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The Ukraine war and European and United States stock market volatility.

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Abstract

This study examines the relationship between the Russia-Ukraine war and volatility in the United States and European financial markets. On aggregate, on the one hand, the European stock market index volatility is more heavily influenced by events or news regarding the war. On the other hand, for the United States index, baseline volatility increased after the start of the war. The US index volatility experienced a limited to negligible influence on events or news regarding the war. While investors' volatility expectations are largely shared between the United States and Europe, the war only affected the expectations of European investors, although this effect is not pronounced. Results have implications for risk managers, policymakers, and investors, among others. Results can furthermore assist in forming volatility expectations during future wars.

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1 Introduction

Besides generating death and destruction, the recent Russia-Ukraine war has instilled uncertainty in financial markets around the world. The Russian invasion of Ukrainian territory resulted from years of rising tensions between Russia and Ukraine. Following the Russian invasion, multiple countries and firms – mostly from Europe, the United Kingdom, and the United States – have imposed sizeable sanctions on Russia (Funakoshi et al., 2022). The purpose of these sanctions is clear: hurting Russia financially and economically such that the war cannot be sustained. Expectedly, the strategy has proven to be a double-edged sword; the sanctions and the following Russian responses significantly and adversely affected the countries that imposed them (Qureshi et al., 2022). Other countries are affected similarly, indicated by declines in worldwide stock market indices (Boungou & Yatié, 2022), these effects are found to be heterogeneous between countries (Boubaker et al., 2022).

Most research in finance regarding the Ukraine war has been dedicated to studying how stock market returns responded to the war. Apart from research such as that of Lo et al. (2022) stock market volatility remains a relatively unexplored topic. This paper aims to fill that gap by studying the relationship between the Ukraine war and stock market volatility in the United States (US) and Europe (EU). The topic deserves investigation for a multitude of reasons.

In the case of war, Yadav (1992) provides multiple inefficiencies that may accompany high volatility. First, high volatility could imply there is an inefficient allocation of resources. Second, high volatility can increase the required risk premium on stocks and possibly result in reduced investor confidence and flow of capital away from financial markets. Third, high volatility tends to increase the risks for market makers, increasing what they charge for their liquidity services. Such inefficiencies and the effects of war on stock market volatility have implications for risk managers and the pricing of options.

It is unclear in what direction wars affect volatility, however. Intuitively, if investors become more hesitant to invest – for example, because bans on Russian imports affect industries varyingly – this would increase volatility by lowering the number of buyers of assets in the financial market. On the other hand, as demonstrated by Schwert (1989), historically, the United States puzzlingly experienced lower or stable stock market volatility during periods of conflict it was involved in. Schwert hypothesizes this is due to investors' expectations of the war having short-term effects only, and hence not sufficiently changing their behavior. Thus, during wars, the level of volatility should reflect how long investors think a war will last. The current paper on volatility during the Ukraine war helps us understand such expectations and overall sentiment.

The effects of the Ukraine war on countries come in different magnitudes. Markets are highly interconnected in the world economy. Especially net oil importers are likely to experience adverse effects from the war (Lo et al., 2022), given that Russia is one of the world's largest oil exporters. After the start of the Ukraine war, the United States stopped importing Russian oil, as did Europe. Especially Europe has been heavily dependent on Russian oil (Paillard, 2010). While the US was relatively less dependent, both are likely to experience adverse effects from the war in terms of stock market returns (Boungou and Yatié, 2022) or volatility (Lo et al., 2022). Furthermore, these effects are found to be stronger for countries bordering Ukraine and Russia, and UN member states that demanded the Russian offensive ended (Boubaker et al., 2022). If wars affect countries to varying extents, expectations of investors concerning the Ukraine war are bound to vary as well. Equivalently, effects on stock market volatility should also vary between countries. As Europe and the United States were large oil importers and some of the main imposers of sanctions on Russia, the effects on volatility in their respective financial markets are expectedly pronounced. Hence the relevance to study these regions.

This paper aims to study the relationship between the Ukraine war and United States and European stock market volatility between September 3, 2021, and August 18, 2022, through OLS regressions. To start, the study of Boungou and Yatié (2022) is replicated to study the war's effect on US and EU stock market index returns. Following, this study examines the relationship between the Ukraine war and three distinct volatility measures: intra-day volatility, which are index price movements on a specific day; between-day – or daily – volatility, which captures index price movements between days; and implied volatility, which captures volatility expectations of investors. Worldwide online search volumes related to the Ukraine war are converted to a rating used as a proxy for the intensity of attention to the war. Attention to the war can drive investors to buy or sell stock, affecting stock market volatility. The relation between this proxy and the specified volatility measures is studied. This approach is similar to that of Boungou and Yatié (2022). Controls are added to account for past volatility and the effect of market shocks and the leverage effect.

Results show no effect of the Ukraine war on US and EU returns. Furthermore, attention to the war significantly affected within-day volatility for US and EU. After the war began, only US experienced a higher base level within-day volatility. Meanwhile, the effect of war attention intensity became negligible. Moreover, the intensity of attention to the war did not affect US between-day volatility, before or after the war started. While base EU between-day volatility remained stable after the war started, the effect of war attention intensity was strong before the start of the war and remained so afterward. Finally, US volatility expectations were not affected by the Ukraine war, while effects on EU volatility expectations were limited and remained unchanged when the war started.

This paper is structured as follows: section two presents an overview and discussion of literature and theory concerning this paper's research objective, followed by hypotheses in section three. Section four describes the data and methodology used in this paper. Section five shows the results, followed by robustness checks in section six. Finally, a concise summary and discussion are presented in section seven.

2 Literature Review

2.1 Stock markets during wars

Literature has extensively studied past armed conflicts. In a highly influential paper, Schwert (1989), found that, counter-intuitively, US stock volatility is lower during periods of conflict. It is argued that this is caused by unchanged investor behavior, under the expectation that the war will be short-term. Cortes et al. (2022) have similar findings for World Wars One and Two. They hypothesize that this is because firm profits are easier to forecast during wartime, because of sharply increased government spending. Contradictory, Choudhry (1997) finds significant short-run increases in volatility in various stock markets of several European countries and the United States during the Second World War. Schneider et al. (2006) find negative and sporadically positive stock market reactions to war. They suggest these reactions signal what traders think the outcome of the war will be. The differences between studies make it difficult to predict what the effects of war are on stock market volatility. Furthermore, no wars are equal, and no countries are affected equally (Choudhry, 1997; Boubaker et al., 2022; Lo et al., 2022). This highlights the need for a comprehensive approach that models the relationship from multiple angles.

Concerning stock prices, Brune et al. (2015) studied stock market reactions to large conflicts since the Second World War. They found stock prices to decrease when the chance for war increases. Yet, when war erupts, they find stock prices to increase. They note an exception for eruptions of war that occur surprisingly, this tends to decrease stock prices. What can be deduced is that there may be an anticipation effect at play. Furthermore, given that stock prices increase after the eruption of war, financial markets seemingly overreact to the news of a potential war. Possibly, both effects apply to stock market volatility as well. Both insights are therefore highly relevant and are to be accounted for.

Proceeding World War Two, Europe experienced a relatively peaceful period in terms of large-scale wars and conflicts. However, just like times of economic prosperity, times of peace never last forever. Fast forward to late 2021, the world experiences a new war boiling in Ukraine. An abstract of the work of Walker (2023) summarises the development of the conflict. On 10 November, reports surfaced of unusual movement of Russian troops near the borders of Ukraine. In the following days, Ukraine reports that nearly 100,000 Russian troops have gathered near Ukrainian borders. On 17 December, Russia presents demands to lessen the current tensions. Including the demand that Ukraine will never be able to become a NATO member state. In 2022, on 24 January, the US readies troops for deployment in Europe. On 10 February, Russia launches proclaimed military exercises of enormous scale. On 21 February, Russian troops are ordered to enter Ukrainian territory for so-called “peacekeeping duties”. Some studies such as that of Ahmed et al. (2022) use this as starting point of the war. On 24 February, Russia launches a full-scale invasion of Ukraine. The bulk of studies, such as those by Umar et al. (2022) and Boubaker et al. (2022), interpret this as the start of the war.

Boungou and Yatié (2022) were the first to explore the impact of the Ukraine-Russia war on world stock markets using panel data analysis. They show a negative relationship between the war and stock market returns of 94 different countries in this period. The study furthermore shows a larger impact on stock market returns at the start of the conflict, mainly during the first two weeks following the invasion on 24 February. Izzeldin et al. (2023)

similarly found an instantaneous, limited reaction of global stock markets to the Russia-Ukraine war. They conclude that expectations were that the war would not be prolonged. This may help understand why the effects were found to be strongest only for two weeks post-invasion. Another possible explanation is that markets overreacted to the news initially, and readjusted quickly after. Lo et al. (2022) used panel regression methodology to study the daily volatility and returns of stocks in Europe and the United States. They find across-the-board significant positive effects on volatility and significant negative effects on returns. The positive effect on volatility is stronger in developed countries, compared to emerging countries. The negative effect on returns is also stronger in developed countries.

Boubaker et al. (2022) conducted an event study and found matching negative heterogeneous effects of the war on world stock markets. They strikingly find higher stock market returns for some NATO countries, given that military preparedness provided a stimulus for the economy. Umar et al. (2022) additionally conducted an event study. They show a significant increase in anomalous returns associated with the renewable energy industry. Besides the gas oil index, no conventional energy or metals market had large abnormal returns on the event day. They found that different markets reacted at different speeds. With a similar event study, Ahmed et al. (2022) add that effects vary based on the size of firms in different industries. Country-wide effects are also heterogeneous in nature.

Distilling from the literature, preliminary predictions can be made regarding the relationship between the Ukraine war and stock market volatility in Europe and the United States. While most literature focuses on returns, it is feasible to assume that the insights these papers provide apply to volatility as well: the effects of the Russian invasion on volatility are likely significant, immediate, and short-lasting. Probably, markets responded too intensely to the event. Furthermore, varying effects between Europe and the United States are to be expected. While some industries – depending on the industry type and the size of the industry and its firms – may benefit from the war, the overall effects on volatility are expectedly positive for both EU and US. Based on current literature, it is difficult to determine in which region the effects of the war on volatility are stronger, given the lack of literature drawing direct comparisons.

