 tells us that the expected increase in the number of shares held when the most pessimistic subject would become the most optimist subject instead is roughly two. Considering that subjects hold five shares on average, the effect of beliefs seems minor. According to our test results (final column) share holdings is predicted by short term beliefs but not long term beliefs. However, we find that this result is due to multicollinearity between (ranked) short and long term beliefs.

A.1.2 Bid and ask orders

Table A.2: Panel analysis of bids and asks

	$rank[Bid]$			$rank[Ask]$		
	Short	Long	All	Short	Long	All
$rank[STB_{mit}]$	0.259***		0.184***	0.202***		0.138***
	(7.33)		(4.41)	(6.04)		(3.29)
$rank[LTB_{mit}]$		0.259***	0.148***		0.262***	0.182***
		(6.63)	(3.22)		(6.36)	(3.83)
Intercept	2.506***	2.505***	2.190***	2.173***	1.952***	1.692***
	(11.96)	(11.18)	(9.50)	(11.99)	(10.25)	(8.28)
Regression type	RE	RE	RE	RE	FE	RE
Observations	580	580	580	493	493	493

Note: A Hausman test is used to choose between Fixed-effects (FE) and Random-effects (RE) regression, with a critical of $p \leq 0.10$. Figures in brackets are t-statistics; * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Observation 6: *Ranked short term beliefs and ranked long term beliefs are significant predictors of ranked bids and ranked asks.*

Table A.2 reports panel regressions of bids and asks on beliefs. We use ranks for both variables to correct for the differential effect of price developments between markets. The significant coefficients indicate a positive correlation between subjects' beliefs and subsequent bid decisions. Subjects with higher ranked beliefs (optmists) tend to place higher ranked bids. The size of the coefficients is similar to Carle (2016), who reports a slightly larger coefficient of $\beta = 0.38$ for short term beliefs. Since we only have beliefs for periods up to $t + 2$, we are unable to test whether long term beliefs are less informative than short term beliefs. With coefficients between 0.15 and 0.30, we find a noisy relationship between beliefs and trading behavior, in line with prior studies (Carle, 2016).

A.2 Stability of beliefs

This appendix comprises tests of the stability of beliefs over time within our experimental markets. The first section provides correlation coefficients of beliefs in the first period with beliefs in latter (averages) of periods. The second section describes the method used to calculate rank changes per market and periods

A.2.1 Belief correlations

We use three measures of beliefs to test the correlation of beliefs between periods. Over the course of the market round, beliefs are influenced by prices and progress differently among markets. By considering relative beliefs, we correct for these market-specific factors. Our first measure is the rank of average beliefs ($rank[\overline{B}_{mit}]$), defined by equation A.3. The second measure is the centered average belief, calculated as the difference between a subject i's belief and the period average belief within a market.

Table A.3: Correlation of beliefs

Periods (t)	2 - 5	6 - 9	10 - 13	13	1 - 13
$rank[B_{mit=1}]$	0.35**	0.18*	0.29**	0.29**	0.39***
$B_{mit=1} - \overline{B}_{mt=1}$	0.52***	0.39***	0.04	-0.07	0.49***

Note: This table includes Pearson correlation coefficients of ranked beliefs and centered $N = 60$ individual subjects. For each set of periods, (ranked) beliefs are averaged. * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

For each measure, we calculate the Pearson correlation coefficient of the value at $t = 1$ and averages of period subsets. We find that The higher correlation coefficients for our second measure suggests that the distribution of beliefs is stable throughout the experimental session.

A.2.2 Rank changes

Over time and in response to observed prices, subjects update their beliefs. Initial distributions and ranked of beliefs change in accordance from one period to the next. Beliefs may cross one another, as initial pessimists could ascribe higher value to the resale option of the asset than the initial optimists Morris (1996). To measure these shifts in ranked beliefs, we calculate the sum of absolute changes in ranked average beliefs.

$$RC_m = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{T-1}\sum_{t=2}^{13}|rank[B_{mit-1}] - rank[B_{mit}]| \tag{A.4}$$

where BS_m is the sum of changes in ranked beliefs of all $i = 8$ subjects in each market between periods $t-1$ and t in each market m. Since beliefs are elicited in period 1 through 13, this results in 12 shifts per subject i for each market m. Average rank changes per market (RC_m) are compared with a theoretical distribution assuming randomness in ranked beliefs. We perform a Monte Carle simulation of 10,000 markets with random belief rankings using the statistical program Stata. The simulation is repeated for markets of different size n, as our markets range from six to eight subjects. In addition, we compute rank changes and benchmark distributions for individual periods (RC_{mt}):

$$RC_{mt} = \frac{1}{n}\sum_{i=1}^{n}|rank[B_{mit-1}] - rank[B_{mit}]| \tag{A.5}$$

