Thesis

The profitability of (most common) dynamic portfolio insurance strategies compared to a simple benchmark; evidence from European stock markets

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Abstract

Although the performance of portfolio insurance strategies has been studied by various academics over the past decades, none of them was able to clarify the effectiveness of portfolio insurance strategies. This study relates to the existing literature by comparing different portfolio insurance strategies by measuring performance, generated by Monte Carlo simulations based on historical data. It goes beyond the existing literature by comparing dynamic portfolio insurance strategies (constant proportion and synthetic put) with a static portfolio insurance strategy (buy-and-hold) with recent post-crisis European data. Hereby, the historical data is based on the volatility and annual returns of the West European stock market from January 2009 to January 2020. This study found although the option-based portfolio insurance intensively adjusted and provides a good floor, it fails to perform better than other strategies in a moderate increasing market. Moreover, we found that the CPPI outperforms the buy-and-hold portfolio, except when it is monthly rebalanced.

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1. Introduction

"Life has an immense analogy with the stock market. It is volatile, but if you stick on long enough, it has the potential to reward you with handsome returns in the long run" (Arora, 2018)

In general, investors tend to be risk-averse and for this reason, they are unwilling to accept unnecessary volatility in their investment portfolios (Fabozzi et al., 2002). According to portfolio theory, the degree of risk aversion is important for the decision-making process of asset allocation since investors seek to achieve their portfolios' target level whilst taking the lowest amount of risk desired (Shefrin and Statman, 2000). The risk of an investment portfolio could be divided into systematic risk and idiosyncratic risk. Markowitz (1952) stated that idiosyncratic risk is the risk that is particular to one or a small number of assets and can significantly be decreased with a portfolio that holds more than thirty assets. In contrast to idiosyncratic risk, systematic risk relates to the overall economy and cannot be influenced by the number of assets.

To overcome short-term losses and stress due to systematic risk, many investors adhere to a static, buy & hold strategy to their portfolio. This is the simplest portfolio managing strategy since the investor only has to decide on the asset allocation at the beginning of the time horizon. With a buy & hold portfolio, an amount is invested in a risky asset while the rest is invested into risk-free assets. No matter what happens to the relative values of the portfolio, the asset allocation will not change until the end of the time horizon, and therefore, it is inexpensive due to the lack of transaction costs. The buy & hold portfolio will be linearly related to the performance of the overall stock market and moreover, the long-term strategy eliminates the short-term focus on the portfolio (Perold and Sharpe, 1988).

Although the buy & hold strategy is chosen by a vast majority of investors, the portfolio will still be exposed to downside systematic risk. This is because, after the discovery that price changes of assets follow a normal distribution, Mandelbrot (1963) and Blattberg and Gonedes (1974) stated that fat tails (significant deviations from the normal curve) exist. This means that events that cause an unprecedented rise or fall of the overall stock market are rare and their probability is low, but they do exist (Agic-Sebata, 2017). With the appearance of dynamic portfolio insurance strategies in the '70s, investors could limit downside risk by ensuring a predicted floor whilst still participating in the upside of a risky asset. Therefore, an investor could maximize the terminal value of a portfolio over a given investment horizon (Pézier and Scheller, 2011). A dynamic portfolio insurance protects the risky assets not only against abrupt falls in the market but also common, general financial downturns. Risky assets include stocks, bonds, commodities, hedge funds, credits, and other alternative assets. The portfolio insurance strategies all have a convex payoff function, which implies they buy assets high and sell them low. Put differently, when the market decreases, an investor switches from a risky to a riskless asset, and vice versa (Ho et al., 2010).

The two most popular dynamic portfolio insurance strategies are the option-based portfolio insurance with a synthetic put option and the constant proportion portfolio insurance (Ho et al., 2010).

The simplest way to have insurance on the downside risk of a portfolio is to buy an at-the-money put option on the assets held by investors. Hereby, the investor will decide at the beginning which investment horizon and floor value they will take. By this means, the value of the portfolio in the worst-case scenario will equal the exercise price of the put option minus the option premium (Do and Faff, 2004). However, Rubinstein and Leland (1981) stated that since the implied volatility usually exceeds the historical volatility of options, the price of the actual option will exceed the price of a synthetic option. Since synthetic options are cheaper and put options are not always available, it is more significant to use a synthetic put option to hedge the downside risk of a portfolio. A synthetic put option is created by creating a portfolio with a risky asset and a risk-free asset, where rebalancing is based on delta; a measure of the amount change in the price of an option when the underlying stock or index changes with €1. To match the synthetic put with the portfolio, it should be continuously rebalanced (Ho et al., 2010; Pézier and Scheller, 2011).

The second most popular dynamic portfolio insurance is the constant proportion portfolio insurance. Perold (1986) and Black and Jones (1987) first introduced the constant proportion portfolio upon which it was further developed by Perold and Sharpe (1988) and extended to fixed income instruments by Hakanoglu, Kopprasch, and Roman (1989). With the constant proportion portfolio insurance, the exposure to the risky asset is always the difference between the portfolio and the floor value, times a constant multiplier (Ho et al., 2010). To implement this strategy, the investor has to decide the multiplier and the floor value at the beginning of the investment period. Since the portfolio will stay in constant proportion, the strategy will sell assets in a decreasing market and buy assets in an increasing market (Black and Jones, 1987). Moreover, the constant proportion portfolio will reduce the exposure in assets to zero when the assets approach the floor, hence, the minimum payoff will be the floor value. The only risk of this insurance is the amount of rebalancing because if the market declines unexpectedly by a huge amount before one has had the chance to rebalance, then the portfolio could do worse than its floor value (Perold and Sharpe, 1988).

After the appearance of dynamic portfolio insurance strategies in the '70s, various studies are conducted on the existence and performance of portfolio insurance strategies (Ho et al., 2010). Hereby, most studies focused on the theoretical properties of continuous-time portfolio insurance (Carvalho et al., 2018). Based on the classical expected utility theory of Von Neumann and Morgenstern (1944), many academics studied the optimality of portfolio insurance strategies with the focus on utility maximization (Benninga and Blume, 1985; Black and Perold, 1992). They stated that the utility maximization of investors with portfolio insurance strategies is only possible under very specific assumptions regarding the preferences of the investor. Balder and Mahayni (2009) add to this that under the assumptions of classical expected utility theory, the constant proportion portfolio insurance will be the optimal strategy since it could sufficiently be matched with investors' preferences. However, the classical expected utility theory of Von Neumann and Morgenstern (1944) assumes rationality among investors, which does not always hold according to behavioral finance theories (Carvalho et al., 2018).

When analysing portfolio insurance strategies based on the cumulative prospect theory, Dierkes, Erner, and Zeisberger (2010) stated that most investors have a preference for portfolio insurance strategies. Dichtl and Drobetz (2011), and Gaspar and Silva (2015) add to this that most investors have a preference for portfolio insurance strategy due to probability weighting and loss aversion. Nonetheless, they did not find evidence supporting the constant proportion portfolio insurance as the most optimal portfolio insurance.

A second stream of literature on portfolio insurance was mainly focused on the stochastic dominance of portfolio insurance strategies (Bookstaber and Langsam, 2000; Carvalho et al., 2018). The stochastic dominance framework provides the probability distribution on the preference or risky assets of risk-averse investors (Clak and Kassimatis, 2014). While using this framework, Bertrand and Prigent (2005) stated that there is no evidence of a strong or weak stochastic dominance between the option-based portfolio insurance and the constant proportion portfolio insurance. However, in the mean-variance sense, there might be a stochastic dominance of the option-based portfolio insurance strategy, but this depends on the multiplier used in the constant proportion portfolio insurance. Annaert, Van Osselaer, and Verstraete (2009) evaluated different portfolio insurance strategies and did not find a dominance relation with the buy-and-hold portfolio. Since many investors are risk-averse, they stated that portfolio insurance strategies are preferred over buy-and-hold portfolios by investors who want to hand in some potential profit in order to obtain a lower risk, but also did not find evidence about the outperformance of a portfolio insurance strategy over another. Zagst and Kraus (2011) conducted their study on portfolio insurance strategies by considering a multiplier higher than one for the constant proportion portfolio insurance and analysed the option-based portfolio insurance using the criteria of the stochastic dominance. They extended the analysis by Annaert, Van Osselaer, and Verstraete (2009) and Bertrand and Prigent (2005), by adding a second and third-order stochastic dominance. Hereby, they stated that there is a higher probability of the constant proportion portfolio insurance dominating the option-based portfolio insurance in the third-order stochastic dominance.

A third stream of literature focused on comparing the performance of portfolio strategies with each other (Benninga, 1990). When conducting a study with the performance of volatile and not-so-volatile markets, Perold and Sharpe (1988) stated that there is no best dynamic strategy that works in all situations since it depends on an investor's preferences. However, Zhu and Kavee (1988) compared the synthetic put option insurance strategy to the constant proportion insurance strategy by assuming a 15% annual mean return. They stated that in a volatile market, the transaction costs of the synthetic put option insurance strategy may become too high for it to be profitable. Although the constant proportion portfolio insurance has lower transaction costs, they could still be sufficient. Contrarily, Black and Rouhani (1989) stated that the constant proportion portfolio insurance only performs better than the option-based portfolio insurance, when there is a moderate market increase. Focusing on the Australian futures market, Loria, Pham, and Sim (1991) that there is no guarantee that the portfolio will go below its floor since it could happen due to the mispricing of futures. Moreover, they stated in severe markets, an option-based portfolio insurance is more profitable than other strategies. This is in accordance with Cesari and Cremonini (2003), who used different performance measures than Loria, Pham, and Sim (1991), but stated that the constant proportion portfolio insurance performs better in bear and sideways markets. As an extension of Cesari and Cremonini (2003) and Loria, Pham, and Sim (1991), Do and Faff (2004) conducted several simulations with SPI futures, and with Australian stock index and bills. In contrast to Cesari and Cremonini (2003) and Loria, Pham, and Sim (1991), and after adding implied volatility to their model, they found no evidence of performance differences between the two portfolio insurance strategies. Hereby they suggested that it seems that the synthetic put-option insurance tends to perform better in protecting downward losses due to the floor protection, but the constant proportion portfolio insurance tends to perform better with upside participation. However, a more recent paper by Pézier and Scheller (2011) stated that the constant proportion portfolio insurance is superior to option-based portfolio insurance in all cases.

Apart from the inconsistencies and mixed results of previous research on portfolio insurance strategies, the vast majority of literature focused on the United States (Ho et al., 2010), Malaysia (Masaar et al., 2012), China (Jiang et. al., 2009), and Turkey (Iscanoglu-Cekic 2016), while Agic-Sebata (2017) states that portfolio insurance strategies are most commonly used in West European stock markets. Furthermore, most studies were conducted before the financial crisis of 2008 although the financial markets changed significantly after 2008. Investors are able to exploit opportunities to employ innovative infrastructures and trading practices due to post-crisis technological innovations, regulatory developments, and growing competitions (ESMA, 2014). Hereby, high-frequency trading plays a big role since it positively affects stock volatility and causes flash crashes, which have an influence on option pricing because of the increased volatility in stock markets (Zhang, 2010).

This study relates to the existing literature by comparing different portfolio insurance strategies by measuring performance, generated by Monte Carlo simulations based on historical data. It goes beyond the existing literature by comparing dynamic portfolio insurance strategies (constant proportion and synthetic put and their variances) with a static portfolio insurance strategy (buy-and-hold) while using post-crisis European stock market data as a basis for simulations. Hereby, the historical data is based on the volatility and annual returns of the West European stock market from June 2009 to January 2020. Moreover, the performance is not only measured by the Sharpe ratio, Calmar ratio, and the Sortino ratio, but also by the expected shortfall, which is more appropriate for portfolio insurances. The key objective of this study is to find the most effective portfolio insurance strategy; simply put, which strategy has the best ratios between risk and return.

Therefore, we strive to answer the following question: How effective are dynamic portfolio insurance strategies in the West European stock markets compared to a static benchmark?

Although the performance of portfolio insurance strategies has been studied by various academics over the past decades, none of them was able to clarify the effectiveness of portfolio insurance strategies in Europe, in recent financial markets (Carvalho et al., 2018). Besides clarifying the theoretical results regarding portfolio insurance strategies, this study also contributes to high-net-worth individuals, private banks, and institutional investors by providing them clarification about the best suitable portfolio insurance to use. Therefore, they will be able to adjust their portfolios to their strategies, and with the minimum amount of risk, get the highest result (Kosowski and Neftci, 2015).

After simulating more than 3 billion daily stock prices through Monte Carlo simulations, this study found that it is important to choose the right portfolio strategy for the situation. Although the option-based portfolio insurance intensively adjusted and provides a good floor, it fails to perform better than other strategies in a moderate increasing market. Moreover, we found that the CPPI outperforms the buy-and-hold portfolio, except when it is monthly rebalanced. Among the CPPI portfolios, the portfolio with a multiplier of 1 outperformed the other strategies.