2.2 Volatility indicators

Volatility is often modeled using volatility of the previous period or implied volatility – which is the market's expectation of future volatility. Both are widely used in literature, and there are various studies with mixed findings regarding which is the best predictor (Canina & Figlewski, 1993; Christensen & Prabhala, 1998; Berestycki et al., 2004; Liang et al., 2020; Chun et al., 2023). As historical volatility is directly observable by looking at the past, it is easily used in modeling by including lagged volatility values. Implied volatility is the expected level of volatility that is not directly observable. Future volatility expectations can result in trading behavior that drives current volatility. Expected volatility is derived from option pricing, for example by inserting the market price of an option into the Black-Scholes formula and solving for volatility (Canina & Figlewski, 1993). Refer to Appendix 9.3 for a more extensive explanation of how this is achieved. Some indices capture such implied volatility estimates: VIX and VSTOXX for US and EU financial markets, respectively. Opposite to historical volatility, the usage of implied volatility in models requires the inclusion of an additional variable. As discussed by Canina and Figlewski (1993), because implied volatility uses market expectations, it can prove as a poor predictor if market expectations are inefficient.

Another commonality among volatility studies is the inclusion of past returns as predictors. Returns of the previous period are commonly found to be negatively related to volatility (Ghysels et al., 2006; Mittnik et al., 2015; Pan & Liu, 2018; Chun et al., 2023). This relationship is referred to as the leverage effect and is most commonly found at the firm level. Financial leverage is the use of debt as a means of financing. When the value of stock drops – which implies negative returns – this increases financial leverage and therefore makes stock riskier, increasing volatility (Pan & Liu, 2018). Canina and Figlewski (1993) demonstrated the leverage effect at the level macroeconomic level, which implies past returns can predict volatility for US and EU indices. Another proposition is that past returns capture the effect of market shocks on investor behavior. Market shocks can result in high or low returns. In response, investors can buy or sell stocks, affecting volatility (Dendramis et al., 2015). Both high and low returns can thus increase volatility. Following the results of Chun et al. (2023), it is most likely that the aggregate effect of past returns on volatility is negative.

Literature that aims to design models that accurately predict future volatility tends to require much more controls besides historical or implied volatility and historical returns. These models typically include many financial and macroeconomic variables as controls. The works of Schwert (1989), Mittnik et al. (2015), and Chun et al., (2023) provide a vast list of potential variables that can be controlled for. For example, financial and macroeconomic drivers may include changes in GDP, the level of economic activity and inflation, high-yield bond spreads, bid-ask spreads, inflation, interest rates, and others. Which variables and the number of variables should be included is up for debate and varies with the purpose of a study; for forecasting purposes, the inclusion of a collection of financial and macroeconomic variables is most sensible. For the current study, a model that controls for the effects of only the strongest volatility drivers: historical volatility and historical returns, is sufficient. This strikes a balance between providing sufficient accuracy and simplicity. More concise models require fewer data and generally have the benefit of being easier interpretable.

3 Hypotheses

Following the review of relevant literature and concerning the stated research question in section two, multiple hypotheses are established.

When attention to the Ukraine war intensifies — either due to news regarding rising tensions between Ukraine and Russia before the war starts or because of news regarding events occurring during the war — it can lead to increased uncertainty among investors and heightened speculation. Meanwhile, investors may fear stock prices dropping, motivating them to trade stock. In both cases, this increases the fluctuation of market index prices, which is equivalent to increased volatility. A reasonable prediction is that such positive effects are present for both intra-day volatility and between-day volatility. The hypotheses that follow concerning intra-day volatility are:

1) “The Russia-Ukraine war increased intra-day volatility in the United States stock market.”
2) “The Russia-Ukraine war increased intra-day volatility in the European stock market.”

Regarding between-day volatility, the hypotheses are:

1) "The Russia-Ukraine war increased volatility in the United States stock market."
2) "The Russia-Ukraine war increased volatility in the European stock market."

Financial markets are driven by investors that trade based on their expectations regarding the future. Similarly to how an investor may predict what the future value of the market index will be, he forms predictions – or expectations – regarding the future level of volatility. The previously stated hypotheses are likely similar to the expectations that investors form regarding the effect of the Ukraine war on stock market volatility. In other words, traders probably expect a higher level of US and EU stock market volatility. The hypotheses regarding future volatility expectations – implied volatility – are:

1) The Russia-Ukraine war increased volatility expectations in the United States stock market."
2) "The Russia-Ukraine war increased volatility expectations in the European stock market."

Implicitly, these hypotheses are formed under the assumption that investors expect lasting effects of the Ukraine war on volatility. If expectations are that effects on volatility are short-term, volatility expectations are expectedly not affected strongly by the war.

4 Research Design

Boungou and Yatié (2022) studied the effects of the Ukraine war on worldwide stock market returns per week. They used a sample of daily data from 22 January 2022 to 24 March 2022 and suggest that further studies could incorporate a broader time horizon. Focusing on the daily US and EU logarithmic (log) stock market returns, $Return_{r,t}$, the current study follows this suggestion. Subscript r refers to the region of analysis, for $r = \{Europe, United\ States\}$, t refers to the trading day.

Then, to study the relationship between the Ukraine war and US and EU stock market volatility, separate OLS regressions are run for both regions. The regressions use three different variables that represent different measures of daily volatility as dependent variables. First, a simple measure using the difference between the highest and lowest index price on a day, relative to the average price of the index, $HML_{r,t}$, referred to as intra- or within-day volatility. Second, daily volatility is calculated from return data, $DVOL_{r,t}$, referred to as between-day volatility. Third, $IVOL_{r,t}$, represents implied volatility or future volatility expectations. Each volatility measure offers a different perspective. They together yield a comprehensive analysis of the relationship between the Ukraine war and volatility.

Similarly to Boungou and Yatié (2022), for the main explanatory variable, War_t, Wikipedia page views are used as a proxy for the intensity of attention to the Ukraine war worldwide. Attention to the war can drive investors to buy and sell stock, affecting volatility. War_t is generated from search terms that are related to the Ukraine war. The intensity of attention for the war is given a score approximately ranging from 0 to 100[i], where 0 equals zero attention for the war, and 100 equals maximum attention.

Per region, each volatility measure is used in two OLS regressions: models 1 and 2. The first model tests for the relation between the war attention intensity variable and volatility over the entire period of September 2, 2021, to August 18, 2022. The second model differentiates between the periods before and after the onset of the war on February 24, 2022, by incorporating an intersect and interaction dummy in the model.

All models control for the effects of past stock index returns and past values of the volatility measures they use.

4.1 Data

Similar to Umar et al. (2022), this study uses daily stock data – only considering trading days – from September 3, 2021, to August 18, 2022. This avoids the covid era, while including the event date and a sufficiently large estimation and post-event window for the event study. For the analysis, daily stock volatility data is used from Euro Stoxx 50 and Wilshire 5000[ii] indices, which are proxies for European and United States stock markets, respectively. Euro Stoxx 50, Willshire 5000, VSTOXX, and VIX stock data are acquired from the Refinitiv Datastream database. Wikipedia page view statistics are publicly available online at Wikipedia.org[iii]. Table 1 contains descriptive statistics of the data. Table 2 contains pairwise correlations between the volatility measures. Appendix 9.1 contains plots of the data.

Table 1. Descriptive statistics

Variables	Obs.	Mean	Std. dev.	Min.	Max.
$Return_{US,t}$	239	-0.000396	0.014012	-0.042686	0.030718
$Return_{EU,t}$	239	0.000063	0.011162	-0.036595	0.044232
$HML_{US,t}$	239	0.017857	0.008742	0.004214	0.047698
$HML_{EU,t}$	239	0.013025	0.006604	0.003710	0.038919
$DVOL_{US,t}$	239	0.000196	0.000298	0	0.001822
$DVOL_{EU,t}$	239	0.000124	0.000242	0	0.001957
$IVOL_{US,t}$	239	23.59	5.11	15.01	36.45
$IVOL_{EU,t}$	239	25.78	6.32	15.97	49.64
War_t	239	5.75	11.06	0.52	98.25

Source: Author database, processed through R.

The total amount of 239 observations is sufficiently large for estimates to be reliable. 118 observations fall within the period before the start of the war and 121 observations fall within the period after. Both periods are of approximately equal size and have a sample that remains adequately large.

$Return_{US,t}$ is plotted in Appendix 9.1.3, it is on average -0.000396 for the used sample. This implies a negative average log return for the US stock market[iv]. Multiplying an investment by $e^{-0.000396}$ yields the total investment value after one trading day, on average. $e^{-0.000396} - 1$ gives the regular return, which is also -0.000396[v]. While negative, the daily log US return is quite small. For example, an investment of 10 million dollars would be worth $10mln \cdot e^{-0.000396} = 9.996mln$ after one trading day. Over multiple days, however, the investment would be worth much less. After 239 days, on average, the investment would be worth $10mln \cdot e^{-0.000396 \cdot 239} = 9.097mln$[vi]. The minimum and maximum daily US log returns are -0.042686 and 0.030718, respectively. An investment of 10 million would be worth between 9.58212 and 10.31195 million, after one day. Both the maximum and minimum daily log returns are substantial, with the largest possible loss exceeding the largest possible win. The standard deviation of 0.014012 shows that there is a fair amount of deviation from the mean log return.

$Return_{EU,t}$ is plotted in Appendix 9.1.4, it is on average 0.000063. Again, this is nearly zero. Using the same example, an investment of 10 million euro would be worth 10.00063 million after one day, and 10.15171 million after 239 days. The minimum and maximum daily EU log returns are -0.036595 and 0.044232, respectively. An investment of 10 million would be worth between 9.64067 and 10.45225 million, after one day. The mean log return is larger for the EU index compared to the US index. Furthermore, the minimum and maximum EU log returns outperform minimum and maximum US log returns. The standard deviation of EU log returns is 0.011162, which is very fairly similar to the US.

$HML_{US,t}$ represents the intra-day volatility of the US index. It gives the difference between the highest and lowest US index prices, respective to the average US index price of 44015.45. It is plotted in Appendix 9.1.5. The mean of 0.017857 implies that, on average, there is a difference of $44015.45 \cdot 0.017857 = 785.98$ dollars between the highest and lowest Willshire 5000 index price on a given day. That is a 1.7857 percent difference in terms of the average index price. The minimum and maximum values of $HML_{US,t}$ are 0.004214 and 0.047698, respectively. They show that there is always some difference between the highest and lowest daily index prices. The maximum difference is nearly 5 percent, which is relatively small. Converted to absolute dollar differences, the minimum and maximum differences are 185.48 and 2099.45, respectively[vii]. The standard deviation of 0.008742 is small; it is substantially smaller than the value of the mean and the difference between the lowest and highest value of $HML_{US,t}$. Thus, data points do not deviate from the mean by much.