A.3 Additional bubble measures

In addition to the widely used measures RD_m and RAD_m in section 5, this appendix contains additional bubble measures as described by Stöckl et al. (2010).[37] All bubble measures are included in Table A.4 Total Dispersion (TD) is defined as the sum of all periods absolute deviations of prices from fundamental value. The measure indicates the extend to which market prices and fundamental value correspond to each other over all periods in the round:

$$TD_m = \sum_{t=1}^{15} |P_{m,t} - FV_t| \tag{A.6}$$

Average Bias (AB_m) provides information about the degree of general overvaluation or undervaluation. It is the average deviation of the market price from fundamental value. A positive value indicates overvaluation, while a negative value indicates undervaluation.

$$AB_m = \frac{1}{15} \sum_{t=1}^{15} (P_{m,t} - FV_t) \tag{A.7}$$

A shortcoming is that AB_m increases with the absolute level of fundamental value and prices. This limits comparability across different setups, while also overweighting early periods' relative to late periods in experiments with a decreasing fundamental value. Relative Deviation (RD_m) corrects AB_m by dividing over the average fundamental value.[38]

$$RD_m = \frac{1}{15} \sum_{t=1}^{15} \frac{(P_{m,t} - FV_t)}{F\bar{V}_m} \tag{A.8}$$

A relative deviation of $RD_m = 0.15$ indicated that on average, the price per period is 15% higher than fundamental value. Relative Absolute Deviation (RAD_m) measures the average absolute level of price deviations relative to the average fundamental value and indicates the magnitude of average relative price deviations.

$$RAD_m = \frac{1}{15} \sum_{t=1}^{15} \frac{|P_{m,t} - FV_t|}{F\bar{V}_m} \tag{A.9}$$

A relative absolute deviation of $RAD_m = 0.20$ indicates that on average, the price per period differs 20% from the average fundamental value. Price Amplitude (PA_m) measures the distance between

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[37] Due to the questionable result from Haessels R^2 and limited application in existing literature, this measure is not included in the analysis.

[38] Note that $F\bar{V}_m$ is the average fundamental value of periods in which trade occurred, which varies among markets. Without this correction, markets with no-trade periods would have significantly smaller values.

the highest and lowest ratio of the period's price deviation to fundamental value, normalized by the fundamental value for the period.

$$PA_m = max\left(\frac{P_{m,t} - FV_t}{FV_t}\right) - min\left(\frac{P_{m,t} - FV_t}{FV_t}\right) \tag{A.10}$$

A high price amplitude indicates a large movement in prices relative to fundamental value, but it provides no information about the duration of bubbles. Finally, we measure trade volume (V_m) as the average number of trades per period $q_{m,t}$ normalized by total shares Q_m to account for the varying number of subjects per market.

$$V_m = \frac{1}{Q_m}\sum_{t=1}^{15}(q_{m,t}) \tag{A.11}$$

Table A.4: Observed Values of Bubble Measures

Market	TD_m	AB_m	RD_m	RAD_m	PA_m	V_m
1	513	24	0.10	0.14	0.60	1.80
2	1198	43	0.18	0.33	0.99	1.30
3	920	52	0.22	0.25	1.20	2.20
4	1534	103	0.43	0.49	2.16	2.23
Homogeneous	1041	56	0.23	0.30	1.24	1.88
5	2323	155	0.64	0.69	3.42	2.13
6	3677	255	1.06	1.17	5.20	1.93
7	1855	114	0.47	0.51	1.19	1.15
8	763	33	0.14	0.21	0.87	1.38
Heterogeneous	2154	139	0.58	0.64	2.67	1.64
p-value	0.15	0.15	0.15	0.15	0.39	0.39

Note: This table reports individual market and averages of observed bubble measures, by treatment. Markets 1-4 have homogeneous initial beliefs and markets 5-8 have heterogeneous initial beliefs. Rows 'Homogeneous' and 'Heterogeneous' denote average measures of respective treatment markets. The bottom row reports p-values of a two-sided Mann-Whitney U test comparing homogeneous and heterogeneous treatment markets. * is significant at the 10% level, ** is significant at the 5% level, *** is significant at the 1% level.