The structure of this study is organized as following. Section 2 provides a preliminary comparison of portfolio insurance strategies, which are relevant to this study, and whereafter the hypothesis will be formulated. In section 3, the data and the methodology will be presented while in section 4, the results will be discussed. At last, the conclusion will be given. A more detailed analysis of the results will be presented in the appendices.

2. An overview and preliminary comparison of portfolio insurance strategies

2.1. Buy-and-hold portfolio

The buy & hold portfolio is the simplest strategy for investors to implement. At the beginning of the time horizon, the investor decides the amount to invest in risky assets and risk-free assets (the initial mix). Until maturity, the portfolio does not make any changes and sticks to its initial plan. Therefore, it is a static approach and the risk/return payoff will be linear with the market, hereby, the upside potential is unlimited. Since this static approach will not make any changes during the time until maturity, this is the cheapest variant of the portfolio strategies since there are no transaction costs during the investment period and can serve as a benchmark to compare other portfolio strategies with (Perold and Sharpe, 1988). To put it in a more mathematical perspective, if w_{risky} is the initial part that is invested in risky assets, the initial floor is:

$$F_0 = (1 - w_{risky})A_0 \tag{1}$$

Where the initial assets are defined by A_0. Therefore, the floor at time t will be described by:

$$F = \left(1 - w_{risky}\right)A_0 e^{rt} = F_0 e^{rt} \tag{2}$$

Where the risk-free rate is defined by r at time t. The payoff at time t will be described by:

$$A = F + w_{risky}A_0 \frac{S}{S_0} \tag{3}$$

Where the stock market at t (total return index) is defined by S, and the initial level of the stock market is defined by S_0. At last, the desired stock position (exposure) E can be described by:

$$E = A - F \tag{4}$$

2.1. Option-based portfolio insurance: synthetic put

Since synthetic options are cheaper and put options are not always available, it is more significant to use a synthetic put option to hedge the downside risk of a portfolio. Rubinstein and Leland (1981) introduced the synthetic put strategy, which is a synthetic put, created by a continuously adjusted portfolio of risky assets and a risk-free asset. Hereby, the rebalancing is based on the delta, according to the option pricing formula of Black and Scholes (1973). The synthetic put strategy will have a convex risk/return payoff curve and since it uses the Black-Scholes option pricing formula, it assumes normally distributed returns. Furthermore, it is a "buy high, sell low" strategy, which implies high transaction costs because of the continuous rebalancing (Do and Faff, 2004; Ho et al., 2010). When protecting the portfolio (S) with a put option (p), the value of the portfolio can be calculated as follows:

$$S + p(S, K, \sigma, r, T, \delta) = S + Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1) \tag{5}$$

Hereby, the strike price is defined by K, the risk-free rate is defined by r, the dividend yield is defined by δ, and the time to maturity is defined by T. Furthermore, the standard normal cumulative distribution function is defined by $N(.\ )$, and $-d_2$ and $-d_1$ are defined as follows:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + (r - \delta + 0.5\sigma^2)T}{\sigma\sqrt{T}} \tag{6}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{7}$$

In the two equations above, the standard deviation of the risky asset is defined by σ. Since we are replicating the portfolio instead of buying options, we need the delta as a measurement of buying and selling risky assets. The delta is calculated by $N(d_1)$. With this delta, we can calculate the weights of the risky and risk-free asset accordingly:

$$w_{risky} = \frac{S * N(d_1)}{S + Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)} \tag{8}$$

$$w_{risk-free} = 1 - w_{risky} \tag{9}$$

Since this strategy is a "buy high, sell low" strategy, it will switch to more risk-free assets in a decreasing market and switch to more risky assets in increasing markets (Dichtl et al., 2017).

2.2. Constant proportion portfolio insurance

The exposure to the risky assets of the constant proportion portfolio insurance is always the difference between the portfolio and the floor value, times a constant multiplier. Investors who invest according to the constant proportion portfolio will have a zero-tolerance for risk beneath a specified floor. As well as the option-based portfolio, the constant proportion portfolio will have a "buy high, sell low" perspective on the stock market. Hereby, if the market decreases, more risky assets will be sold, and risk-less assets will be bought. If the market increases, more risk-less assets will be sold and more risk-less assets will be bought, in order to maintain the constant proportion of the initial mix between risky and risk-less assets. Because of this, the transaction costs will be higher, but lower than with the option-based portfolio insurance. Moreover, this dynamic strategy also has a convex risk-return payoff curve since it will buy high and sell low. Unlike the synthetic put, the constant proportion portfolio insurance does not have any assumptions regarding normal distribution (Black and Jones, 1987; Ho et al., 2010; Perold, 1986); Perold and Sharpe (1988). To put it in a mathematical perspective, the constant proportion portfolio insurance contains a constant multiplier m. Hereby, the floor (F) at t is given by:

$$F = F_0 e^{rt} \quad (10)$$

The constant proportion strategy corresponds to the following special case:

$$F = F_0 = 0 \quad (11)$$

The payoff for the constant proportion portfolio (A) at t is given by:

$$A = F + (A_0 - F_0)\left(\frac{S}{S_0}\right)^m e^{(1-m)(r+0.5m\sigma^2)t} \quad (12)$$

Whereby Where the stock market at t (total return index) is defined by S, and the initial level of the stock market is defined by S_0. Moreover, the variance is given by σ^2. Therefore, the exposure (E) to the market will be described as:

$$E = m(A - F) \quad (13)$$

The exposure will be limited to the total assets A, if there is no leverage allowed, thus:

$$E = \min\,(A, m(A - F)) \quad (14)$$

2.3. Theoretical predictions

An overview of the three portfolio strategies is displayed in the table below.

	Buy & Hold	**Synthetic put**	**Constant proportion**
Operational costs	Low	High	Moderate
Insurance strategy	Static	Dynamic	Dynamic
Risk/return payoff	Linear	Convex	Convex
Investment philosophy	Do nothing	Buy High / Sell Low	Buy High / Sell Low
Protection target	-	Initial wealth	Initial wealth
Type of insurance	-	Strong floor compliance	Strong floor compliance
Model assumptions	-	Normal distribution	No assumptions

Table 1. Summary of three portfolio strategies

The table above describes static portfolio strategies as “do nothing” and dynamic portfolio strategies as “buy high, sell low”. Because of this, a static portfolio strategy is the cheapest to implement since investors will not change the portfolio after deciding about the initial mix until the end of their investment period. Contrarily, dynamic portfolio insurance strategies are more complex and thus, harder to implement. Even though the higher costs, they could be tailed to the preferences of the investor (Ho et al., 2010). The synthetic put and the constant proportion strategy may be more suitable for high risk-averse investors in volatile markets since they have a floor. However, there are some complications with these strategies.

Firstly, the transaction costs might be an enormous flaw while using dynamic strategies since periodic rebalancing implies performing a lot of costly trades. Therefore, there is a constant trade-off between limiting the transaction costs and guaranteeing the effectiveness of the strategy. If there are high transaction costs, a buy-and-hold portfolio might be preferable, especially when the dynamic portfolio insurances are adjusted on a daily basis. The formula below displays that the profit of the Buy-and-hold strategy could be equal or larger to the profit of the dynamic strategy minus the transaction costs over the periodic changes in the value of the whole period. It shows the effect of transaction costs on the profit of dynamic strategies (Pézier and Scheller, 2011).

$$B\&H_{profit} \geq Dynamic_{profit} - (C * \Delta w_{risky} * T) \tag{15}$$

Secondly, the occurrence of sudden price jumps may be catastrophic for dynamic strategies, since they may not be able. to adjust their portfolio in time, particularly for the constant proportion portfolio insurance strategy Therefore, the dynamic strategies could go below their floors. Moreover, if the price first decreased and thereafter jumps, most dynamic strategies will not fully obtain the profit of the jump since they already sold (a part of) their risky assets. Hereby, the buy-and-hold strategy is also preferable, this is visualized in the equation below (Do and Faff, 2004).

$$B\&H_{profit} \geq Dynamic_{profit} - loss_{jumps} \tag{16}$$

Thirdly, issues could occur with the option-based portfolio insurance when replicating options due to the estimation of the volatility. As a result, the volatility estimation, which will be assumed constant, could differ from the actual volatility, since in reality, the volatility fluctuates and is not a constant. Therefore, while using the Black-Scholes formula, which assumes a constant volatility, estimated option prices might differ from the real option prices, and thus, the delta differs (Ho et al., 2010). The equation below explains that the profit of the buy-and-hold portfolio could be larger than dynamic portfolio insurance strategies when the costs of assuming a constant volatility when it fluctuates.

$$B\&H_{profit} \geq OBPI_{profit} - costs_{N(d_1)} \tag{17}$$

The European stock market had ups and downs, but in general, it had an increasing trend over the past years, we expect the dynamic portfolio insurance strategies to perform better since they can profit more by the rise in the stock market after a shortfall because they can adjust and invest more in risky assets. In this study, performance will be measured by the Sharpe ratio, Calmar ratio, Sortino ratio, and expected shortfall (Ho et al., 2010). Therefore, the following hypothesis is developed:

H1. Dynamic portfolio insurances perform better than static portfolios in recent West-European markets based on stock market data from January 2009 until January 2020.

Moreover, when comparing both dynamic portfolio insurance strategies, Zhu and Kavee (1988) found that the constant proportion portfolio may perform better than the option-based portfolio when there are no abrupt jumps in prices. Due to the high transaction costs due to the constant rebalancing, the transaction costs for the synthetic put option might be too high to be profitable. Contrarily, Black and Rouhani (1989) stated that the constant proportion portfolio insurance only performs better than the option-based portfolio insurance, when there is a moderate market increase. This is because option-based portfolio insurance includes forward-looking model parameters (Zhu and Kavee, 1988). Because of this, the downward protection is of higher quality, but this will impact the upward movement of a moderate market increase. Loria, Pham, and Sim (1991) also suggested that the synthetic put-option insurance tend to perform better in protecting downward losses due to the floor protection, but the constant proportion portfolio insurance tend to perform better with upside participation. This is in accordance with Cesari and Cremonini (2003), who used different performance measures but stated that the constant proportion portfolio insurance performs better in bear and sideways markets. Since in recent years, the European stock market experienced moderate increases, the following hypothesis is developed:

H2. The constant proportion portfolio strategy performs better, in recent West-European markets, based on stock market data from January 2009 until January 2020, compared to the option-based portfolio insurance.

3. Study design

3.1. Data

This study will focus on the West European stock market data. Pain and Rand (2008) stated that the use of portfolio insurance strategies is more prevalent in Europe than in the United States. Moreover, Agic-Sebata (2017) stated that portfolio insurance strategies are most commonly used in West European stock markets. Even though, most studies conducted were focused on the United States (Ho et al., 2010), Malaysia (Masaar et al. 2012), China (Jiang et. al. 2009), and Turkey (Iscanoglu-Cekic 2016). Besides, most studies on the performance differences of portfolio insurance strategies are conducted before the financial crisis. This means that the significant change in the financial market due to crisis technological innovations, regulatory developments, and growing competition, were not considered yet (ESMA, 2014).

This study will focus on comparing portfolio insurance strategies based on the historical returns from the West European stock markets from January 2009 to January 2020 by using an index tracker. The best option to track the West European stock market is via an exchange-traded fund. This is an investment fund that traded on stock exchanges and follows an index as closely as possible. Therefore, the historical data will be based on the performance of the Lyxor EURO STOXX 50 UCITS ETF, which is an exchange-traded fund that tracks the STOXX Europe 50 Index as closely as possible. The STOXX

Europe 50 Index consists of the fifty largest European blue-chip companies with an exposure of 99 percent in West Europe, which provides a good indication of the overall market return (Franceschetto, 2018). The data will be derived from a combination of Thomson Reuters Eikon and the Factset Mergerstat database and is summarized in appendix A.

3.2. Implementation variables

3.2.1. Floor value

At the beginning of the investment period, a floor value needs to be selected. For both the synthetic put and the constant proportion portfolio, the floor value grows at a constant risk-free rate. Therefore, the initial floor value, chosen at the start of the investment period, is discounted back from maturity, as a constant risk-free rate. The annual risk-free rate of 0.85% is calculated by taking the average of the Libor-free rate of the West-European countries over the previously mentioned sample period (Khuman et al., 2008).