$HML_{EU,t}$ is plotted in Appendix 9.1.6. Its mean is 0.013025, which is slightly lower compared to the US. The average EU index price is 3643.75. Therefore, the average absolute euro difference between the highest and lowest index prices is $3643.75 \cdot 0.013025 = 47.46$. The minimum and maximum values of $HML_{EU,t}$ are 0.003710 and 0.038919, respectively. Both are lower compared to the US. The standard deviation of 0.006604 is relatively small and smaller than US standard deviation.

$DVOL_{US,t}$ is the squared US log return on a specific day and gives the variance of log returns. It is plotted in Appendix 9.1.9. The higher $DVOL_{US,t}$, the higher volatility in the US stock market. Note that taking the square root of $DVOL_{US,t}$ does not return $Return_{US,t}$ because negative returns are turned positive when they are squared. Taking the square root of

$DVOL_{US,t}$ does however allow us to study the daily changes in index prices. For example, the mean daily volatility is 0.000196. $e^{\sqrt{0.000196}} - 1 = 0.014$ gives the average change in index price when moving from one day to the other[viii]; on average, in the data, US index prices move by 1.4 percent of the current index price between days. This movement can be either positive or negative. $DVOL_{US,t}$ has a standard deviation of 0.000298, which is the average difference between mean daily volatility of 0.000196 and log returns. The standard deviation is larger than the mean but much lower than the maximum US daily volatility of 0.001822, suggesting that there is a lot of variation from the mean. The maximum furthermore indicates that there are strong outliers in the data. Reviewing Appendix 10.1.9, the outliers lie in the period after the start of the Ukraine war, although they do not occur directly after the Russian invasion. Because the standard deviation tends to increase with time, the standard deviation of 0.000298 annually is $0.000298 \cdot \sqrt{252} \cong 0.004731$, which is the average difference between US yearly volatility and mean volatility of 0.000196. The minimum of $DVOL_{US,t}$ is 0, implying that at the least there is one moment in time where US returns are zero.

$DVOL_{EU,t}$ is plotted in Appendix 9.1.10. The variable has a mean of 0.000124, which is lower than the mean of the $DVOL_{US,t}$. The EU standard deviation of 0.000242 is lower compared to the US. Yearly, the standard deviation is $0.000242 * \sqrt{252} \cong 0.003842$. While on average daily volatility and its variance around the mean are both lower in the EU, there has been a stronger spike in volatility. The plot in Appendix 9.1.10 shows this spike to be in the period shortly after the Russian invasion on 24 February 2022.

$IVOL_{US,t}$, and $IVOL_{EU,t}$ are implied volatility proxies that come from the VIX and VSTOXX indices. These indices are derived from option prices and their values capture investor sentiment. The values of these indices do not translate directly to changes in stock prices or returns. Instead, the lower (higher) the index price, the lower (higher) expected volatility. The variables are plotted in Appendices 9.1.7 and 9.1.8.

$IVOL_{US,t}$ has a mean of 23.59 in the used sample. This is larger than the average of 19.66, calculated over the years 1990 to 2023. $IVOL_{EU,t}$ has a mean of 25.78, which is larger than the historical average of 23.83, calculated over the years 1999 to 2023. In conclusion, for both US and EU, investors expected higher volatility than they do on average.

$IVOL_{US,t}$ is lowest at 15.01 and highest at 36.45. Values between the mean of 23.59 and the lowest or highest value indicate relatively low or relatively high expected volatility. $IVOL_{EU,t}$ is lowest at 15.97 and highest at 49.64. While both variables are not directly comparable, the highest value of $IVOL_{EU,t}$, relatively, lies further from the mean of 25.78 than the highest value of $IVOL_{US,t}$ lies from its mean. This is indicative of higher peaks in volatility expectations of EU, compared to the US. Possibly, because of Europe's closer proximity to Ukraine.

Both standard deviations of $IVOL_{US,t}$, and $IVOL_{EU,t}$ are relatively low, compared to the mean. This indicates that data points on average lie close to the respective means of the indices.

War_t is plotted in Appendix 9.1.11. and has a mean of 5.75; on average, attention for the war was relatively low in intensity. The minimum and maximum are 0.52 and 98.25, respectively. The intensity of attention for the war is lower (higher) when the value of War_t is closer to its minimum (maximum). On the one hand, in comparison to these extremes, the standard deviation of 11.06 is relatively low. On the other hand, this standard deviation is roughly twice the size of the mean, which is relatively high. This suggests that the maximum war attention intensity of 98.25 is a strong outlier[ix].

Table 2. Pairwise correlations of volatility measures

	$HML_{US,t}$	$IVOL_{US,t}$	$DVOL_{US,t}$	$HML_{EU,t}$	$IVOL_{EU,t}$	$DVOL_{EU,t}$
$HML_{US,t}$	1.000					
$IVOL_{US,t}$	0.699	1.000				
$DVOL_{US,t}$	0.684	0.460	1.000			
$HML_{EU,t}$	0.602	0.683	0.342	1.000		
$IVOL_{EU,t}$	0.538	0.843	0.317	0.741	1.000	
$DVOL_{EU,t}$	0.444	0.385	0.335	0.715	0.440	1.000

Source: Author database, processed through R. All correlations are significant at the 1% level.

Table 2 shows how $HML_{US,t}$ is strongly correlated to $IVOL_{US,t}$ and $DVOL_{US,t}$: for the US, the simple volatility measure using daily highest and lowest prices is fairly similar to the measures of volatility expectations and daily volatility. The correlations between $HML_{EU,t}$

and $IVOL_{EU,t}$ and $DVOL_{EU,t}$ show even stronger connections for EU. While the correlations between $IVOL_{US,t}$ and $DVOL_{US,t}$, and $IVOL_{EU,t}$, and $DVOL_{EU,t}$ are smaller in comparison, they are still somewhat sizeable. Expectedly, there are no too large differences between all volatility variables. The coefficient between $IVOL_{US,t}$, and $IVOL_{EU,t}$ is very large; evidently, volatility expectations are largely shared between US and EU, in the sample period.

4.2 Methodology

As a point of departure, a slight variation of the model of Boungou and Yatié (2022) is used:

$$Vol_{r,t} = a_r + \beta_1 War_t + \varepsilon_{r,t} \tag{0}$$

The dependent variable $Vol_{r,t}$ denotes daily volatility measures of the market index in region r at time t, for $Vol_{r,t} = \{HML_{r,t}, DVOL_{r,t}, IVOL_{r,t}\}$. a_r is a constant per region r. It gives the part of $Vol_{r,t}$ that cannot be explained by the independent variable. $\varepsilon_{r,t}$ gives the difference between observations of $Vol_{r,t}$, and the predicted values by the model over time.

4.2.1 Intra-day volatility

I propose the use of volatility measure $HML_{r,t}$, which is calculated by taking the difference between the highest index price, $H_{r,t}$, and lowest index price, $L_{r,t}$, for region r at day t, and dividing by the average of $n = 239$ US or EU index closing prices, $C_{r,t}$, over the period September 3, 2021, to August 18, 2022:

$$HML_{r,t} = (H_{r,t} - L_{r,t}) / \frac{\sum_{t=1}^{n} C_{r,t}}{n}$$

While not widely used in literature, this simple measure gives allows for a new perspective on the effects of the Ukraine war. $HML_{r,t}$ is referred to as intra-day volatility; it captures volatility within specific days.

4.2.2 Daily volatility

$DVOL_{r,t}$ captures volatility between days. It is calculated using US and EU log returns. Log returns, $Return_{r,t}$, are first calculated from US and EU index closing prices, $Index_{r,t}$:

$$Return_{r,t} = \ln\left(1 + \frac{Index_{r,t} - Index_{r,t-1}}{Index_{r,t-1}}\right)$$

Logarithmic returns are widely used in finance literature, mainly because they are additive over time. That is to say, summing the logarithmic returns of a collection of days gives the total logarithmic return over those days. Furthermore, log returns tend to better follow normal distributions, compared to regular returns. Although this difference is minimal for the returns used in this study. Because log returns capture the size of the changes in index prices, they are implicitly a measure of volatility. In literature, for daily returns, returns are usually squared (Patton, 2011). The formula for daily volatility that follows is denoted:

$$DVOL_{r,t} = Return_{r,t}^2$$

Squaring returns turns all returns into positive values, similar to how variance is always positive. Because of this feature, it does not matter whether returns are positive or negative, their effects on volatility are equal. Furthermore, squaring returns magnifies large price movements more compared to small price movements. This helps in studying the effects of the Ukraine war on volatility.

4.2.3 Implied volatility

$IVOL_{r,t}$ is equal to the United States VIX index price and equal to Europe's VSTOXX index price. Volatility expectations are often used in studies to explain volatility (Lo et al., 2022). Given that volatility is driven by investor behavior, expectations of high volatility can cause investors to buy or sell assets, resulting in higher actual volatility.

4.2.4 War attention intensity

War_t denotes worldwide fear regarding the Ukraine war at time t. It is derived from Wikipedia page view data of the total amount of daily online searches regarding the Ukraine war and acts as a proxy for the worldwide intensity of attention to the war, which is explained similarly in Lo et al. (2022). As in Boungou and Yatié (2022), the terms “Ukraine”, ”Russia”, "War", "Vladimir", and "Putin" are used[x]. Page views are normalized. Mathematically, for the term "Ukraine":

$$Ukraine_{norm,t} = \frac{Ukraine_t - Ukraine_{min}}{Ukraine_{max} - Ukraine_{min}} \cdot 100$$

min and max refer to the minimum and maximum amount of page views over the entire sample period, respectively. Figure 1. depicts the normalized search terms.