Table reports these bubble measures for our experimental markets. In general, most measures indicate the same distribution of bubbles among markets. Close to all periods had prices in excess of fundamental value, which causes normal metrics (AB_m, RD_m) to yield similar results to absolute measures (TD, RAD_m). In fact, the non-parametric Mann-Whitney U test reports the same insignificant test result of $p = 0.15$ for all four bubble metrics.

A.4 Subject characteristics

A.4.1 Subjects characteristics

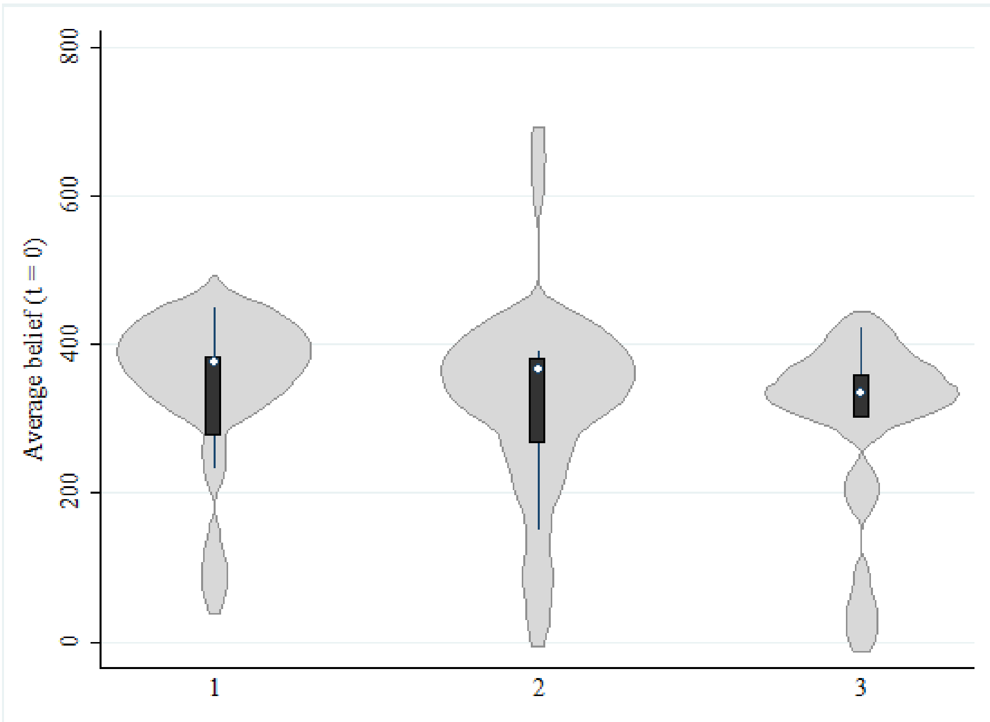
Table A.5: Descriptive statistics of individual subjects

	Radboud University	Rhine-Waal University		All
	Session 1	*Session 2*	*Session 3*	*Sessions 1-3*
Age	21.31	23.43	23.63	23.05
Gender	0.46	0.52	0.63	0.55
Year of study	2.31	3.13	3.13	2.95
Economics	0.23	0.61	0.58	0.52
Business administration	0.07	0.04	0.00	0.03
Political science	0.00	0.00	0.04	0.02
Geography	0.38	0.22	0.25	0.27
Other studies	0.31	0.13	0.13	0.17
CRT (correct)	4.08	3.43	3.58	3.63
CRT (intuitive)	1.77	1.87	1.92	1.87
CRT (wrong)	1.15	1.70	1.50	1.50
Risk tolerance	5.18	5.94	5.33	5.53
Payment (euro)	18.07	21.19	21.31	20.56
Number of participants	13	23	24	60

Note: Descriptive statistics of subject characteristics by session.