3.2.2. Rebalancing interval

When implementing an analysis between dynamic strategies, transaction costs will play a huge role. Since the amount of transaction costs paid is influenced by the rebalancing interval, the choice for the interval will be important for the overall results. This means that more often rebalancing, will cause the portfolio to stick closer to the initial strategy, but the overall transaction costs will also increase a lot. Investors typically choose to rebalance every day, week, month, or quarter, in order to keep their constant proportion portfolio up to date (Ho et al., 2010). Although Ho, Cadle, and Theobald (2010) state that the least amount of errors occurs with weekly rebalancing, several studies used other rebalancing intervals. Since the rebalancing periods of studies comparing the performance of constant proportion portfolio insurance strategies are somewhat mixed, this study will use daily, weekly and monthly rebalancing (Do and Faff, 2004). Therefore, different portfolio strategies may also be compared while using different rebalancing intervals.

3.2.3. Transaction costs

As previously mentioned, the transaction costs depend on the rebalancing interval. In this study, the transaction costs will be considered in two ways. Firstly, Dichtl and Drobetz (2011) stated that the cost over the total value of the traded assets should be $c = 0.1\%$. Moreover, Leland (1985) stated that the volatility of the delta must be corrected for transaction costs. This will be done by using the following formula:

$$\sigma_{Legland} = \sigma \sqrt{1 + \sqrt{\left(\frac{2}{\pi}\right) * \left(\frac{k}{\sigma\sqrt{\Delta t}}\right)}} \tag{18}$$

Hereby, the adjusted volatility is given by σ^2_{adj}, the initial volatility is given by σ, the round-trip transaction costs are given by k and the length of the readjustment period is given by Δt.

3.3. The simulation setup

3.3.1. The simulation process

To simulate different equity market scenarios, from which the behaviour of different portfolio strategies will be analysed, Monte Carlo simulations will be used and conducted using Python (Cesari and Cremonini, 2003). A Monte Carlo simulation uses randomness to generate the probability of different outcomes. The simulations of Monte Carlo are based on stock price processes and follows a geometric Brownian; they include a drift rate (a constant expected return) and a variance rate (constant volatility). Whilst using the historical returns of the Lyxor EURO STOXX 50 UCITS ETF, the periodic daily return can be calculated:

$$r_{1/365} = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{19}$$

Hereby, the lognormal difference between the latest value (S_t), and the value with one period less (S_{t-1}) is calculated. Since the logarithm is normally distributed and the variance is equivalent to time, Agić-Šabeta (2016) states that we are able to calculate the standard deviation by the following equation:

$$d\ln(S_t) = \sigma X\sqrt{dt} \tag{20}$$

Whereby a random number drawn is described by X, the variance is described by σ and the passing of time is given by dt. We are now able to form the normal distribution since Deutsch and Eller (1998) states that this is formed by the sum of the independent random numbers. When an independent variable drift and a mean μt, random walks can be generated for the stock prices:

$$d\ln(S_t) \; = \; \mu dt \; + \; \sigma\, X\sqrt{dt} \tag{21}$$

In this formula, we have the drift rate, given by the mean μ, and the variance rate, given by σ times an independent random number. However, the creation of continuous numbers is preferred while conducting a Monte Carlo analysis. According to Hull (2003), this can be accomplished by using Ito's lemma from the stochastic analysis:

$$dS(t) = S(t)\left(\mu + \frac{\sigma^2}{2}\right)dt + S(t)\sigma\, X\sqrt{dt} \tag{22}$$

This can be rewritten as the stock price path of $S(t + dt)$:

$$S(t + dt) = S(t)Xe^{\left(\mu+\frac{\sigma^2}{2}\right)+S(t)\sigma\, X\sqrt{dt}} \tag{23}$$

Python will be used to generate normally distributed random numbers, and to repeat this entire process many times to implement the different portfolio strategies and its variables and provide returns.

3.4. Performance ratios

The performance of the portfolio insurance strategies will be compared on the basis of four performance ratios; Sharpe ratio, Sortino ratio, Calmar ratio and Expected shortfall. Apart from the traditional performance measures like the Sharpe ratio, Calmar ratio and the Sortino ratio, the Expected Shortfall is added since it is a more appropriate ratio in terms of portfolio insurance strategies (Annaert et al., 2009).

3.4.1. Sharpe ratio

The Sharpe ratio will be used to compare the return of an investment to its risk. It will be calculated with the following formula:

$$Sharpe\ ratio = \frac{\mu - r_f}{\sigma} \tag{24}$$

Hereby, μ is the mean return, r_f is the risk-free rate, and σ is the standard deviation. A Sharpe ratio of 1 is acceptable, 2 is very good and 3 is excellent (Agić-Šabeta, 2016).

3.4.2. Calmar ratio

The Calmar ratio is another measure for risk-adjusted returns and compares the annual return to the maximum drawdown. The maximum drawdown is the maximum loss from a peak of a portfolio until a new peak is obtained. It can be considered as a measure for the downside risk of the portfolio over a given period (Agić-Šabeta, 2016).

$$Calmar\ ratio = \frac{\mu}{maximum\ drawdown} \tag{25}$$

3.4.3. Sortino ratio

Since the Sharpe ratio does not focus on the downside deviation, the Sortino ratio uses downside deviation instead of volatility and a threshold (=0) instead of the risk-free rate:

$$Sortino\ ratio = \frac{\mu}{downside\ deviation} \tag{26}$$

Whereby the downside deviation is created by using the standard deviation of negative portfolio returns (Agić-Šabeta, 2016).

3.4.4. Expected Shortfall

The Expected Shortfall is important to know the tail risk of the investment portfolios. Tail risk is a risk that has a small probability of occurring but has huge consequences when occurs. The small probability concerns that the investment will move at least three standard deviations from its mean.

$$Expected\ Shortfall = \frac{1}{1-c}\int_{-1}^{VaR} xp(x)\,dx \tag{27}$$

Hereby, the cut-off point on the distribution is given by c, the agreed-upon value at risk level is given by VaR, and the probability density of getting return x, is given by $p(x)\,dx$. VaR (Value at Risk) measures, with a certain confidence level, the maximum expected loss on an investment in a specified time period (Acerbi and Tasche, 2002).

4. Results

In this chapter, the results of the Monte Carlo simulations of the different portfolio insurance strategies will be discussed. Moreover, not only the differences between portfolio insurance strategies will be discussed, also the results of using different parameters within a portfolio insurance strategy will be presented. The Monte Carlo simulations have been conducted over a period of 10 years and 7 months, from the 1st June 2009 until the 31th December 2019. To provide effective estimates, the Monte Carlo simulations have 50,000 iterations each. The Monte Carlo simulations are conducted based on the Lyxor Euro STOXX 50 ECITS ETF, of which detailed description can be found in Appendix A. Based on the Lyxor Euro STOXX 50 ECITS ETF and the sample period, the Monte Carlo simulations were conducted with the following statistics:

Parameter	Value
Total days	2,712
Total years	10.5833
Days in a year	256
Risk-free rate	0.85%
Standard deviation	20.26%
Annual return	3.07%
Total return	32.45%
Starting value	€26.23

Table 2. Input Monte Carlo simulations.

Besides that, an overview of all the results is presented in appendix M-O.

4.1. Maximum and minimum values

In the table below, the Monte Carlo results regarding the profit and variance are presented. Since the standard deviation of the Lyxor Euro STOXX 50 ECITS ETF is 20.26%, which is used as an input for the Monte Carlo analyses, the terminal values of the stock prices range between €1.70 and €641.50. Moreover, due to the lack of protection, the table presents the buy-and-hold portfolio like the one with the lowest minimum value. Most of the strategies have a minimum price which does not exceed the starting value of the portfolio of €26.23, except the constant proportion portfolio insurance strategy (CPPI) with a multiplier of 1, and the CPPI with a floor of 1.

Monte Carlo results insurance strategies – minimum and maximum value

	B&H	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1
Minimum value	€ 1.70	€ 33.42	€ 22.32	€ 21.52	€ 25.22	€ 27.82
Maximum value	€ 377.81	€ 388.61	€ 462.02	€ 594.21	€ 582.03	€ 400.56

	CPPI W	CPPI M	OBPI K25	OBPI K225	OBPI K20
Minimum value	€ 21.23	€ 21.09	€ 24.17	€ 22.77	€ 19.36
Maximum value	€ 347.39	€ 641.50	€ 385.11	€ 498.88	€ 453.45

Table 3. Monte Carlo results insurance strategies – minimum and maximum value.

4.2. Profit and variance

The results of the profit and variance for each strategy are presented in the table below. While the annual return of the Lyxor Euro STOXX 50 ECITS ETF is 3.07%, the annual return of the buy-and-hold portfolio is 3.01%, which is approximately equal. Overall, all the portfolios produced positive results, between 1.82% for the option-based portfolio insurance (OBPI) with a strike price of €25 and 9.98% for the constant proportion portfolio insurance (CPPI) with a multiplier of 1. Moreover, the standard deviation of the portfolio's ranges from 6.65% for the weekly adjusted CPPI, and 20.27% for the buy-and-hold portfolios.

When comparing the static portfolio insurance strategies with dynamic portfolio insurance strategies, the static buy-and-hold portfolio has the highest annual standard deviation compared to other strategies. This could be a result of the lack of a floor when stock prices decrease. Other strategies include a floor (the option-based portfolio insurance strategy through its strike price), whereby the volatility could be toned down. Moreover, the CPPI strategy seems to generate the highest returns, except for the monthly-rebalanced CPPI. The option-based portfolio insurance (OBPI) performs overall the worst. This could be because the European stock market is was moderately increasing by 3.07% and thus, having a very strong floor compliance, and a low standard deviation is less helpful.

Furthermore, there seems to be a relationship between the multiplier and the performance of the CPPI. Hereby, a lower multiplier decreases the volatility, and increases (almost doubles) the annual return. Moreover, also an increasing floor of the CPPI, tends to generate higher returns while decreasing the volatility. Although a higher strike price decreases the annual standard deviation of the option-based portfolio insurance, the annual returns decrease as well.

Monte Carlo results insurance strategies – profit/variance

	B&H	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1
Annual standard deviation	20.27%	11.02%	17.13%	17.94%	16.35%	15.70%
Variance	4.11%	1.21%	2.93%	3.22%	2.67%	2.47%
Annual return	3.01%	9.98%	5.12%	4.48%	5.85%	6.53%
Total return	31.86%	105.62%	54.14%	47.44%	61.91%	69.13%

	CPPI W	CPPI M	OBPI K25	OBPI K225	OBPI K20
Annual standard deviation	6.65%	12.00%	9.41%	12.70%	15.16%
Variance	0.44%	1.44%	0.88%	1.61%	2.30%
Annual return	3.24%	2.22%	1,82%	2.27%	2.51%
Total return	34.28%	23.48%	19.22%	23.98%	26.61%

Table 4. Monte Carlo results insurance strategies – profit/variance.

4.3. Normal distribution and downside risk

In the table presented below, we see that the stock prices are heavy-tailed and outliers. As presented in table 3, there is a wide range between minimum and maximum values. Moreover, the maximum drawdown is the highest for the buy-and-hold portfolio (-94.16%) and the lowest for the CPPI with a multiplier of 1. This means that the downside risk over the time period is the highest for the buy-and-hold portfolio.

For comparing the parameters within different portfolio strategies, there is not a high difference, apart from the CPPI with a multiplier of 1. The rest of the portfolios have a maximum drawdown in the range of -75.63% to -81.04%.

Monte Carlo results insurance strategies – normal distribution and downside risk

	B&H	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1
Maximum drawdown	(94.16%)	(55.01%)	(78.15%)	(76.76%)	(77.36%)	(75.63%)
Skewness	2.51	1.87	2.73	3.26	3.00	2.72
Kurtosis	14.64	10.22	16.00	24.57	21.60	15.99
Downside deviation	38.06%	-	3.19%	4.61%	0.85%	-
	CPPI W	CPPI M	OBPI K25	OBPI K225	OBPI K20	
Maximum drawdown	(77.07%)	(77.93%)	(78.52%)	(79.22%)	(81.04%)	
Skewness	3.24	4.09	4.66	3.65	2.97	
Kurtosis	20.31	38.11	36.84	24.31	17.27	
Downside deviation	4.99%	5.61%	0.86%	2.81%	6.41%	

Table 5. Monte Carlo results insurance strategies – normal distribution and downside risk.

4.4. Predicted upper and lower limits

Presented in the table below are the predicted upper and lower limits with a confidence interval of 95%. Since there were 50,000 iterations per Monte Carlo simulations, the values are quite predictable. The values are quite consistent with the returns for each portfolio. With these market conditions, the OPBI portfolios have the lowest predicted upper and lower limits. Also, there is a clear difference between CPPI portfolios, especially the portfolio with a multiplier of 1 and the portfolio with a floor of 1 are performing well.

Monte Carlo results insurance strategies – predicted upper and lower limits

	B&H	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1
Upper limit (95% confidence)	€ 36.31	€ 75.63	€ 45.28	€ 42.37	€ 48.94	€ 52.59
Lower limit (95% confidence)	€ 35.84	€ 75.20	€ 44.86	€ 41.94	€ 48.50	€ 52.15
	CPPI W	CPPI M	OBPI K20	OBPI K225	OBPI K20	
Upper limit (95% confidence)	€ 37.14	€ 33.36	€ 31.95	€ 33.54	€ 34.44	
Lower limit (95% confidence)	€ 36.77	€ 32.99	€ 31.62	€ 33.13	€ 34.01	

Table 6. Monte Carlo results insurance strategies – predicted upper and lower limits.