Following expectations, page views for all terms increase on the day of the invasion. After the invasion – and therefore during the war – page views seem to be higher. This is indicative of the lasting fear and attention for the war. Furthermore, it appears there is a strong correlation between the search terms. Table 3 confirms this for all variables. Once more, this conforms to expectations. To be used in the analysis, the normalized search terms are averaged:

$$War_t = \frac{Ukraine_{norm,t} + Russia_{norm,t} + Putin_{norm,t} + Vladimir_{norm,t} + War_{norm,t}}{5}$$

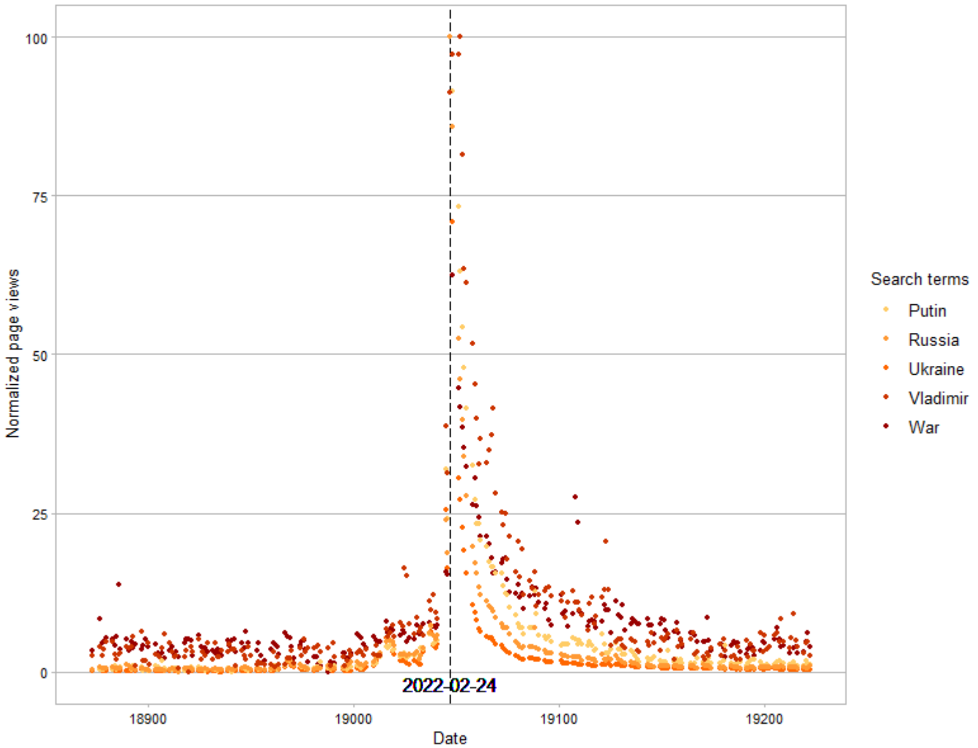
War_t is depicted in Figure 2**.**

Table 3. Pairwise correlations Wikipedia search terms

	Ukraine	Russia	Putin	Vladimir	War
Ukraine	1.000				
Russia	0.972	1.000			
Putin	0.915	0.980	1.000		
Vladimir	0.795	0.902	0.961	1.000	
War	0.895	0.932	0.939	0.895	1.000

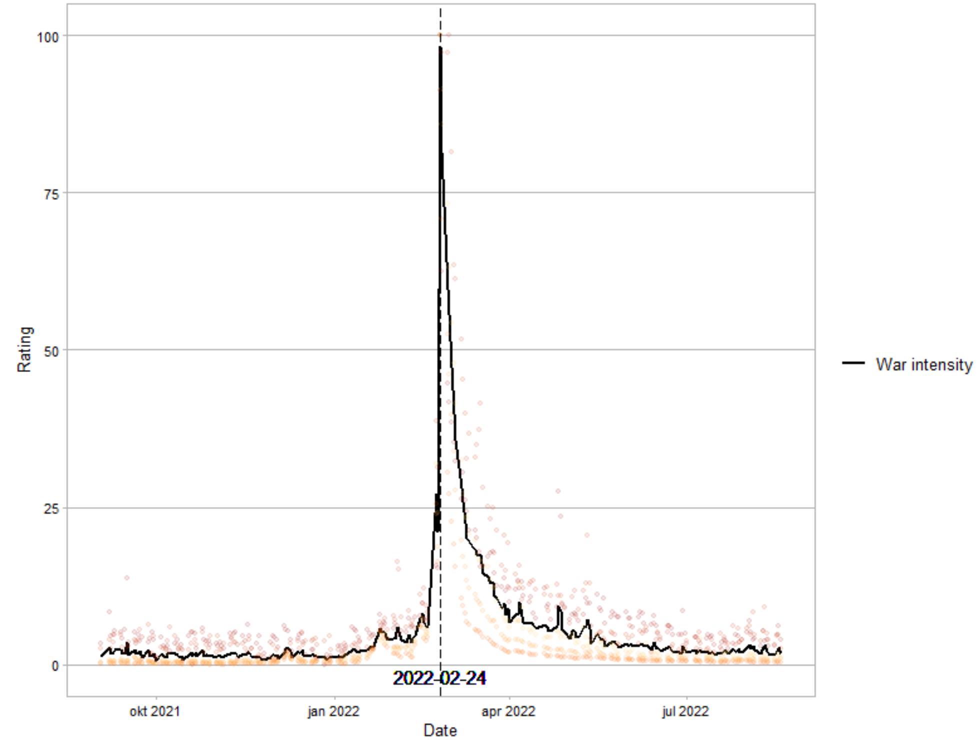
Source: Author database, processed through R. All correlations are significant at the 1% level.

Figure 1. Normalized Wikipedia page views



Notes: Page views of 100 indicate the maximum views of the sample period. The vertical line marked by 2022-02-24 gives the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

Figure 2. Ukraine war fear score



Notes: Page views of 100 indicate the maximum views of the sample period. The vertical line marked by 2022-02-24 gives the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

4.2.5 Control variables

For this study, which is not to directly forecast volatility but rather to study the relationship between the Ukraine war on volatility, it serves to create concise models; that is to say, only the most important drivers of volatility must be accounted for. Staying close to the work of Lo et al. (2022), who studied the effects of the Ukraine war on volatility as well, historical volatility is included as a control variable. To be precise, for each volatility measure $Vol_{r,t} = \{HML_{r,t}, DVOL_{r,t}, IVOL_{r,t}\}$, a one-day lagged value $Vol_{r,t-1} = \{HML_{r,t-1}, DVOL_{r,t-1}, IVOL_{r,t-1}\}$ is included in the model. This is to account for the clustering of volatility: there tend to be periods of low or high volatility (Berkes et al., 2003). Furthermore, one-day lagged values of returns, $Return_{r,t-1}$, are controlled to account for the leverage effect (Pan & Liu, 2018), and to account for market shocks (Dendramis et al., 2015).

Following the discussion in section 2 of this paper, controls for other macroeconomic and financial factors are excluded, as this would overcomplicate the model.

Volatility models frequently include historical volatility or volatility expectations as controls. Using both as controls would lead to multicollinearity issues, given that volatility expectations and historical volatility strongly correlate with each other. As discussed by Canina and Figlewski (1993), if market expectations are incorrect, implied volatility measures are incorrect as well. Historical volatility is used to avoid the issue. Furthermore, this allows the volatility models to be reliant on fewer variables.

With the addition of the discussed controls, model 1 is established:

$$(1) \qquad Vol_{r,t} = a_r + \beta_1 War_t + \beta_2 Vol_{r,t-1} + \beta_3 Return_{r,t-1} + \varepsilon_{r,t}$$

Model 1 is the first model used to test the relationship between the Ukraine war and stock market volatility. the coefficient β_1 takes a positive (negative) value if the Russia-Ukraine conflict has a positive (negative) effect on stock market volatility. Reviewing the plots in appendices 9.1.5 – 9.1.10, volatility generally seems to spike at the date of the Russian invasion and remains higher in the period thereafter. This is a first indication of a possible positive effect of the Ukraine war on stock market volatility. The relationship requires further exploration. While there are studies that include more controls in their models, model 1 is concise and captures the essence of the volatility relationship. With regards to each specific volatility measure $Vol_{r,t} = \{HML_{r,t}, IVOL_{r,t} DVOL_{r,t}\}$, model 1 takes the following forms:

(1) $Vol_{r,t} = a_r + \beta_1 War_t + \beta_2 Vol_{r,t-1} + \beta_3 Return_{r,t-1} + \varepsilon_{r,t}$

(1a) $HML_{r,t} = a_r + \beta_1 War_t + \beta_2 HML_{r,t-1} + \beta_3 Return_{r,t-1} + \varepsilon_{r,t}$

(1b) $DVOL_{r,t} = a_r + \beta_1 War_t + \beta_2 DVOL_{r,t-1} + \beta_3 Return_{r,t-1} + \varepsilon_{r,t}$

(1c) $IVOL_{r,t} = a_r + \beta_1 War_t + \beta_2 IVOL_{r,t-1} + \beta_3 Return_{r,t-1} + \varepsilon_{r,t}$

Model 1 tests for a general relationship between the Ukraine war and stock market volatility in EU and the US. To broaden our understanding of how volatility behaved before and after the Russian invasion on 24 February 2022 a dummy, D, is introduced. $D = 0$ for the period before the invasion, and $D = 1$ for the period after the invasion. Model two is denoted as follows:

(2) $Vol_{r,t} = a_r + \beta_1 War_t + \beta_2 (War_t D) + \beta_3 D + \beta_4 Vol_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t}$

Where:

$$Vol_{r,t} = \begin{cases} a_r + \beta_1 War_t + \beta_4 Vol_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t} & if\ D = 0 \\ a_r + (\beta_1 + \beta_2) War_t + \beta_3 + \beta_4 Vol_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t} & if\ D = 1 \end{cases}$$

β_2 is the coefficient of the interaction effect. It captures the change in the relationship between the war and volatility, after 24 February. β_3 is a constant, it captures deviations in the base level of volatility. $\beta_2 > 0$ indicates that the Ukraine war's effect on volatility has become stronger post-invasion, and $\beta_2 < 0$ indicates the opposite. $\beta_3 > 0$ indicates that the base level of volatility increased post-invasion. As for model 1, model 2 takes different forms when using different measures of volatility:

(2) $Vol_{r,t} = a_r + \beta_1 War_t + \beta_2 (War_t D) + \beta_3 D + \beta_4 Vol_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t}$

(2a) $HML_{r,t} = a_r + \beta_1 War_t + \beta_2 (War_t D) + \beta_3 D + \beta_4 HML_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t}$

(2b) $DVOL_{r,t} = a_r + \beta_1 War_t + \beta_2 (War_t D) + \beta_3 D + \beta_4 DVOL_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t}$

(2c) $IVOL_{r,t} = a_r + \beta_1 War_t + \beta_2 (War_t D) + \beta_3 D + \beta_4 IVOL_{r,t-1} + \beta_5 Return_{r,t-1} + \varepsilon_{r,t}$

Table 4 contains all the used variables and their expected coefficients. The following section of this paper presents and discusses the results of models 1 and 2.