A.4.2 Initial beliefs by session

Figure A.1: Violin plots of initial beliefs by session



A.5 Experiment instructions

1. Experimental instructions

2. Table of fundamental values and histograms of holding values per period

Instructions

This is an experiment on decision making in markets. The instructions are simple and if you follow them carefully and make good decisions, you might earn a considerable amount of money, which will be paid to you in cash at the end of the experiment. The experiment consists of a sequence of trading Periods in which you will have the opportunity to buy and sell in a market. The currency used in the market is Gulden. All trading will be done in terms of Gulden. At the end of the experiment Gulden will be converted to Euro and you will be paid immediately. The conversion rate is: 200 Gulden = 1 Euro.

1 Market Platform

In this experiment you can trade shares for cash (referred to as Gulden). In each period, you will see a computer screen like the one you should see right now.

On the top of your screen you see your cash available (*Number of Gulden*) and the shares you hold in your inventory (*Number of Shares*).

- If you would like to buy shares, you can submit a buy order. Your buy order indicates the number of shares you would like to buy (field *Number of shares you would like to buy*) and the highest price that you are willing to pay for each share that you buy (field *Highest Price for which you would like to buy*). If you buy in this period, the price you pay for each share is exactly the highest price submitted or – even better – a lower price.
- If you would like to sell shares, you can submit a sell order. Your sell order indicates the number of shares you would like to sell (field *Number of shares you would like to sell*) and the lowest price that you are willing to accept for each share that you sell (field *Lowest Price for which you would like to sell*). In case you buy, you buy for that price or for a lower price. If you sell in this period, the price you receive for each share is exactly the lowest price submitted or – even better – a higher price.

In each period, you may submit both a buy order and a sell order. The price at which you offer to buy must be less than the price at which you offer to sell. The price that you specify in your order is a per-share price, at which you are willing to buy or sell each share. In case you don't want to submit a buy or sell offer – number of shares to offer = zero – click the respective button to hide the input window (click again to unhide). Do so now to see what happens.

PLEASE ENTER ARBITRAY NUMBERS (between 300 and 500 ONLY in trial) AND CLICK CONFIRM!

Once you click the *Confirm* button, your buy and sell orders for this period are final and can no longer be revised. The computer will tell you when you are not able to make a particular offer (either not enough shares to sell or not enough money to buy).

The computer will then organize the buy and sell orders and use them to determine the market price at which shares will be bought and sold in this period. All transactions in the period will occur at this market price. The market price will be a price such that the number of shares with sell order prices at or below this price is equal to the number of shares with buy order prices at or above this price.

At the end of each period, you will see a results screen as the one you see now.

This screen provides information about your offers, the market price, and the change of your share inventory and your Gulden balance.

When your buy price is higher than the market price, you buy at that market price. When your sell price is lower than the market price, you sell at the market price. It is rarely possible that you do not buy or sell although your order would allow for a transaction; in this situation, similar orders have been placed, and the computer randomly had to choose which of those orders will be filled.

In the trial period only, you also see the "Order Book". The order book shows the offer prices (*Price*) together with the cumulated quantities of sell orders (*Sell Volume*) and buy orders (*Buy Volume*) for a particular price.

The market price is at or around the price(s) for which the sell volume and the buy volume together allow for the highest number of transactions.

We will now have three more repetitions such that you can get comfortable with the trading platform. Raise your hand in case you have questions.

2 Dividend and Buyback

The actual experiment will consist of two sequences of 15 periods. In each period, there will be a market in which you may buy and sell shares, operating under the rules explained in section 1. Shares are assets with a life of 15 periods, and the shares that you hold will carry over from one period to the next within each 15-period sequence.

At the beginning of period one you have 2100 Gulden available with 1500 being a loan which you have to pay back after period 15.

At the end of each period you may earn a dividend for each share that you hold. The computer randomly determines the same dividend for each share after every period. Hence, each share pays

- a dividend of 0 Gulden with probability ¼ (25%)
- a dividend of 8 Gulden with probability ¼ (25%)
- a dividend of 28 Gulden with probability ¼ (25%)
- a dividend of 60 Gulden with probability ¼ (25%)

Since each of the four dividend values is equally likely, the expected dividend for each share in each period is 24 Gulden. Dividends will be added to your money balance automatically after each period. After the dividend has been paid at the end of period 15, the sequence ends. You sell each of your shares for 50 Gulden ("Buyback value") to the computer. Your final Gulden amount is your payoff for that sequence.

Your sequence payment in Gulden equals

	Your endowment of 2100 Gulden
–	Loan of 1500 Gulden
+	Amount you received for selling shares
–	Amount you paid for purchasing shares
+	Dividend payments
+	Buyback payment

After the first sequence of 15 periods has finished, a second sequence will begin. The amount of money and shares will be the same as at the beginning of the first sequence.