4.5. Risk-return ratios

The risk-return ratios are presented in the table below. Again, the buy-and-hold portfolio has the highest negative values; value at risk of -30.33% and an expected shortfall of -38.80%. Both are tested with a confidence level of 95%. Furthermore, the option-based portfolio insurance strategies are performing almost as bad as the buy-and-hold portfolio. The expected shortfall decreases drastically when taking a lower multiplier. Moreover, weekly-adjusted CPPI has the lowest expected shortfall. The Sharpe ratio of portfolios is the highest of the CPPI with a multiplier of 1. The Sharpe ratio of the buy-and-hold portfolio is approximately equal to the Sharpe ratio of the OPBI portfolios.

When looking at the Sortino ratio, the option-based portfolio insurance with a strike price of €20 stands out with a Sortino ratio of 2.5895. Given their downside deviations (-0.86%), it is earning its returns more effectively than the other portfolios. Moreover, the Calmar ratio is really low for every portfolio, most probably due their high maximum drawdowns.

Monte Carlo results insurance strategies - Ratios

	B&H	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1
Value at Risk (95% confidence)	(30.33%)	(8.14%)	(23.06%)	(25.02%)	(21.04%)	(19.29%)
Expected shortfall (95%)	(38.80%)	(12.74%)	(30.22%)	(32.51%)	(27.88%)	(25.85%)
Sharpe ratio	0.1068	0.8291	0.2492	0.2028	0.3060	0.3622
Sortino ratio	0.0583	-	0.6956	0.4814	2.6136	-
Calmar ratio	(0.0230)	(0.1660)	(0.0546)	(0.0474)	(0.0647)	(0.0752)

	CPPI W	CPPI M	OBPI K20	OBPI K225	OBPI K20
Value at Risk (95% confidence)	(7.70%)	(17.53%)	(13.66%)	(18.63%)	(22.41%)
Expected shortfall (95%)	(10.48%)	(22.54%)	(17.59%)	(23.93%)	(28.75%)
Sharpe ratio	0.3598	0.1144	0.1031	0.1117	0.1100
Sortino ratio	0.4445	0.3959	2.5895	0.7904	0.3466
Calmar ratio	(0.0311)	(0.0176)	(0.0123)	(0.0179)	(0.0206)

Table 7. Monte Carlo results insurance strategies - Ratios.

5. Conclusion

Although the performance of portfolio insurance strategies has been studied by various academics over the past decades, none of them was able to clarify the effectiveness of portfolio insurance strategies. This study relates to the existing literature by comparing different portfolio insurance strategies by measuring performance, generated by Monte Carlo simulations based on historical data. It goes beyond the existing literature by comparing dynamic portfolio insurance strategies (constant proportion and synthetic put) with a static portfolio insurance strategy (buy-and-hold) with recent post-crisis European data. Hereby, the historical data is based on the volatility and annual returns of the West European stock market from January 2009 to January 2020. Moreover, it not only measures performance by the Sharpe ratio, Calmar ratio and the Sortino ratio, but it also includes the expected shortfall, which is more appropriate for portfolio insurances. The key objective is to know which portfolio insurance strategy is the most effective. Besides clarifying the theoretical results regarding portfolio insurance strategies, this study also contributes to high-net-worth individuals, private banks, and institutional investors by providing them clarification about the best suitable portfolio insurance to use. Therefore, they will be able to adjust their portfolios to their strategies, and with the minimum amount of risk, get the highest result.

After simulating more than 3 billion daily stock prices through Monte Carlo simulations, this study found that it is important to choose the right portfolio strategy for the situation. Although the option-based portfolio insurance intensively adjusted and provides a good floor, it fails to perform better than other strategies in a moderate increasing market. But if the strike price of the option-based portfolio decreases, the standard deviation and the return increases in a moderate increasing market. Moreover, we found that the CPPI outperforms the buy-and-hold portfolio, except when it is monthly rebalanced.

Since the buy-and-hold portfolio does not have a floor value, it has the biggest maximum drawdown and the lowest minimum value, and the portfolio may have a lower return because of this. Among the CPPI portfolios, the portfolio with a multiplier of 1 outperformed the other strategies. When increasing the floor or decreasing the multiplier of the CPPI, this study showed that the standard deviation and return of the portfolios will also increase. Besides that, when rebalancing the CPPI portfolio weekly or monthly, the returns and volatility will decrease.

Although some previous literature contradicts these findings (Annaert et al., 2009; Cesari and Cremonini, 2003), they are in line with Rouhani (1989), Costa and Gaspar (2014), Dichtl et al. (2017), Khuman et al. (2008), and Pézier and Scheller (2011), who stated that the constant proportion portfolio insurance is superior to option-based portfolio insurance in all cases.

One of the limitations of this study is the number of Monte Carlo iterations. For every simulation, 50,000 iterations were used. Despite the number of iterations, the results changed slightly when conducting the same analysis twice. Moreover, in order to improve the effectiveness of portfolio insurance strategies, a dynamic floor could be used (Lee at al., 2009). This means that when stock prices decrease, the floor will be frozen, but when stock prices increase, the floor will be increased as well. At last, another limitation was the lack of multiple variables. Many factors are influencing investment portfolios, like interest rates, deposits and withdrawals, investment horizons, and tail risks. Incorporating these factors into algorithms could be quite challenging, but this could be a challenge for future research.

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Appendices

Appendix A: Analysis of Lyxor Euro STOXX 50 UCITS ETF

This appendix presents an extensive analysis on the Lyxor Euro STOXX 50 UCITS ETF. Since the Lyxor Euro STOXX 50 UCITS ETF describes best the overall European stock markets, it has been used in this study to conduct several Monte Carlo analyses on. The Euro STOXX 50 index is an index of the fifty largest European companies within Europe by market capitalization. Currently, it holds stocks of eleven countries in Europe and is reviewed annually for rebalancing purposes. The table below presents the fifty holdings of the ETF as of 2020, as well as their percentage in the total index and market value in million.

Holdings by market value as of 2020					
Name of holding	% of index	Market Value (MM)	Name of holding	% of index	Market Value (MM)
Asml Holding	6.01	330.67	Kering	1.55	85.35
Sap	5.81	319.42	Banco Santander	1.54	84.53
Sanofi (Sanofi Aventis)	4.51	248.05	EssilorLuxottica	1.50	82.26
Lvmh Louis Vuitton Moet Hen	4.49	246.98	Muenchener Rueckver	1.42	78.03
Linde Plc	4.44	244.36	Safran	1.36	74.85
Total	3.99	219.25	Deutsche Post	1.35	73.98
Siemens	3.55	195.10	Deutsche Boerse	1.32	72.77
Allianz	3.24	178.04	Daimler	1.28	70.47
L’Oréal	3.02	165.77	Intesa Sanpaolo	1.18	64.96
Unilever Nv	2.94	161.39	Royal Ahold Delhaize	1.15	63.31
Bayer Ag	2.86	157.32	Industria De Diseno Textil	1.15	63.29
Air Liquide	2.60	143.04	Volkswagen	1.06	58.52
Enel Spa	2.59	142.28	Crh	1.05	57.95
Iberdrola	2.58	142.04	Ing Groep	1.02	56.25
Schneider Electric	2.33	128.21	Nokia	0.96	52.77
Deutsche Telekom	2.06	113.16	Eni	0.93	51.14
Vinci	1.97	108.39	Orange	0.90	49.32
Basf	1.95	107.24	Engie	0.88	48.58
Adidas	1.87	102.90	Amadeus It Group	0.88	48.20
Danone (Groupe Danone)	1.79	98.62	Banco Bilbao Vizcaya	0.87	47.93
Bnp Paribas	1.73	95.35	Vivendi	0.87	47.65
Axa	1.67	91.88	Telefonica	0.86	47.04
Koninklijke Philips	1.63	89.73	Fresenius Se & Co	0.79	43.34
Anheuser Busch Inbev	1.59	87.45	BMW	0.78	42.85
Airbus	1.56	85.83	Societe Generale	0.54	29.64

Table 8. Holdings by market value as of 2020 (FactSet Research System, n.d.).

Since the recession of 2008 officially ended in June 2009, this study focuses on the post-recession stock market returns 2020 (NBER, 2010). In the figure below, the lognormal returns of the Lyxor Euro STOXX 50 UCITS ETF have been presented from June 2009, until January 2020. Where the price of the ETF was €26.23 at the 1st of June 2009, it has increased with 31.86% to €36.07 at 31 December 2019. The figure below presents the path of the ETF, and thus the index. Although there is a dip at the end of 2011, the index recovers quickly, and it gains momentum until the end of the sample period.

Figure 1. Lyxor Euro STOXX 50 Ucits ETF returns (FactSet Research System, n.d.).

Index ETFs can be bought and sold throughout the day on several stock exchanges since the index ETFs try to track an index as closely as possible. With this index tracker, an investor can get exposure to a diversified portfolio of fifty companies with only one transaction. Therefore, trading in index trackers is much cheaper than performing each transaction separately. The table below presents the comparison of the Euro STOXX index 50 with the Lyxor Euro STOXX 50 UCITS ETF. Hereby, we see that the ETF tracks the index as closely as possible, but there are some minor differences in the paths. Also, the figure shows the trading volume of the Lyxor Euro STOXX 50 UCITS ETF, whereby the trading volume increases when the volatility increases. Moreover, there is a decrease in trading volume at the end of the sample period.

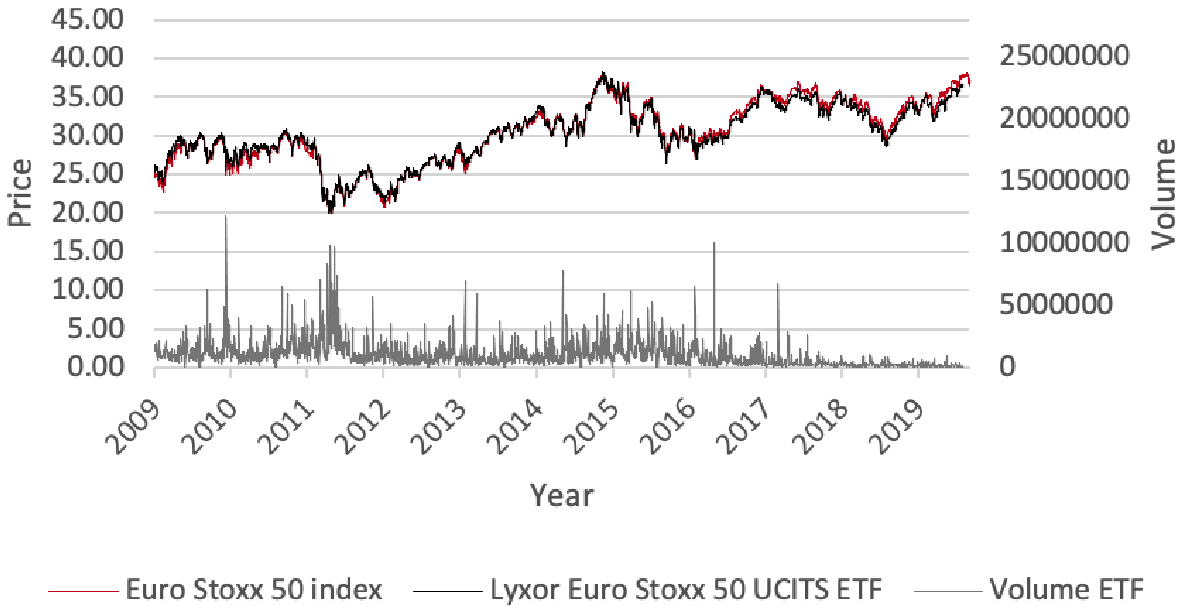

Figure 2. Overview Lyxor Euro STOXX 50 UCITS ETF and Euro STOXX 50 index (FactSet Research System, n.d.).

As showed in the previous table with market holdings as of 2020, the Euro STOXX 50 index is well diversified among several industries. The figure below shows a graphical representation of the revenue exposure per industry. It shows that the financial, consumer non-cyclicals and industrials has the most revenue exposure of the Euro STOXX 50 index. This means that if there is a fluctuation in stock prices of companies in the financial sector, the Euro STOXX 50 index will fluctuate a lot more than with a fluctuation in the stock prices of companies in the telecommunications sector.