Table 4. Overview of variables

Main variables	Indicator	Expected coefficient
War_t	Ukraine war attention intensity	$\beta_1 \geq 0$
War_tD, Interaction effect	Interaction effect	$-1 \leq \beta_2 \leq 1$
D, War indicator	Base volatility after 2022-02-24	$\beta_3 \geq 0$
Control variables		
$Return_{r,t-1}$	Market shocks, Leverage effect	$\beta \leq 0$
$Vol_{r,t-1}$	Historical volatility	$\beta \geq 0$

5 Results

This section contains the results of the models described in section 4 of this paper. First, as a slight detour, the relationship between the Ukraine war and US and EU market returns is studied. Second, as is the central aim of this paper, results are presented regarding the relationship between the war and the three volatility measures: intra-day volatility, between-day volatility, and implied volatility. All findings are discussed and related to relevant literature.

5.1 Returns models results

Stock market volatility is inherently linked to returns[xi]. As an introductory analysis that substantiates current literature, the basic model of Boungou and Yatié (2022) is replicated using this study's sample, with the addition of dummy D, marking the start of the war when $D = 1$. Models 1r and model 2r are used:

(1r) $$Return_{r,t} = a_r + \beta_1 War_t + \varepsilon_{r,t}$$

(2r) $$Return_{r,t} = \beta_1 War_t + \beta_2 (War_t D) + \beta_3 D + \varepsilon_{r,t}$$

The main differences between the used data and that of Boungou and Yatié (2022) lie within the sample of countries, period[xii], and derivation of the War_t variable. Boungou and Yatié (2022) used a sample of 94 countries around the world, and a period ranging from 22 January 2022 to 24 March 2022. They continuously found a significant negative relationship between the Ukraine war and stock market returns, before and after the start of the war. Table 5 presents the results of models 1r and 2r.

Strikingly, none of the coefficients of the variables used in either model are significant[xiii]. The addition of the dummy does not improve results. Contradictory to Boungou and Yatié (2022), the Ukraine war did not significantly affect stock market index returns. The low R-squared values show that returns must be driven by other factors outside of the model. Differences between these findings and those of Boungou and Yatié (2022) may be due to a multitude of reasons. First, the current study runs separate regressions using daily returns of US and EU market indices, while their study conducted panel data analysis with a much broader range of countries. The benefit of using panel data is that this allows for a study of a shorter time horizon; using daily data of 94 countries over one month, as in Boungou and Yatié (2022), yields approximately 1800 data points. When running separate regressions, as is the case for this study, one month would yield 20 data points per region, which is too small to provide reliable results. The use of panel data has a drawback in that this does not allow for a study of specific countries or regions. Because of the different time horizons used, results between studies may differ. Another possibility is that, while the returns of the US and EU were not affected, returns of most of the other countries around the world did experience negative effects, which drives results. Contradictory, Ahmed et al. (2022) did find negative effects of the war on European stock markets. A less probable explanation would be that the derivation of the variable War_t in both studies is too different, and hence affects results. There is no evidence to suggest this, however.

In conclusion, the relationship between the Ukraine war and United States and European stock index returns requires additional attention. It seems most likely that if there were adverse effects of the war on US and EU returns, these effects were brief[xiv].

Table 5. Market return models

VARIABLES	(1r) $Return_{US,t}$	(2r) $Return_{US,t}$	(1r) $Return_{EU,t}$	(2r) $Return_{EU,t}$
War_t	0.000069	-0.000591	-0.000093	-0.000385
	(0.000082)	(0.000401)	(0.000065)	(0.000320)
War indicator		-0.001447		-0.000259
		(0.002230)		(0.001778)
Interaction effect		0.000684*		0.000298
		(0.000410)		(0.000327)
Constant	-0.000793	0.000684	0.000599	0.001081
	(0.001023)	(0.001662)	(0.000812)	(0.001325)
Observations	239	239	239	239
R-squared	0.0030	0.0024	0.0085	0.0001

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

5.2 Intra-day volatility models results

The results of models 1a and 2a are presented in Table 6. The R-squared values give the part of the variance of $HML_{r,t}$ that can be explained by the used models. For US models 1a and 2a, and EU models 1a and 2a, 21.6, 23.8, 41.6, and 41.8 percent of variance is captured by the models, respectively. In comparison, the models seem to better fit the data for Europe. Presumably, over the used sample, HML_t – and by extension volatility – of Europe is driven more by the attention to the Ukraine war, compared to the HML_t of the United States.

Recall that intra-day volatility in this study is defined as the difference between the highest and lowest price on a given day, respective to the average index price of the entire period: 44015.45 dollars for the US, 3643.75 euro for EU. The significantly positive constants of all models suggest that when there is no influence of the Ukraine war – zero war attention – or the control variables, the expected difference between the highest and lowest index price on a given day is larger than zero. For US model 1a, the constant of 0.010485 translates to a $0.010485 \cdot 44015.45 = 461.50$ absolute difference between the highest and lowest daily prices. For US model 2a and EU models 1a and 2a, the coefficients translate to absolute differences of 397.72, 24.55, and 22.45, respectively.

Table 6. Intra-day volatility models

VARIABLES	(1a) $HML_{US,t}$	(2a) $HML_{US,t}$	(1a) $HML_{EU,t}$	(2a) $HML_{EU,t}$
War_t	0.000124*** (0.000047)	0.000592*** (0.000222)	0.000197*** (0.000033)	0.000343** (0.000148)
War indicator		0.003558*** (0.001256)		0.001338 (0.000822)
Interaction effect		-0.000509** (0.000226)		-0.000160 (0.000149)
$HML_{US,t-1}$	0.373560*** (0.060929)	0.329733*** (0.062072)		
$Return_{US,t-1}$	-0.068809* (0.036621)	-0.070260* (0.036239)		
$HML_{EU,t-1}$			0.396087*** (0.056236)	0.378174*** (0.057235)
$Return_{EU,t-1}$			-0.074738** (0.030019)	-0.075090** (0.030059)
Constant	0.010485*** (0.001163)	0.009036*** (0.001251)	0.006737*** (0.000750)	0.006161*** (0.000834)
Observations	238	238	238	238
R-squared	0.216	0.238	0.416	0.418

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

$HML_{US,t-1}$ is the value of HML_{US} of the previous trading day. The coefficient of 0.373560 for US model 1a is substantially high, suggesting the presence of autocorrelation; a one-unit increase in the last day's intra-day volatility equals a 0.373560 increase in current-day price fluctuation. For US model 2a and EU models 1a and 2a, the coefficients are 0.329733, 0.396087, and 0.378174, respectively. All coefficients suggest that there are clusters of low or high intra-day volatility in both the US and EU, as can be seen in Appendices 10.1.5 and 10.1.6.

For the United States, returns of the previous trading day, $Return_{US,t-1}$, do not affect HML_{US}. There are significant negative effects found for past EU returns, $Return_{EU,t-1}$. For EU model 1a, an increase of one in log returns of the previous trading day leads to a decrease of current day intra-day volatility of -0.074738, or -272.33 euro. Given that log EU returns range between -0.036595 and 0.044233, this effect is small; the effect of log returns on intra-day volatility ranges from $0.044233 \cdot -0.074738 = -0.0033$ to $-0.036595 \cdot -0.074738 = 0.0027$, or from -12.02 to 9.84 euro. The coefficient of $Return_{EU,t-1}$ for model 2a is -0.075090, which is of an equally small size.

Coefficients of War_t are significantly positive for all US and EU models. This shows that Ukraine's war attention intensity has a positive effect on intra-day volatility for both regions, regardless of whether the distinction is made between the period before and after the Russian invasion on 24 February 2022. For the US, without incorporation of the dummy variable, in model 1a, War_t has a significant coefficient of 0.000124. This implies that for every one-point increase in Ukraine war attention intensity, US intra-day volatility increases by 0.0124 percent or $0.000124 \cdot 44015.45 = 5.46$ dollars. A maximum score of 100 would yield a 1.24 percent of the average US index price increase, equivalent to a 545.79 dollar increase. In US model 2a, War_t has a coefficient of 0.000592, which is roughly five times larger compared to model 1a. In the period before the war, a one-point increase in war attention intensity gives a 0.0592 percent or 26.06 dollar increase in US intra-day volatility. Maximum war attention intensity implies a 5.92 percent or 2605.71 dollars increase, which is quite large. In the period before the war's start, the constant is 0.009036, which is lower than the constant estimated in model 1a. Summing the constant with the coefficient of "War indicator" gives the constant level of intra-day volatility for the period after the start of the war: $0.009036 + 0.003558 = 0.012594$, which is higher than the constant in model 1a. Summing coefficient of War_t with the coefficient of "Interaction effect" gives the effect of Ukraine war attention intensity on intra-day volatility for the period after the start of the war: $0.000592 - 0.000509 = 0.000083$. This coefficient is so low that it is negligible.

In short, while the base level of US intra-day volatility has increased after the start of the war, the effect of the attention intensity of the war has decreased to nearly zero. The best fitting explanation is that before the start of the war on 24 February 2022, investors were uncertain regarding whether the war would break out. Therefore, the US market intra-day volatility was highly influenced by news or attention regarding Ukraine, which is captured by the War_t variable. With the start of the war, this uncertainty disappeared. Following, the effect of news or attention regarding the war decreases. Because the Ukraine war was a reality after 24 February, a higher base level of intra-day volatility entered the market.

For EU model 1a, War_t has a significant coefficient of 0.000197, which is higher compared to the US. For every increase in War_t, EU intra-day volatility increases by 0.0197 percent or 0.72 euro. A maximum score of war attention intensity translates to a 1.97 percent or 71.78 euro increase. EU model 2a shows the coefficient of War_t to be 0.000343 for the period before the onset of the war, which is higher compared to EU model 1a. Opposite to the US, the constant level of intra-day volatility and the effect of war attention intensity do not differ significantly after the start of the war. Possibly, it is the case that a higher level of intra-day volatility already entered the market in the period before the war broke out.