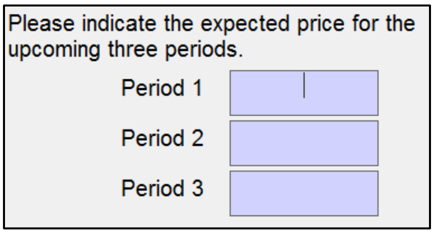
3 Information available and the Holding Value

After each period, you find similar information you saw in the trial period, plus the dividend information and a price history. Make also use of the graphs and the tables in the appendix to make your decisions (HAVE A LOOK AT IT NOW). The appendix provides information on the "Holding Value" which is the sum of all remaining dividend payments plus the buyback value, i.e. for each period the appendix indicates how much each share pays if it is held until the end of the 15th period.

We simulate one million holding values for each period such that you can see a distribution of holding values for each period. The higher a bar, the higher the probability that that particular holding value realizes (the red line indicates the mean holding value). The table summarizes the information by showing certain statistics together with the explanations.

4 Price Forecast

Additionally, you can make money by accurately forecasting the market price for each of the upcoming three periods. At the beginning of each period (not period 14 and 15), you submit your estimate on the market price in this period, in the next period, and the period after the next period (see figure below). If the market price is within the interval [Your estimate – 10, Your estimate price + 10], you earn 10 Eurocents.



Example (far below the actual prices): You estimate the market price in the next period to be 25. Hence the relevant interval equals [15, 35]. If the market price equals 40 you earn zero Eurocents (you estimate was too low). If the market price equals 32 you earn 10 Eurocents (market price is in the interval).

Your earnings from the forecasts will be added to your sequence payment at the end of a sequence.

5 End of the experiment

Right after sequence two, we would like you to fill an on-screen questionnaire. Relevant instructions will be on the screen.

After the questionnaire you find information about all your payments. One sequence will be paid out in cash. You flip a coin to determine which of the two sequences. The final cash payment equals the sequence payment plus a four Euro show up fee plus additional earning possibilities during the questionnaire.

Please enter the payment on the receipt and sign the receipt.

Note that you take part in this experiment to earn money. Hence, try to earn as much as possible during the two sequences and by making correct forecasts.

Any questions left?

In Period	Remaining	Min	25%<	50%	25%>	Max		Average	+/- on standard deviation
1	15	82	346	406	470	866		410	(320 500)
2	14	74	326	382	446	798		386	(299 473)
3	13	58	302	358	418	746		362	(278 445)
4	12	50	282	334	390	738		338	(258 418)
5	11	58	258	310	366	678		314	(237 391)
6	10	58	238	290	338	650		290	(217 363)
7	9	50	218	266	310	590		266	(197 336)
8	8	50	194	242	286	530		242	(177 308)
9	7	50	174	214	262	470		218	(157 279)
10	6	50	154	194	234	410		194	(137 251)
11	5	50	134	170	206	350		170	(118 222)
12	4	50	114	146	178	290		146	(100 192)
13	3	50	86	118	146	230		122	(82 162)
14	2	50	66	86	118	170		98	(65 131)
15	1	50	58	78	78	110		74	(51 97)

Example: Suppose you are in period five with eleven dividend payments left plus the buyback value. We simulated 1,000,000 holding values in period eleven. The lowest holding value was 58 (Min), 25% of all holding values were below 258, 50% of all holding values are below 310, and 75% of all holding values are below 366, i.e. 25% are higher than 366, and the highest draw yielded 678. The average of the 1,000,000 holding values is 314. Most of the values are within one standard deviation above and below the mean, i.e. within 237 and 391 in period five.

Example: Suppose you are in period five with eleven dividend payments left plus the buyback value. Hence, if you hold a share from period five until after period 15 you earn the sum of all remaining, eleven, dividend payments and the buyback value which is the 'Holding Value'. The respective graphs shows 1,000,000 randomly drawn holding values.. The graph displays the histogram with the holding values on the x-axis (the lowest number possible is 100 and the highest number possible is 11x60+100) and the frequency on the y-axis. The higher a golden bar, the higher the probability that the holding value equals the respective value on the x-axis. The red line points to the mean of 1,000,000 holding values which is about equal to 11x24+50=314.