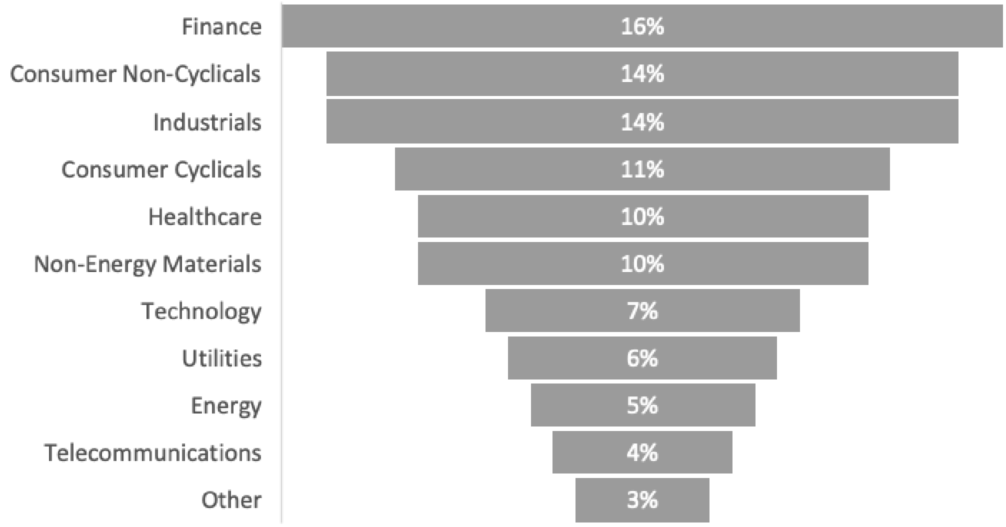

Figure 3. Revenue exposure by industry (FactSet Research System, n.d.).

Furthermore, the revenue exposure could also be measured by geography and by type of economy. The geographic revenue exposure displays the revenue of the Lyxor Euro STOXX 50 UCITS ETF that is impacted by geopolitical, macroeconomic and market risk of certain regions. Although the index contains only European countries, there is a revenue exposure of 30% to America. This is due, among other things, to globalization and the exposure to foreign markets of these European companies (Leijonhufvud, 2007). Moreover, the second figure displays that although most of the revenue exposure is located in developed economies, there is still 25% of the revenue exposure to emerging markets like China, India, Brazil and Russia.

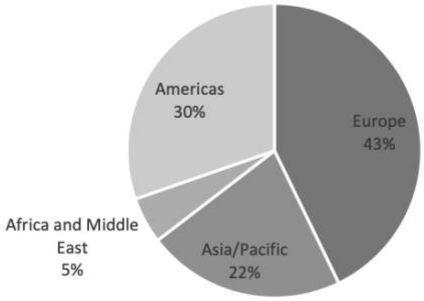

Figure 4. Revenue exposure by geographic revenue

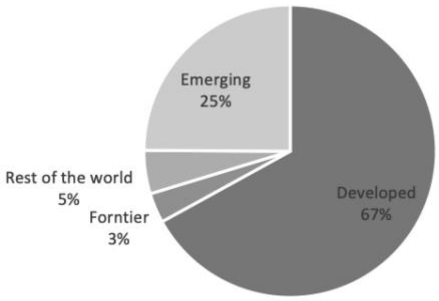

Figure 5. Revenue exposure by economy

At last, since the Lyxor Euro STOXX 50 UCITS ETF is very diversified among fifty companies, the shareholder activism of the fifteen biggest shareholders is overall very low. This is mainly due to the fact that it is very diversified, and most shareholder activism happens with a certain company or industry, but it is less likely with an index (Hadani, Goranova, & Khan, 2011).

Type	% of shares	Market Value (MM)	Activism
Total	7.84%	436.03	-
Bankia Fondos SGIIC SA	2.68%	149.20	Very Low
SIGNAL IDUNA Asset Management GmbH	1.99%	110.43	Very Low
Mediolanum Gestione Fondi SGRpA	1.46%	80.95	Very Low
Deka Investment GmbH	0.25%	13.78	Low
Santander Private Banking Gestion SA SGIIC	0.20%	11.09	Very Low
Mapfre Asset Management SGIIC SA	0.19%	10.32	Very Low
Haas Gestion SAS	0.14%	7.98	Very Low
Rothschild Asset Management SCS	0.14%	7.57	Very Low
Habbel, Pohlig und Partner - Vermögensverwaltung	0.14%	7.53	Very Low
PROMEPAR Asset Management SA	0.13%	7.01	Very Low
Generali Investments LLC	0.12%	6.70	Very Low
Bankinter Gestion de Activos SA SGIIC	0.11%	6.34	Very Low
La Francaise Asset Management SAS	0.11%	6.22	Very Low
Monega Kapitalanlagegesellschaft mbH	0.10%	5.65	Very Low
Compagnie de Gestion Privee Monegasque SAM	0.09%	5.27	Very Low

Table 9. Top 15 stakeholders of the Lyxor Euro STOXX 50 UCITS ETF (FactSet Research System, n.d.).

Appendix B: Results of buy-and-hold portfolio

This appendix presents the results of the Monte Carlo simulations of the buy-and-hold portfolio. In total, 50,000 simulations were conducted over 2,712 days, which means that the figure below contains 135.6 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean and the purple line explains the median of the buy-and-hold portfolio.

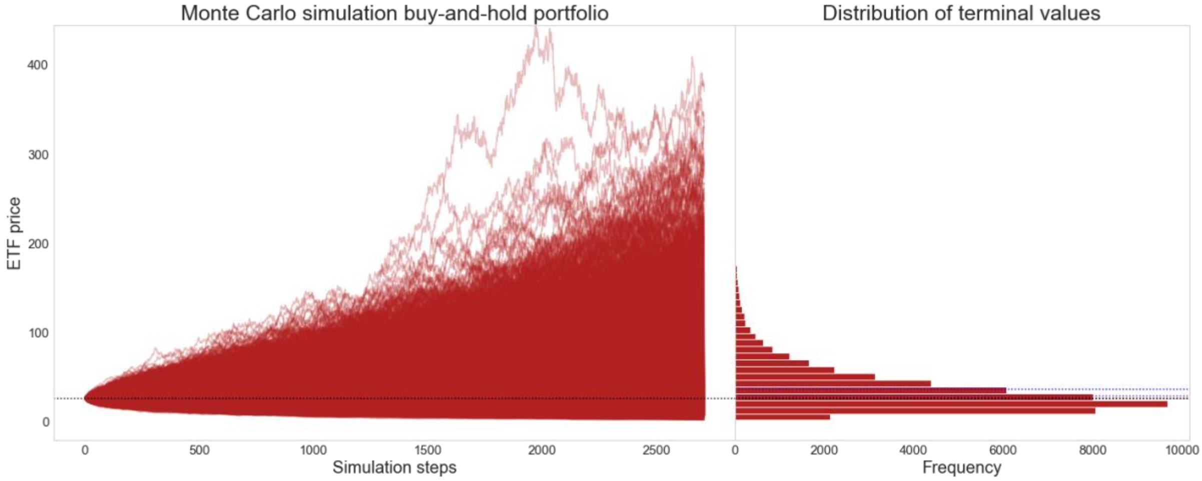

Figure 6. Monte Carlo simulation of the buy-and-hold portfolio.

The table below displays the relevant statistics of the Monte Carlo simulation of the buy-and-hold portfolio. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the buy-and-hold portfolio results	
Simulations	50,000
Annual standard deviation	20.27%
Variance	4.11%
Annual return	3.01%
Total return	31.86%
Mean price	€ 36.07
Median price	€ 27.12
Mode	€ 21.30
Minimum value	€ 1.70
Maximum value	€ 377.81
Maximum drawdown	(0.94%)
Skewness	2.51
Kurtosis	14.64
Downside deviation	38.06%
Upper limit (95% confidence)	€ 36.31
Lower limit (95% confidence)	€ 35.84
Value at Risk (95% confidence)	(30.33%)
Expected shortfall (95% confidence)	(38.80%)
Sharpe ratio	0.1068
Sortino ratio	0.0583
Calmar ratio	(0.0230)

Table 10. Monte Carlo simulation of the buy-and-hold portfolio results.

Appendix C: Results of constant proportion portfolio

This appendix presents the results of the Monte Carlo simulations of the daily rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.8. In total, 50,000 simulations were conducted over 2,712 days, which means that the figure below contains 135.6 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

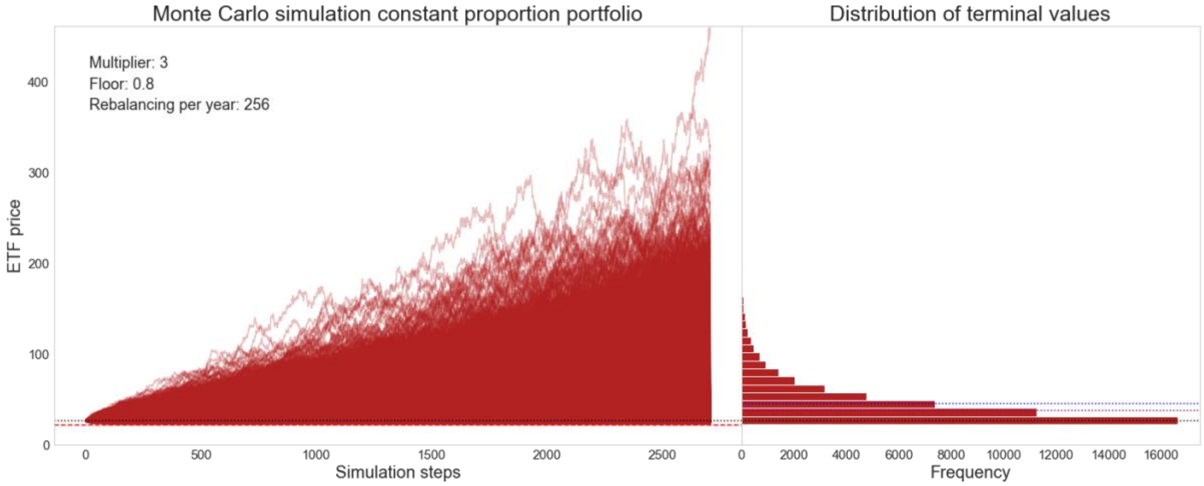

Figure 7. Monte Carlo simulation of the constant proportion portfolio.

The table below displays the relevant statistics of the Monte Carlo simulation of the daily rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.8. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 3; floor = 0.8; rebalancing per year = 256*	
Simulations	50,000
Annual standard deviation	17.13%
Variance	2.93%
Annual return	5.12%
Total return	54.14%
Mean price	€ 45.07
Median price	€ 30.51
Mode	€ 60.29
Minimum value	€ 22.32
Maximum value	€ 462.02
Maximum drawdown	(78.15%)
Skewness	2.73
Kurtosis	16.00
Downside deviation	3.19%
Upper limit (95% confidence)	€ 45.28
Lower limit (95% confidence)	€ 44.86
Value at Risk (95% confidence)	(23.06%)
Expected shortfall (95% confidence)	(30.22%)
Sharpe ratio	0.2492
Sortino ratio	0.6956
Calmar ratio	(0.0546)

Table 11. Monte Carlo simulation of the constant proportion portfolio.

Appendix D: Results of constant proportion portfolio (M = 1)

This appendix presents the results of the Monte Carlo simulations of the daily rebalanced constant proportion portfolio with a multiplier of 1 and a floor of 0.8. In total, 50,000 simulations were conducted over 2,712 days, which means that the figure below contains 135.6 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

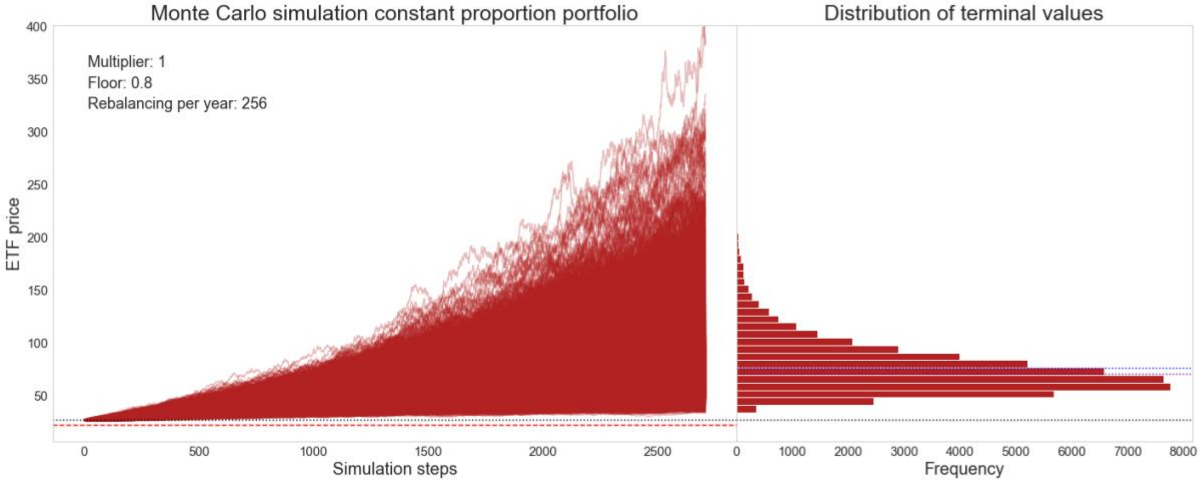

Figure 8. Monte Carlo simulation of the constant proportion portfolio (M=1).