Price fluctuation in the European stock market still largely depends on news regarding the Ukraine war, while they do not in the United States stock market. This is likely because of Europe's closer proximity to Ukraine; events in the war have a more direct effect on Europe and its stock market, compared to the US. Hence, Ukraine war news remains more relevant in EU and continues to drive intra-day volatility there. In the US, news regarding the war was mostly relevant before the start of the war, because investors were uncertain if the war would break out.

The Ukraine war has had differing effects on US and EU intra-day volatility. This price fluctuation is driven by investors that buy and sell stock. Among various other factors, decisions to buy and sell stock are influenced by stock market returns; there is a strong link between stock market returns and volatility. Literature often derives volatility measures from stock market returns, as is done in this study. While we concluded in section 5.1 of this paper that returns were not affected by the war for the used sample, we have yet to study the effects of the Ukraine war on daily volatility directly. The following section of this paper is attributed to this.

5.3 Daily volatility models results

The results of models 1b and 2b are presented in Table 7. For US models 1b and 2b, and EU models 1b and 2b, 0.2, 5.5, 19, and 18.7 percent of variance of $DVOL_{r,t}$ is captured by the models, respectively. The R-squared values are substantially lower compared to those of the intra-day volatility models. There is a lot of variance not captured by the models; daily volatility has strong drivers that are not specified in the model, especially for the US stock market. Similarly to models 1a and 2a, the models seem to better fit the data for Europe. This may again be an indication that, over the used sample, the volatility of Europe is driven more by the intensity of attention for the Ukraine war, compared to the United States.

Recall that daily volatility is calculated by squaring log returns, $DVOL_{r,t} = Return_{r,t}^2$. The significant constant 0.0001716 of US model 1b suggests that when all variables in the model are equal to zero – and there is no effect of the Ukraine war – there is a difference between daily US index prices of $e^{\sqrt{0.0001716}} - 1 = 0.013186$ or 1.3186 percent. For US model 2b and EU models 1b and 2b, the significant constants translate to differences in daily index prices of 1.0899, 0.8686, and 0.8480 percent, respectively. The lower constants for EU suggest that there is a lower base level of volatility there, similar to $HML_{r,t}$ models.

The insignificance of lagged US and EU daily volatility, $DVOL_{US,t-1}$, and $DVOL_{EU,t-1}$, indicate that differences between index prices on one day do not drive differences on the following day. This is opposite to what was previously demonstrated in the $HML_{r,t}$ models. The differences between the $DVOL_{r,t}$ and $HML_{r,t}$ models substantiate that between-day and within-day volatility are disconnected.

For the US, the coefficients of one-day lagged returns are insignificant; there is no evidence of the leverage effect or the effect of market shocks on US volatility. This was also found in the HML_r models. A further similarity to the $HML_{r,t}$ models is that for EU, lagged returns do predict volatility. In EU model 1b, the coefficient of $Return_{EU,t-1}$ is -0.0030992, which is evidence of the presence of the leverage effect. A one-unit increase in $Return_{EU,t-1}$ would yield a $-(e^{\sqrt{0.0030992}} - 1) = -0.057249$ increase in between-day prices. As explained previously, EU returns range from -0.036595 to 0.044233. The effect of $Return_{EU,t-1}$ on $DVOL_{EU,t}$ would be between $0.044233 \cdot -0.057249 = -0.002532$ and $-0.036595 \cdot -0.057249 = 0.002095$. For EU model 2b, a one-unit increase in lagged returns would yield a -0.0031082 decrease in between-day prices. As before, the effects of past returns on volatility are barely noticeable. This is true for EU models 1b and 2b.

Table 7. Daily volatility models

VARIABLES	(1b) $DVOL_{US,t}$	(2b) $DVOL_{US,t}$	(1b) $DVOL_{EU,t}$	(2b) $DVOL_{EU,t}$
War_t	0.0000025	0.0000008	0.0000092***	0.0000091***
	(0.0000018)	(0.0000018)	(0.0000014)	(0.0000014)
War indicator		0.0001525***		0.0000081
		(0.0000406)		(0.0000296)
Interaction effect		-0.0000100		0.0000057
		(0.0000085)		(0.0000064)
$DVOL_{US,t-1}$	0.0556064	-0.0135701		
	(0.0674515)	(0.0681764)		
$Return_{US,t-1}$	0.0013648	0.0008658		
	(0.0014237)	(0.0013918)		
$DVOL_{EU,t-1}$			-0.0249594	-0.0256957
			(0.0619231)	(0.0621049)
$Return_{EU,t-1}$			-0.0030992**	-0.0031082**
			(0.0012775)	(0.0012805)
Constant	0.0001716***	0.0001175***	0.0000748***	0.0000713***
	(0.0000249)	(0.0000282)	(0.0000168)	(0.0000210)
Observations	238	238	238	238
R-squared	0.002	0.055	0.190	0.187

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

For US models 1b and 2b, the Ukraine war does not have a significant effect on between-day volatility, which is a striking difference compared to US models 1a and 2a. Seemingly, within-day volatility is affected by the war in the US, while volatility between days is not. Because news regarding the war is published at a specific moment on a given day, it could be the case that stock indices respond strongly on that day, while returning to normal before the day ends. This would cause the difference between the highest and lowest index price on that day to be large, while not affecting returns. For EU model 1b, the Ukraine war is found to have a significant effect on volatility over the entire period. The coefficient of 0.0000092 indicates that one unit of additional war attention intensity increases the difference between daily EU index prices by $e^{\sqrt{0.0000092}} - 1 = 0.003038$ or 0.3038 percent. A maximum war attention intensity of 100 increases this difference by 30.38 percent, which is a lofty increase. In EU model 2b, the war has a nearly equivalent significant effect of 0.0000091; the effect of the Ukraine war on between-day volatility of the period before the Russian invasion is roughly equal to the effect of the war on volatility over the entire period. The insignificant coefficients of “war indicator” and “interaction effect” for EU show that the base level of between-day volatility and the effect of news regarding the war remained unchanged after the start of the war. This was also true for EU models 1a and 2a. For US model 2b, the significant positive coefficient of “war indicator” is 0.0001525. This suggests that after the start of the war, the base level of between-day volatility was $0.0001175 + 0.0001525 = 0.0003275$, which is higher than the period before the war and than the volatility over the entire period. While in the US, the relation between news and between-day volatility is not significant and has not changed comparing the period before and after the war's beginning. The base level of volatility has increased after the start of the war. Similar to the results of US models 1a and 2a, this supports the hypothesis that higher volatility entered the market after the war broke out.

To summarize, in the United States, the base level of intra-day and between-day volatility increased after the start of the Ukraine war. After the war’s start, the effect of war attention intensity or news regarding the war on intra-day volatility became weaker but remains significant. Effects of the war attention intensity on between-day volatility are not significant before or after the war broke out. In Europe, findings are more consistent between the volatility measures. Base intra-day and between-day volatility remained unchanged after the war broke out, the significant effect of the war’s attention intensity on both volatility measures also remained the same. For both the United States and Europe, stock market volatility measures are driven by investors that buy and sell stock based on what they expect will happen in the future. Studying these expectations is key to understanding the war-volatility relationship and the final piece of the puzzle. The next part of this study is dedicated to studying the effects of the Ukraine war on investors’ volatility expectations.

5.4 Implied volatility models results

The results of models 1c and 2c are presented in Table 8. For US models 1b and 2b, and EU models 1b and 2b, 84.4, 84.4, 88.1, and 88.1 percent of the variance of $DVOL_{r,t}$ is captured by the models, respectively. The R-squared values are incredibly high, especially compared to those of the intra-day and between-day volatility models. This suggests that the most important drivers of implied volatility are captured in the model. The differences in R-squared values between US and EU are not pronounced.

Recall that implied volatility captures the – primarily between-day – volatility expectations of investors in the financial market. An increase (decrease) in $IVOL_{r,t}$ is to be interpreted as an increase (decrease) in volatility expectations. Values of $IVOL_{r,t}$ are however not directly convertible to index price changes between days. Instead, they allow us to study changes in the sentiment of investors.

The significant constants of US models 1c and 2c, and EU models 1c and 2c are 2.226, 2.491, 2.571, and 2.966, respectively. These constants give the value of $IVOL_{r,t}$ when all explanatory variables in the models are zero. $IVOL_{US,t}$ is equal to the VIX index price, and $IVOL_{EU,t}$ is equal to the VSTOXX index price. Table 1 showed us that $IVOL_{US,t} \in [15.01, 36.45]$ and $IVOL_{EU,t} \in [15.97, 49.64]$. The constants of the models are quite low in comparison to these ranges.

The low constants, along with the high R-squared values for both US and EU are understood by reviewing the significant coefficients of the lagged implied volatility measures, $IVOL_{r,t-1}$. For US model 1c the coefficient of $IVOL_{US,t-1}$ is 0.903, indicating that for every one-unit increase in the last trading day's implied volatility, current-day volatility increases by 0.903, which is close to a one-to-one increase. Similar coefficients are found for US model 2c and EU models 1c and 2c. We can conclude that volatility expectations are sticky: when investors expect a certain level of volatility, they do this over a prolonged period. Investors' volatility expectations thus do not vary by much between days, which is quite intuitive.

For both US and EU, it seems clear that volatility expectations are not formed based on past returns. The leverage effect or market shocks do not affect investors' expectations, at least in the used sample.

US models 1c and 2c show there is no effect found of the Ukraine war attention intensity on implied volatility, over either the pre-and post-war period, or both. On the contrary, from

EU model 1c we find over the full period that attention intensity of the war has a significant coefficient of 0.049. For every one-unit increase in war attention intensity, the VIX index price increases by 0.049. A maximum attention intensity of 100 yields a 4.9 unit increase in VIX price. This effect is small, given $IVOL_{EU,t} \in [15.97, 49.64]$. EU Model 2c shows this effect to be unchanged after the start of the war.