The table below displays the relevant statistics of the Monte Carlo simulation of the daily rebalanced constant proportion portfolio with a multiplier of 1 and a floor of 0.8. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 1; floor = 0.8; rebalancing per year = 256*	
Simulations	50,000
Annual standard deviation	11.02%
Variance	1.21%
Annual return	9.98%
Total return	105.62%
Mean price	€ 75.42
Median price	€ 45.61
Mode	€ 62.34
Minimum value	€ 33.42
Maximum value	€ 388.61
Maximum drawdown	(55.01%)
Skewness	1.87
Kurtosis	10.22
Downside deviation	-
Upper limit (95% confidence)	€ 75.63
Lower limit (95% confidence)	€ 75.20
Value at Risk (95% confidence)	(8.14%)
Expected shortfall (95% confidence)	(12.74%)
Sharpe ratio	0.8291
Sortino ratio	-
Calmar ratio	(0.1660)

Table 12. Monte Carlo simulation of the constant proportion portfolio (M=1).

Appendix E: Results of constant proportion portfolio (M = 5)

This appendix presents the results of the Monte Carlo simulations of the daily rebalanced constant proportion portfolio with a multiplier of 5 and a floor of 0.8. In total, 50,000 simulations were conducted over 2,712 days, which means that the figure below contains 135.6 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

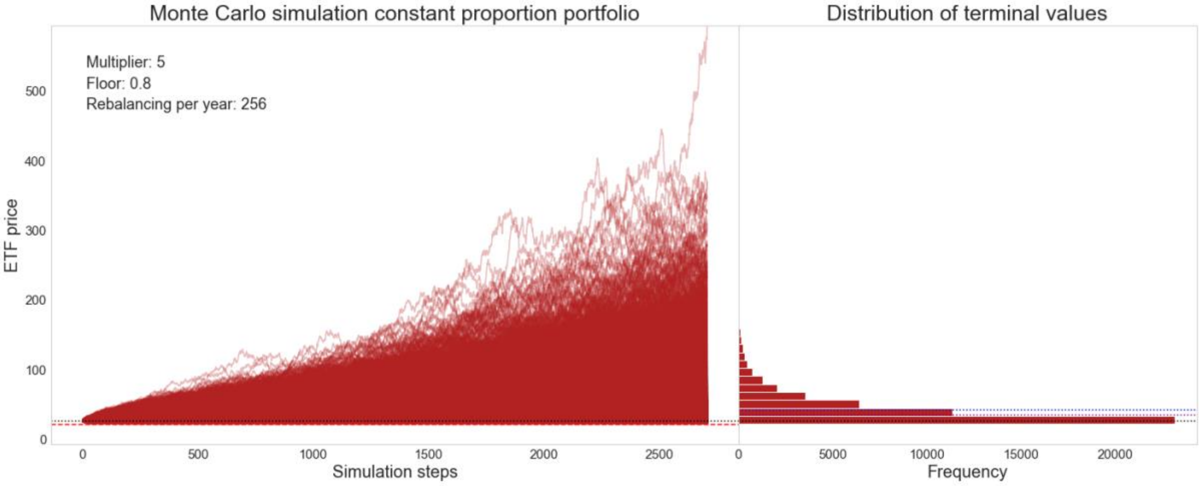

Figure 9. Monte Carlo simulation of the constant proportion portfolio (M=5).

The table below displays the relevant statistics of the Monte Carlo simulation of the daily rebalanced constant proportion portfolio with a multiplier of 5 and a floor of 0.8. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 5; floor = 0.8; rebalancing per year = 256*	
Simulations	50,000
Annual standard deviation	17.94%
Variance	3.22%
Annual return	4.48%
Total return	47.44%
Mean price	€ 42.15
Median price	€ 28.96
Mode	€ 62.83
Minimum value	€ 21.52
Maximum value	€ 594.21
Maximum drawdown	(76.76%)
Skewness	3.26
Kurtosis	24.57
Downside deviation	4.61%
Upper limit (95% confidence)	€ 42.37
Lower limit (95% confidence)	€ 41.94
Value at Risk (95% confidence)	(25.02%)
Expected shortfall (95% confidence)	(32.51%)
Sharpe ratio	0.2028
Sortino ratio	0.4814
Calmar ratio	(0.0474)

Table 13. Monte Carlo simulation of the constant proportion portfolio (M=5).

Appendix F: Results of constant proportion portfolio (floor = 0.9)

This appendix presents the results of the Monte Carlo simulations of the daily rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.9. In total, 50,000 simulations were conducted over 2,712 days, which means that the figure below contains 135.6 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

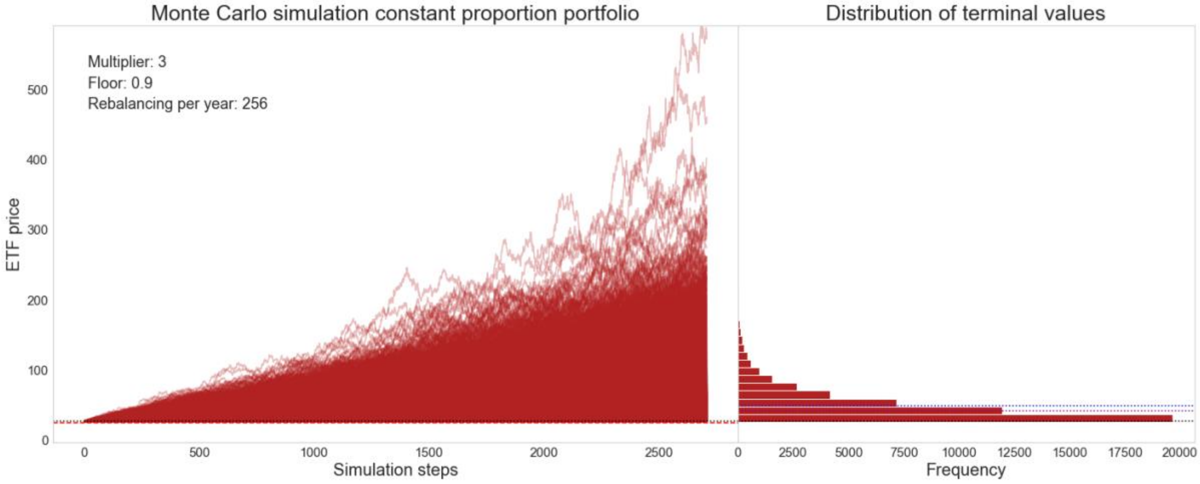

Figure 10. Monte Carlo simulation of the constant proportion portfolio (floor = 0.9)

The table below displays the relevant statistics of the Monte Carlo simulation of the daily rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.9. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 3; floor = 0.9; rebalancing per year = 256*	
Simulations	50,000
Annual standard deviation	16.35%
Variance	2.67%
Annual return	5.85%
Total return	61.91%
Mean price	€ 48.72
Median price	€ 32.86
Mode	€ 29.75
Minimum value	€ 25.22
Maximum value	€ 582.03
Maximum drawdown	(77.36%)
Skewness	3.00
Kurtosis	21.60
Downside deviation	0.85%
Upper limit (95% confidence)	€ 48.94
Lower limit (95% confidence)	€ 48.50
Value at Risk (95% confidence)	(21.04%)
Expected shortfall (95% confidence)	(27.88%)
Sharpe ratio	0.3060
Sortino ratio	2.6136
Calmar ratio	(0.0647)

Table 14. Monte Carlo simulation of the constant proportion (floor = 0.9)

Appendix G: Results of constant proportion portfolio (floor = 1.0)

This appendix presents the results of the Monte Carlo simulations of the daily rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 1.0. In total, 50,000 simulations were conducted over 2,712 days, which means that the figure below contains 135.6 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

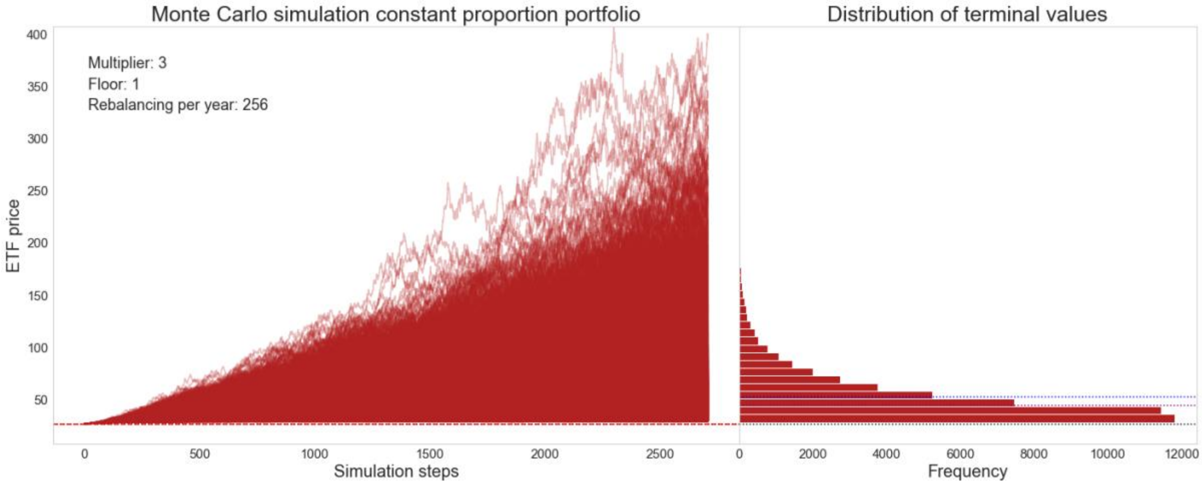

Figure 11. Monte Carlo simulation of the constant proportion (floor = 1.0)

The table below displays the relevant statistics of the Monte Carlo simulation of the daily rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 1.0. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 3; floor = 1.0; rebalancing per year = 256*	
Simulations	50,000
Annual standard deviation	15.70%
Variance	2.47%
Annual return	6.53%
Total return	69.13%
Mean price	€ 52.37
Median price	€ 35.52
Mode	€ 46.86
Minimum value	€ 27.82
Maximum value	€ 400.56
Maximum drawdown	(75.63%)
Skewness	2.72
Kurtosis	15.99
Downside deviation	-
Upper limit (95% confidence)	€ 52.59
Lower limit (95% confidence)	€ 52.15
Value at Risk (95% confidence)	(19.29%)
Expected shortfall (95% confidence)	(25.85%)
Sharpe ratio	0.3622
Sortino ratio	-
Calmar ratio	(0.0752)

Table 15. Monte Carlo simulation of the constant proportion (floor = 1.0).

Appendix H: Results of constant proportion portfolio (weekly rebalancing)

This appendix presents the results of the Monte Carlo simulations of the weekly rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.8. In total, 50,000 simulations were conducted over 551 weeks, which means that the figure below contains 27.55 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

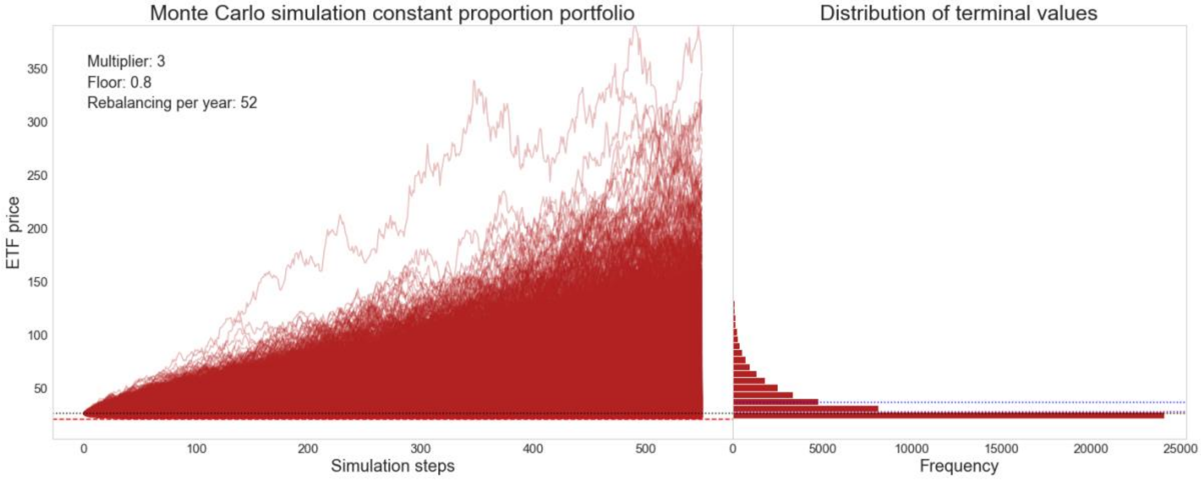

Figure 12. Monte Carlo simulation of the constant proportion portfolio (weekly rebalancing).

The table below displays the relevant statistics of the Monte Carlo simulation of the weekly rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.8. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 3; floor = 0.8; rebalancing per year = 52*	
Simulations	50,000
Annual standard deviation	6.65%
Variance	0.44%
Annual return	3.24%
Total return	34.28%
Mean price	€ 36.96
Median price	€ 26.61
Mode	€ 29.08
Minimum value	€ 21.23
Maximum value	€ 347.39
Maximum drawdown	(77.07%)
Skewness	3.24
Kurtosis	20.31
Downside deviation	4.99%
Upper limit (95% confidence)	€ 37.14
Lower limit (95% confidence)	€ 36.77
Value at Risk (95% confidence)	(7.70%)
Expected shortfall (95% confidence)	(10.48%)
Sharpe ratio	0.3598
Sortino ratio	0.4445
Calmar ratio	(0.0311)

Table 16. Monte Carlo simulation of the constant proportion portfolio (weekly rebalancing).

Appendix I: Results of constant proportion portfolio (monthly rebalancing)

This appendix presents the results of the Monte Carlo simulations of the monthly rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.8. In total, 50,000 simulations were conducted over 128 months, which means that the figure below contains 6.4 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the constant proportion portfolio and the red line explains the floor times the initial stock price of (€26.23).

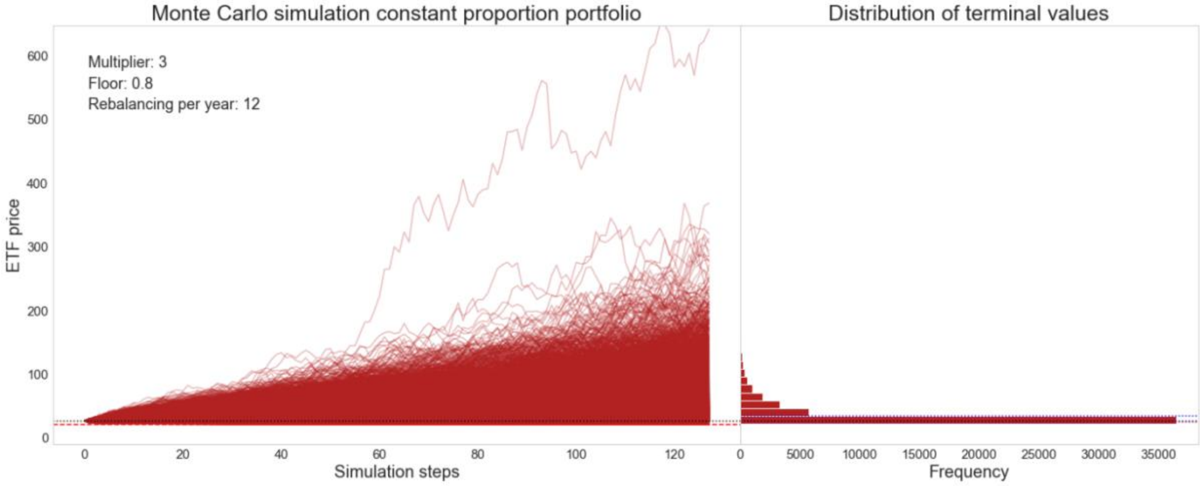

Figure 13. Monte Carlo simulation of the constant proportion portfolio (monthly rebalancing).

The table below displays the relevant statistics of the Monte Carlo simulation of the monthly rebalanced constant proportion portfolio with a multiplier of 3 and a floor of 0.8. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different portfolio insurance strategies.

Monte Carlo simulation of the constant proportion portfolio *Multiplier = 3; floor = 0.8; rebalancing per year = 12*	
Simulations	50,000
Annual standard deviation	12.00%
Variance	1.44%
Annual return	2.22%
Total return	23.48%
Mean price	€ 33.17
Median price	€ 25.30
Mode	€ 26.02
Minimum value	€ 21.09
Maximum value	€ 641.50
Maximum drawdown	(77.93%)
Skewness	4.09
Kurtosis	38.11
Downside deviation	5.61%
Upper limit (95% confidence)	€ 33.36
Lower limit (95% confidence)	€ 32.99
Value at Risk (95% confidence)	(17.53%)
Expected shortfall (95% confidence)	(22.54%)
Sharpe ratio	0.1144
Sortino ratio	0.3959
Calmar ratio	(0.0176)

Table 17. Monte Carlo simulation of the constant proportion portfolio (monthly rebalancing).

Appendix J: Results of option-based portfolio

This appendix presents the results of the Monte Carlo simulations of the quarterly rebalanced option-based portfolio with a strike price of €25. In total, 50,000 simulations were conducted over 43 periods of 3 months, which means that the figure below contains 2.15 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the option-based portfolio and the red line explains the floor times the initial stock price of (€26.23).

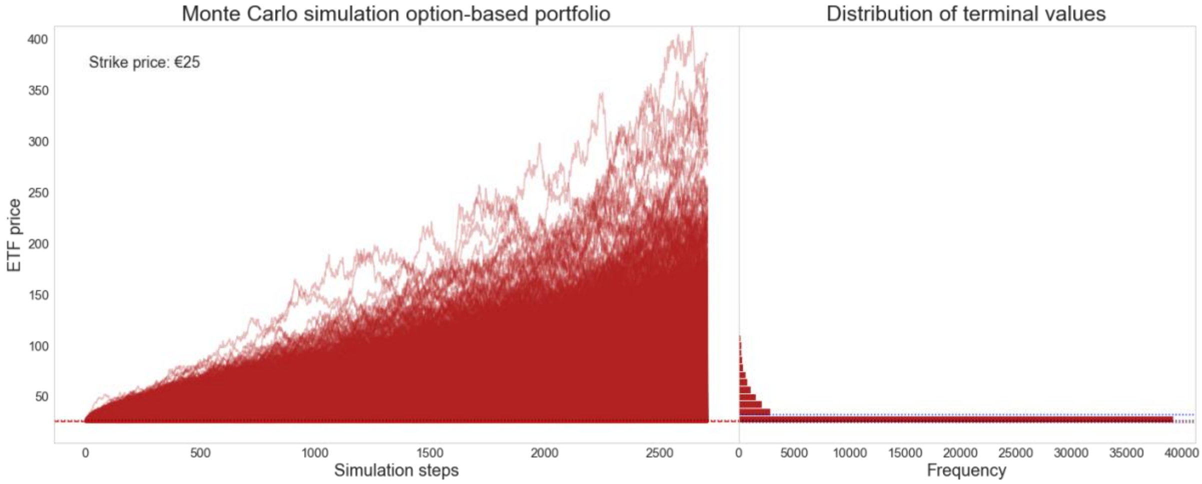

Figure 14. Monte Carlo simulation of the option-based portfolio.

The table below displays the relevant statistics of the Monte Carlo simulation of the quarterly rebalanced option-based portfolio with a strike price of €25. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different strategies.

Monte Carlo simulation of the option-based portfolio *Strike price = €25; rebalancing per year = 4*	
Simulations	50,000
Annual standard deviation	9.41%
Variance	0.88%
Annual return	1,82%
Total return	19.22%
Mean price	€ 31.79
Median price	€ 24.53
Mode	€ 24.47
Minimum value	€ 24.17
Maximum value	€ 385.11
Maximum drawdown	(78.52%)
Skewness	4.66
Kurtosis	36.84
Downside deviation	0.86%
Upper limit (95% confidence)	€ 31.95
Lower limit (95% confidence)	€ 31.62
Value at Risk (95% confidence)	(13.66%)
Expected shortfall (95% confidence)	(17.59%)
Sharpe ratio	0.1031
Sortino ratio	2.5895
Calmar ratio	(0.0123)

Table 18. Monte Carlo simulation of the option-based portfolio.

Appendix K: Results of option-based portfolio (strike price = €22.5)

This appendix presents the results of the Monte Carlo simulations of the quarterly rebalanced option-based portfolio with a strike price of €22.5. In total, 50,000 simulations were conducted over 43 periods of 3 months, which means that the figure below contains 2.15 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the option-based portfolio and the red line explains the strike price.

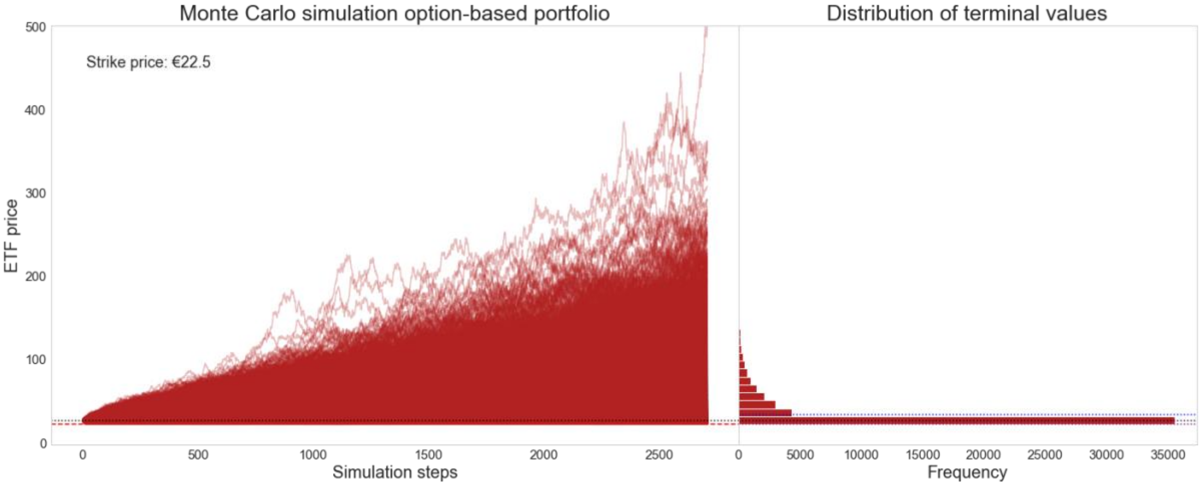

Figure 15. Monte Carlo simulation of the option-based portfolio (strike price = €22.5).

The table below displays the relevant statistics of the Monte Carlo simulation of the quarterly rebalanced option-based portfolio with a strike price of €22.5. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different strategies.

Monte Carlo simulation of the option-based portfolio *Strike price = €22.5; rebalancing per year = 4*	
Simulations	50,000
Annual standard deviation	12.70%
Variance	1.61%
Annual return	2.27%
Total return	23.98%
Mean price	€ 33.34
Median price	€ 22.25
Mode	€ 22.00
Minimum value	€ 22.77
Maximum value	€ 498.88
Maximum drawdown	(79.22%)
Skewness	3.65
Kurtosis	24.31
Downside deviation	2.81%
Upper limit (95% confidence)	€ 33.54
Lower limit (95% confidence)	€ 33.13
Value at Risk (95% confidence)	(18.63%)
Expected shortfall (95% confidence)	(23.93%)
Sharpe ratio	0.1117
Sortino ratio	0.7904
Calmar ratio	(0.0179)

Table 19. Monte Carlo simulation of the option-based portfolio (strike price = €22.5).

Appendix L: Results of option-based portfolio (strike price = €20)

This appendix presents the results of the Monte Carlo simulations of the quarterly rebalanced option-based portfolio with a strike price of €20. In total, 50,000 simulations were conducted over 43 periods of 3 months, which means that the figure below contains 2.15 million simulated stock prices. On the left side of the figure, a graph is displayed with an increase or decrease in the price over the days (simulation steps). On the right side, the distribution of terminal values has been displayed. In the figure, the black line explains the stock price on the 1st of June 2009 (€26.23), the blue line explains the overall mean, the purple line explains the median of the option-based portfolio and the red line explains the strike price.