Table 8. Implied volatility models

VARIABLES	(1c) $IVOL_{US,t}$	(2c) $IVOL_{US,t}$	(1c) $IVOL_{EU,t}$	(2c) $IVOL_{EU,t}$
War_t	0.014	0.013	0.049***	0.050***
	(0.013)	(0.013)	(0.015)	(0.015)
War indicator		0.339		0.490
		(0.318)		(0.356)
Interaction effect		-0.118		-0.107
		(0.061)		(0.065)
$IVOL_{US,t-1}$	0.903***	0.884***		
	(0.029)	(0.034)		
$Return_{US,t-1}$	-3.020	-4.939		
	(9.680)	(9.843)		
$IVOL_{EU,t-1}$			0.890***	0.865***
			(0.027)	(0.032)
$Return_{EU,t-1}$			3.994	1.262
			(12.934)	(13.062)
Constant	2.226***	2.491***	2.571***	2.966***
	(0.664)	(0.709)	(0.665)	(0.721)
Observations	238	238	238	238
R-squared	0.844	0.844	0.881	0.881

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

In conclusion, volatility expectations in both US and EU are primarily driven by past expectations. A significant effect of the Ukraine war on expected volatility is only found in EU. Table 2 showed that $IVOL_{US,t}$, and $IVOL_{EU,t}$ are strongly correlated, implying that volatility expectations are shared between the United States and Europe. Contrarily, a significant effect of the Ukraine war on US volatility expectations is not found. Although the effect of the war on EU volatility is relatively small, it is not negligible. Findings support the hypothesis that the Ukraine war increased European investors' volatility expectations. Volatility expectations in the US remained unaffected.

6 Robustness checks

To ensure the results from models 1 and 2 are reliable, various robustness tests are conducted. To be precise, this study tests for possible delayed or expedited effects of the Ukraine war on returns and volatility, and whether results remain consistent. Furthermore, the effects of removing control variables, are studied to ensure that results are not the product of specific model specifications. Finally, possible problematic day/night effects are discussed.

6.1 Delayed and expedited effects of the Ukraine war

There may be delayed effects of the US and EU stock markets to the Ukraine war (Kaplanski & Levy, 2010; Hudson & Urquhart, 2022; Boungou et al., 2022). In order words, attention to the war does not instantly affect stock markets. To account for this, lags of War_t, the variable that captures war attention intensity, up to the second order are tested; if there are delayed market responses, it is unlikely that this delay exceeds two trading days. Forward-shifted values of War_t up to two days in the future are included to account for a possible anticipation effect; markets may respond in anticipation before news regarding the war has been publicized. Findings are displayed in Tables 8 to 11. War_tD denotes the interaction effect, which captures the change in the coefficient of War_t, after the start of the Ukraine war. D denotes the change in intersect after the start of the war, previously referred to as the "war indicator".

Table 9 contains the effect of time-shifted war attention intensity on US and EU returns. There is no sign of anticipation effects for the US or EU. War_t is shown to have no immediate effect on US and EU returns. However, we find that there are significant one-day delayed effects of the war on EU returns, similar to Boungou and Yatié (2022). War_{t-1} has a coefficient of -0.000607. In the period before the start of the war, a one-unit increase in war attention intensity on the previous day leads to a -0.000607 decrease in current-day EU log returns. A maximum war attention intensity of 100 leads to a -0.0607 or -6.07 percent decrease in log returns after one day, which is a drastic decrease in returns. The coefficient of $War_{t-1}D$ is 0.000659, indicating that after the start of the war, the one-day delayed effect of the Ukraine war on returns is $-0.000607 + 0.000659 = 0.000052$. Surprisingly, the effect is positive. A maximum war attention intensity of 100 leads to a 0.0052 increase in log returns or an $e^{0.0052} - 1 \cong 0.0052$ increase in normal returns. This increase is not pronounced. To summarize, there was a strong negative effect of the Ukraine war on EU returns before the war broke out. After the start of the war, the effect of the war becomes positive, although only slightly so. Positive effects on returns are in line with the findings of Boubaker et al. (2022). Findings suggest heterogeneous effects between US and EU, similar to what Ahmed et al. (2022) found.

Table 10 contains the effect of time-shifted war attention intensity on US and EU intra-day volatility. Compared to the normal model where War_t is the explanatory variable, all coefficients are similar in terms of direction and size. For the US, there is evidence of both anticipation effects and delayed market reactions to the war. For EU, there is evidence of delayed market reactions to the war, but there is no evidence of anticipation effects. In all cases, the coefficient of the interaction effect, War_tD, is smaller than the respective coefficient of War_t. This indicates that the effect of war attention intensity on intra-day volatility remains positive after the start of the war, but reduces drastically. This was similarly concluded for US model 1b.

Table 11 shows that there is no evidence of delayed or expedited effects of the Ukraine war on US between-day volatility. There is evidence for both delayed and expedited effects of the war on EU between-day volatility. Findings regarding the significance of the war indicator variable, D, are consistent.

Table 12 shows that there is no evidence of delayed effects of the Ukraine war's attention intensity on US implied volatility; US investors' expectations do not have a delayed response to war attention intensity. There is an indication of anticipation effects for US volatility expectations; volatility expectations change two days before the Ukraine war attention intensity changes. War_t showed no direct effects on US volatility expectations. For EU, delayed and expedited effects are both found for one and two days before or after the war changes in attention intensity. These findings are more consistent with War_t.

To summarize, first, there may be a delayed effect of the Ukraine war's attention intensity on US returns. This is not found in EU. Second, there are possible anticipation effects and delayed responses of US intra-day volatility to the war attention intensity and possible delayed responses for EU. Third, there is no delayed response or anticipation effect found in US between-day volatility, while both are found in EU. Fourth, for the US, the anticipation effect of war attention intensity on volatility expectations is found. For EU, both anticipation effects and delayed responses may be present. Further studies are required to inspect these properties further, as this was not the main target of the current study. In any case, the directions of coefficients in the presented models are consistent; coefficients do not change signs or become significant or insignificant unexpectedly. There are minimal differences in R-squared values between the models. This suggests that the findings are robust. Further robustness tests are required, however.

Table 9. Market return delayed and expedited effects

VARIABLES	$Return_{US,t}$	$Return_{US,t}$	$Return_{US,t}$	$Return_{US,t}$	$Return_{US,t}$	$Return_{EU,t}$	$Return_{EU,t}$	$Return_{EU,t}$	$Return_{EU,t}$	$Return_{EU,t}$
War_{t+2}	-0.000664*					-0.000338				
	(0.000402)					(0.000320)				
$War_{t+2}D$	0.000606					0.000236				
	(0.000404)					(0.000322)				
War_{t+1}		-0.000441					-0.000455			
		(0.000403)					(0.000320)			
$War_{t+1}D$		0.000434					0.000369			
		(0.000408)					(0.000324)			
War_{t}			-0.000591					-0.000385		
			(0.000401)					(0.000320)		
$War_{t}D$			0.000684*					0.000298		
			(0.000410)					(0.000327)		
War_{t-1}				-0.000178					-0.000607**	
				(0.000389)					(0.000307)	
$War_{t-1}D$				0.000219					0.000659**	
				(0.000396)					(0.000313)	
War_{t-2}					0.000475					-0.000286
					(0.000374)					(0.000298)
$War_{t-2}D$					-0.000534					0.000179
					(0.000378)					(0.000301)
D	-0.000389	-0.000200	-0.001447	0.000166	0.002298	-0.000279	-0.000624	-0.000259	-0.001915	0.000357
	(0.002139)	(0.002183)	(0.002230)	(0.002167)	(0.002086)	(0.001704)	(0.001731)	(0.001778)	(0.001711)	(0.001659)
Constant	0.000942	0.000292	0.000684	-0.000429	-0.001909	0.001104	0.001331	0.001081	0.001616	0.000750
	(0.001669)	(0.001673)	(0.001662)	(0.001618)	(0.001555)	(0.001330)	(0.001326)	(0.001325)	(0.001278)	(0.001236)
Observations	237	238	239	238	237	237	238	239	238	237
R-squared	0.001	-0.007	0.002	-0.001	-0.002	0.001	0.002	0.000	0.007	0.000

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

Table 10. Intra-day volatility delayed and expedited effects

VARIABLES	$HML_{US,t}$	$HML_{US,t}$	$HML_{US,t}$	$HML_{US,t}$	$HML_{US,t}$	$HML_{EU,t}$	$HML_{EU,t}$	$HML_{EU,t}$	$HML_{EU,t}$	$HML_{EU,t}$
War_{t+2}	0.000564**					0.000254*				
	(0.000221)					(0.000151)				
$War_{t+2}D$	-0.000461**					-0.000122				
	(0.000221)					(0.000150)				
War_{t+1}		0.000348					0.000365**			
		(0.000225)					(0.000150)			
$War_{t+1}D$		-0.000245					-0.000233			
		(0.000226)					(0.000151)			
War_t			0.000592***					0.000343**		
			(0.000222)					(0.000148)		
War_tD			-0.000509**					-0.000160		
			(0.000226)					(0.000149)		
War_{t-1}				0.000830***					0.000483***	
				(0.000213)					(0.000146)	
$War_{t-1}D$				-0.000821***					-0.000356**	
				(0.000215)					(0.000146)	
War_{t-2}					0.000683***					0.000648***
					(0.000211)					(0.000140)
$War_{t-2}D$					-0.000651***					-0.000520***
					(0.000212)					(0.000139)
D	0.003743***	0.002992**	0.003558***	0.004554***	0.003746***	0.001823	0.001971	0.001338	0.00201*	0.002099*
	(0.001213)	(0.001240)	(0.001256)	(0.001198)	(0.001185)	(0.000820)	(0.000822)	(0.000822)	(0.000805)	(0.000765)
$HML_{r,t-1}$	0.322597***	0.329882***	0.329733***	0.327049***	0.311012***	0.423868***	0.411468***	0.378174***	0.384828***	0.370735***
	(0.062028)	(0.062761)	(0.062072)	(0.061764	(0.063162)	(0.057838)	(0.058177)	(0.057235)	(0.061143)	(0.059482)
$Return_{r,t-1}$	-0.071653**	-0.070901*	-0.070260*	-0.064676*	-0.076645**	-0.076043**	-0.074976**	-0.075090**	-0.076961**	-0.085483***
	(0.036183)	(0.036470)	(0.036239)	(0.036060)	(0.036215)	(0.031048)	(0.030872)	(0.030059)	(0.030757)	(0.030578)
Constant	0.009081***	0.009598***	0.009036***	0.008596***	0.009405***	0.005686***	0.005625***	0.006161***	0.005802***	0.005734***
	(0.001239)	(0.001241)	(0.001251)	(0.001212)	(0.001193)	(0.000855)	(0.000857)	(0.000834)	(0.000857)	(0.000820)
Observations	236	237	238	238	237	236	237	238	238	237
R-squared	0.242	0.230	0.238	0.254	0.236	0.381	0.387	0.418	0.389	0.412