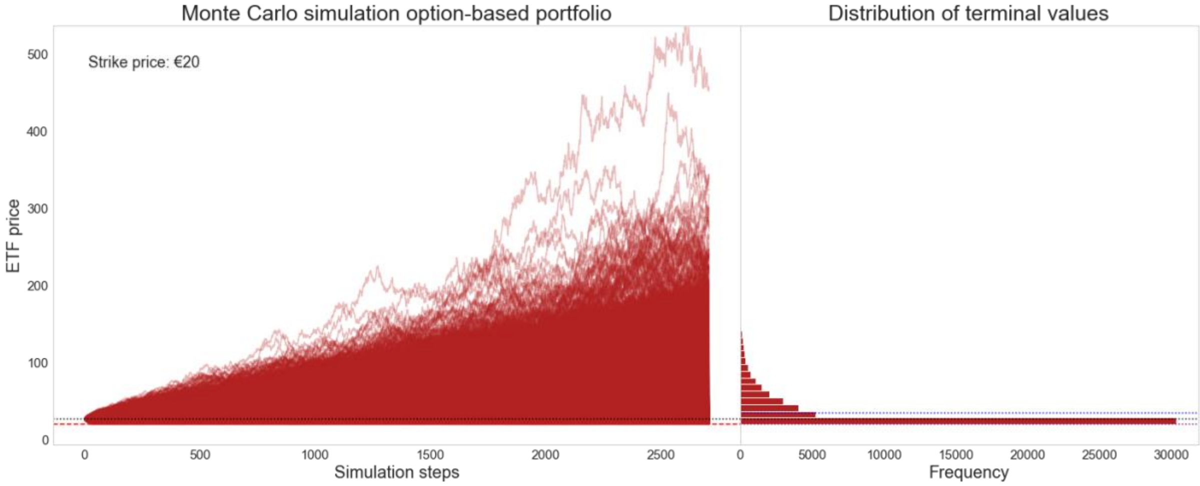

Figure 16. Monte Carlo simulation of the option-based portfolio (strike price = €20).

The table below displays the relevant statistics of the Monte Carlo simulation of the quarterly rebalanced option-based portfolio with a strike price of €20 and a floor of 0.8. In total, 50,000 simulations were conducted. Among other statistics, the upper and lower limit of the price at the end of the period are given with a 95% confidence interval. Moreover, both the value at risk and the expected shortfall are also conducted with a 95% confidence interval. In the end, the Sharpe ratio, Sortino ratio, and Calmar ratio are given, which are relevant ratios used to compare different strategies.

Monte Carlo simulation of the option-based portfolio *Strike price = €20; rebalancing per year = 4*	
Simulations	50,000
Annual standard deviation	15.16%
Variance	2.30%
Annual return	2.51%
Total return	26.61%
Mean price	€ 34.23
Median price	€ 25.16
Mode	€ 19.49
Minimum value	€ 19.36
Maximum value	€ 453.45
Maximum drawdown	(81.04%)
Skewness	2.97
Kurtosis	17.27
Downside deviation	6.41%
Upper limit (95% confidence)	€ 34.44
Lower limit (95% confidence)	€ 34.01
Value at Risk (95% confidence)	(22.41%)
Expected shortfall (95% confidence)	(28.75%)
Sharpe ratio	0.1100
Sortino ratio	0.3466
Calmar ratio	(0.0206)

Table 20. Monte Carlo simulation of the option-based portfolio (strike price = €20).

Appendix M: Overview results of all portfolio insurance strategies

This table summarizes the Monte Carlo simulations displayed in appendices B-N. The table includes the following portfolio insurance strategies: **B&H** (buy-and-hold), **CPPI M1** (daily constant proportion: multiplier = 1, floor = 0.8), **CPPI M3** (daily constant proportion: multiplier = 3, floor = 0.8), **CPPI M5** (daily constant proportion: multiplier = 5, floor = 0.8), **CPPI F09** (daily constant proportion: multiplier = 3, floor = 0.9), **CPPI F1** (daily constant proportion: multiplier = 3, floor = 1), **CPPI W** (weekly rebalanced constant proportion: multiplier = 3, floor = 0.8), **CPPI M** (monthly rebalanced constant proportion: multiplier = 3, floor = 0.8), **OBPI** (quarterly rebalanced option-based: strike price = 25), **OBPI K225** (quarterly rebalanced option-based: strike price = 22.5), and **OBPI K20** (quarterly rebalanced option-based: strike price = 20).

	B&H	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1	CPPI W	CPPI M	OBPI	OBPI K225	OBPI K20
Simulations	50,000	50,000	50,000	50,000	50,000	50,000	50,000	50,000	50,000	50,000	50,000
Annual SD	20.27%	11.02%	17.13%	17.94%	16.35%	15.70%	6.65%	12.00%	9.41%	12.70%	15.16%
Variance	4.11%	1.21%	2.93%	3.22%	2.67%	2.47%	0.44%	1.44%	0.88%	1.61%	2.30%
Annual return	3.01%	9.98%	5.12%	4.48%	5.85%	6.53%	3.24%	2.22%	1,82%	2.27%	2.51%
Total return	31.86%	105.62%	54.14%	47.44%	61.91%	69.13%	34.28%	23.48%	19.22%	23.98%	26.61%
Mean price	€ 36.07	€ 75.42	€ 45.07	€ 42.15	€ 48.72	€ 52.37	€ 36.96	€ 33.17	€ 31.79	€ 33.34	€ 34.23
Median price	€ 27.12	€ 45.61	€ 30.51	€ 28.96	€ 32.86	€ 35.52	€ 26.61	€ 25.30	€ 24.53	€ 22.25	€ 25.16
Mode	€ 21.30	€ 62.34	€ 60.29	€ 62.83	€ 29.75	€ 46.86	€ 29.08	€ 26.02	€ 24.47	€ 22.00	€ 19.49
Minimum value	€ 1.70	€ 33.42	€ 22.32	€ 21.52	€ 25.22	€ 27.82	€ 21.23	€ 21.09	€ 24.17	€ 22.77	€ 19.36
Maximum value	€ 377.81	€ 388.61	€ 462.02	€ 594.21	€ 582.03	€ 400.56	€ 347.39	€ 641.50	€ 385.11	€ 498.88	€ 453.45
Maximum drawdown	(0.94%)	(55.01%)	(78.15%)	(76.76%)	(77.36%)	(75.63%)	(77.07%)	(77.93%)	(78.52%)	(79.22%)	(81.04%)
Skewness	2.51	1.87	2.73	3.26	3.00	2.72	3.24	4.09	4.66	3.65	2.97
Kurtosis	14.64	10.22	16.00	24.57	21.60	15.99	20.31	38.11	36.84	24.31	17.27
Downside deviation	38.06%	-	3.19%	4.61%	0.85%	-	4.99%	5.61%	0.86%	2.81%	6.41%
Upper limit (95% confidence)	€ 36.31	€ 75.63	€ 45.28	€ 42.37	€ 48.94	€ 52.59	€ 37.14	€ 33.36	€ 31.95	€ 33.54	€ 34.44
Lower limit (95% confidence)	€ 35.84	€ 75.20	€ 44.86	€ 41.94	€ 48.50	€ 52.15	€ 36.77	€ 32.99	€ 31.62	€ 33.13	€ 34.01
Value at Risk (95% confidence)	(30.33%)	(8.14%)	(23.06%)	(25.02%)	(21.04%)	(19.29%)	(7.70%)	(17.53%)	(13.66%)	(18.63%)	(22.41%)
Expected shortfall (95% confidence)	(38.80%)	(12.74%)	(30.22%)	(32.51%)	(27.88%)	(25.85%)	(10.48%)	(22.54%)	(17.59%)	(23.93%)	(28.75%)
Sharpe ratio	0.1068	0.8291	0.2492	0.2028	0.3060	0.3622	0.3598	0.1144	0.1031	0.1117	0.1100
Sortino ratio	0.0583	-	0.6956	0.4814	2.6136	-	0.4445	0.3959	2.5895	0.7904	0.3466
Calmar ratio	(0.0230)	(0.1660)	(0.0546)	(0.0474)	(0.0647)	(0.0752)	(0.0311)	(0.0176)	(0.0123)	(0.0179)	(0.0206)

Table 21. Overview results of all portfolio insurance strategies.

Appendix N: Overview of the constant proportion portfolios

This table summarizes the Monte Carlo simulations of the constant proportion portfolio insurance strategy, displayed in appendices C-I. The table includes the following portfolio insurance strategies: **CPPI M1** (daily constant proportion: multiplier = 1, floor = 0.8), **CPPI M3** (daily constant proportion: multiplier = 3, floor = 0.8), **CPPI M5** (daily constant proportion: multiplier = 5, floor = 0.8), **CPPI F09** (daily constant proportion: multiplier = 3, floor = 0.9), **CPPI F1** (daily constant proportion: multiplier = 3, floor = 1), **CPPI W** (weekly rebalanced constant proportion: multiplier = 3, floor = 0.8), and **CPPI M** (monthly rebalanced constant proportion: multiplier = 3, floor = 0.8).

	CPPI M1	CPPI M3	CPPI M5	CPPI F09	CPPI F1	CPPI W	CPPI M
Simulations	50,000	50,000	50,000	50,000	50,000	50,000	50,000
Annual SD	11.02%	17.13%	17.94%	16.35%	15.70%	6.65%	12.00%
Variance	1.21%	2.93%	3.22%	2.67%	2.47%	0.44%	1.44%
Annual return	9.98%	5.12%	4.48%	5.85%	6.53%	3.24%	2.22%
Total return	105.62%	54.14%	47.44%	61.91%	69.13%	34.28%	23.48%
Mean price	€ 75.42	€ 45.07	€ 42.15	€ 48.72	€ 52.37	€ 36.96	€ 33.17
Median price	€ 45.61	€ 30.51	€ 28.96	€ 32.86	€ 35.52	€ 26.61	€ 25.30
Mode	€ 62.34	€ 60.29	€ 62.83	€ 29.75	€ 46.86	€ 29.08	€ 26.02
Minimum value	€ 33.42	€ 22.32	€ 21.52	€ 25.22	€ 27.82	€ 21.23	€ 21.09
Maximum value	€ 388.61	€ 462.02	€ 594.21	€ 582.03	€ 400.56	€ 347.39	€ 641.50
Maximum drawdown	(55.01%)	(78.15%)	(76.76%)	(77.36%)	(75.63%)	(77.07%)	(77.93%)
Skewness	1.87	2.73	3.26	3.00	2.72	3.24	4.09
Kurtosis	10.22	16.00	24.57	21.60	15.99	20.31	38.12
Downside deviation	-	3.19%	4.61%	0.85%	-	4.99%	5.61%
Upper limit (95%)	€ 75.63	€ 45.28	€ 42.37	€ 48.94	€ 52.59	€ 37.14	€ 33.36
Lower limit (95%)	€ 75.20	€ 44.86	€ 41.94	€ 48.50	€ 52.15	€ 36.77	€ 32.99
Value at Risk (95%)	(8.14%)	(23.06%)	(25.02%)	(21.04%)	(19.29%)	(7.70%)	(17.53%)
Expected shortfall (95%)	(12.74%)	(30.22%)	(32.51%)	(27.88%)	(25.85%)	(10.48%)	(22.54%)
Sharpe ratio	0.8291	0.2492	0.2028	0.3060	0.3622	0.3598	0.1144
Sortino ratio	-	0.6956	0.4814	2.6136	-	0.4445	0.3959
Calmar ratio	(0.1660)	(0.0546)	(0.0474)	(0.0647)	(0.0751)	(0.0311)	(0.0176)

Table 22. Overview of the constant proportion portfolios.

Appendix O: Overview of the option-based portfolios

This table summarizes the Monte Carlo simulations of the option-based portfolio insurance strategy, displayed in appendices J-N. The table includes the following portfolio insurance strategies: **OBPI** (quarterly rebalanced option-based: strike price = 25), **OBPI K225** (quarterly rebalanced option-based: strike price = 22.5), and **OBPI K20** (quarterly rebalanced option-based: strike price = 20).

	OBPI	OBPI K225	OBPI K20
Simulations	50,000	50,000	50,000
Annual SD	9.41%	12.70%	15.16%
Variance	0.88%	1.61%	2.30%
Annual return	1,82%	2.27%	2.51%
Total return	19.22%	23.98%	26.61%
Mean price	€ 31.79	€ 33.34	€ 34.23
Median price	€ 24.53	€ 22.25	€ 25.16
Mode	€ 24.47	€ 22.00	€ 19.49
Minimum value	€ 24.17	€ 22.77	€ 19.36
Maximum value	€ 385.11	€ 498.88	€ 453.45
Maximum drawdown	(78.52%)	(79.22%)	(81.04%)
Skewness	4.66	3.65	2.97
Kurtosis	36.84	24.31	17.27
Downside deviation	0.86%	2.81%	6.41%
Upper limit (95% confidence)	€ 31.95	€ 33.54	€ 34.44
Lower limit (95% confidence)	€ 31.62	€ 33.13	€ 34.01
Value at Risk (95% confidence)	(13.66%)	(18.63%)	(22.41%)
Expected shortfall (95% confidence)	(17.59%)	(23.93%)	(28.75%)
Sharpe ratio	0.1031	0.1117	0.1100
Sortino ratio	2.5895	0.7904	0.3466
Calmar ratio	(0.0123)	(0.0179)	(0.0206)

Table 23. Overview of the option-based portfolios.