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

Table 11. Daily volatility delayed and expedited effects

VARIABLES	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$
War_{t+2}	0.0000017					0.0000052***				
	(0.0000017)					(0.0000014)				
$War_{t+2}D$	-0.000013					0.000001				
	(0.000008)					(0.000007)				
War_{t+1}		0.0000014					0.0000063***			
		(0.0000017)					(0.0000014)			
$War_{t+1}D$		-0.000005					0.000001			
		(0.000008)					(0.000007)			
War_t			0.0000008					0.0000091***		
			(0.0000018)					(0.0000014)		
War_tD			-0.0000100					0.0000057		
			(0.0000085)					(0.0000064)		
War_{t-1}				0.0000006					0.0000073***	
				(0.0000018)					(0.0000015)	
$War_{t-1}D$				-0.000013					-0.000009	
				(0.000008)					(0.000006)	
War_{t-2}					-0.0000003					0.0000056***
					(0.0000018)					(0.0000015)
$War_{t-2}D$					-0.000011					-0.000027
					(0.000008)					(0.000006)
D	0.0001566***	0.0001536***	0.0001525***	0.0001534***	0.0001583***	0.0000449	0.0000333	0.0000081	0.0000170	0.0000222
	(0.0000398)	(0.0000401)	(0.0000406)	(0.0000408)	(0.0000412)	(0.0000307)	(0.0000305)	(0.0000296)	(0.0000308)	(0.0000317)
$DVOL_{r,t-1}$	-0.0203520	-0.0179499	-0.0135701	-0.0136102	-0.0131825	0.0408079	0.0203807	-0.0256957	-0.0366564	0.0132752
	(0.0684005)	(0.0682483)	(0.0681764)	(0.0682062)	(0.0683476)	(0.0642769)	(0.0640051)	(0.0621049)	(0.0670754)	(0.0663319)
$Return_{r,t-1}$	0.0008511	0.0008439	0.0008658	0.0008329	0.0008654	-0.0033013**	-0.0032244**	-0.0031082**	-0.0033035**	-0.0040206***
	(0.0013958)	(0.0013948)	(0.0013918)	(0.0013944)	(0.0013956)	(0.0013573)	(0.0013389)	(0.0012805)	(0.0013250)	(0.0013530)
Constant	0.0001134***	0.0001151***	0.0001175***	0.0001180***	0.0001210***	0.0000675***	0.0000692***	0.0000713***	0.0000782***	0.0000801***
	(0.0000287)	(0.0000284)	(0.0000282)	(0.0000281)	(0.0000283)	(0.0000227)	(0.0000222)	(0.0000210)	(0.0000218)	(0.0000223)
Observations	236	237	238	238	237	236	237	238	238	237
R-squared	0.061	0.059	0.055	0.055	0.054	0.096	0.117	0.187	0.128	0.094

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

Table 12. Implied volatility delayed and expedited effects

VARIABLES	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$
War_{t+2}	0.028**					0.052***				
	(0.013)					(0.015)				
$War_{t+2}D$	-0.132**					-0.134**				
	(0.059)					(0.063)				
War_{t+1}		0.023*					0.052***			
		(0.013)					(0.015)			
$War_{t+1}D$		-0.071					-0.144**			
		(0.060)					(0.063)			
War_t			0.013					0.050***		
			(0.013)					(0.015)		
War_tD			-0.118					-0.107		
			(0.061)					(0.065)		
War_{t-1}				0.014					0.031**	
				(0.013)					(0.016)	
$War_{t-1}D$				-0.081					-0.134	
				(0.059)					(0.063)	
War_{t-2}					0.025*					0.068***
					(0.013)					(0.015)
$War_{t-2}D$					0.006					-0.050
					(0.055)					(0.059)
D	0.429	0.380	0.339	0.334	0.292	0.656*	0.597*	0.490	0.467	0.510
	(0.320)	(0.318)	(0.318)	(0.318)	(0.318)	(0.367)	(0.361)	(0.356)	(0.362)	(0.351)
$IVOL_{r,t-1}$	0.868***	0.874***	0.884***	0.884***	0.878***	0.862***	0.861***	0.865***	0.882***	0.841***
	(0.034)	(0.034)	(0.034)	(0.033)	(0.033)	(0.033)	(0.033)	(0.032)	(0.034)	(0.033)
$Return_{r,t-1}$	-5.530	-5.246	-4.939	-5.714	-6.234	1.451	1.464	1.262	1.419	-7.803
	(9.807)	(9.824)	(9.843)	(9.916)	(9.836)	(13.090)	(13.073)	(13.062)	(13.264)	(13.087)
Constant	2.741***	2.655***	2.491***	2.507***	2.593***	2.949***	3.002***	2.966***	2.643***	3.483***
	(0.712)	(0.711)	(0.709)	(0.710)	(0.707)	(0.722)	(0.723)	(0.721)	(0.747)	(0.737)
Observations	236	237	238	238	237	236	237	238	238	237
R-squared	0.846	0.845	0.844	0.844	0.845	0.882	0.882	0.881	0.878	0.885

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.

Source: Author calculations, processed through R.

6.2 Effects of omitted control variables

To be certain that the demonstrated significant relations between explanatory variables and volatility measures found in the used models are not the result of specific model specifications, but rather because of actual existing relationships, the inclusion of control variables varied within the models. The return models used no control variables, and are hence not used in this analysis.

Results are consistent in Table 13; adding and removing control variables does not turn the main explanatory variables significant or insignificant unexpectedly. Removing the control

$HML_{r,t-1}$ causes the coefficient and significance of the constant, $Return_{r,t-1}$, and D to increase substantially for both US and EU. This is to be expected as the effect of the omitted control will be captured by other variables. Similar effects are found when $Return_{r,t-1}$ is omitted, although to a lesser extent. This highlights the importance of the inclusion of lagged intra-day volatility and lagged returns as a control in the model.

The results in Table 14 are consistent. We find that omitting control variables has weaker effects on the coefficients and significance of the other variables that predict between-day volatility, compared to what is demonstrated for intra-day price volatility. This is expected because the relation between the dependent variable $DVOL_{r,t}$, and the controls is relatively weak or non-existent.

The results in Table 15 are consistent with expectations. Crucially, we find that omitting $IVOL_{r,t-1}$ heavily increases the significance and strength of the other explanatory variables. Furthermore, the R-squared values of models where $IVOL_{r,t-1}$ is omitted, drop sharply. This once again highlights how strongly volatility expectations depend on past expectations and emphasizes the need for including lagged volatility expectations as a control in models.

Overall, findings from the above analysis support that results from the used models are robust and are not the result of specific model specifications.

Table 13. Intra-day volatility with omitted control variables

VARIABLES	$HML_{US,t}$	$HML_{US,t}$	$HML_{US,t}$	$HML_{US,t}$	$HML_{EU,t}$	$HML_{EU,t}$	$HML_{EU,t}$	$HML_{EU,t}$
War_t	0.000592***	0.000792***	0.000608***	0.000858***	0.000343**	0.000521***	0.000363**	0.000570***
	(0.000222)	(0.000231)	(0.000223)	(0.000235)	(0.000148)	(0.000158)	(0.000149)	(0.000161)
War_tD	-0.000509**	-0.000665***	-0.000528**	-0.000731***	-0.000160	-0.000257	-0.000180	-0.000297*
	(0.000226)	(0.000237)	(0.000227)	(0.000240)	(0.000149)	(0.000162)	(0.000151)	(0.000164)
D	0.003558***	0.005239***	0.003468***	0.005436***	0.001338	0.002390***	0.001302	0.002445***
	(0.001256)	(0.001285)	(0.001263)	(0.001305)	(0.000822)	(0.000878)	(0.000831)	(0.000894)
$HML_{r,t-1}$	0.329733***		0.356682***		0.378174***		0.404418***	
	(0.062072)		(0.060852)		(0.057235)		(0.056892)	
$Return_{r,t-1}$	-0.070260*	-0.113367***			-0.075090**	-0.111544***		
	(0.036239)	(0.037324)			(0.030059)	(0.032139)		
Constant	0.009036***	0.013612***	0.008624***	0.013430***	0.006161***	0.009970***	0.005806***	0.009830***
	(0.001251)	(0.000959)	(0.001241)	(0.000973)	(0.000834)	(0.000656)	(0.000831)	(0.000666)
Observations	238	238	238	239	238	238	238	239
R-squared	0.238	0.149	0.229	0.122	0.418	0.311	0.404	0.279

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

Table 14. Daily volatility with omitted control variables

VARIABLES	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{US,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$	$DVOL_{EU,t}$
War_t	0.0000008	0.0000008	0.0000008	0.0000007	0.0000091***	0.0000089***	0.0000093***	0.0000092***
	(0.0000018)	(0.0000018)	(0.0000018)	(0.0000018)	(0.0000014)	(0.0000013)	(0.0000014)	(0.0000013)
$War_t D$	-0.0000100	-0.0000099	-0.0000096	-0.0000095	0.0000057	0.0000057	0.0000046	0.0000045
	(0.0000085)	(0.0000085)	(0.0000085)	(0.0000084)	(0.0000064)	(0.0000064)	(0.0000065)	(0.0000065)
D	0.0001525***	0.0001503***	0.0001549***	0.0001519***	0.0000081	0.0000075	0.0000062	0.0000063
	(0.0000406)	(0.0000390)	(0.0000404)	(0.0000388)	(0.0000296)	(0.0000295)	(0.0000299)	(0.0000297)
$DVOL_{r,t-1}$	-0.0135701		-0.0250256		-0.0256957		-0.0165644	
	(0.0681764)		(0.0655559)		(0.0621049)		(0.0626355)	
$Return_{r,t-1}$	0.0008658	0.0009406			-0.0031082**	-0.0030762**		
	(0.0013918)	(0.0013373)			(0.0012805)	(0.0012759)		
Constant	0.0001175***	0.0001161***	0.0001182***	0.0001144***	0.0000713***	0.0000694***	0.0000695***	0.0000679***
	(0.0000282)	(0.0000271)	(0.0000281)	(0.0000269)	(0.0000211)	(0.0000205)	(0.0000213)	(0.0000206)
Observations	238	238	238	239	238	238	238	239
R-squared	0.055	0.059	0.058	0.062	0.187	0.189	0.170	0.173

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

Table 15. Implied volatility with omitted control variables

VARIABLES	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{US,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$	$IVOL_{EU,t}$
War_t	0.013	0.110***	0.013	0.113***	0.050***	0.236***	0.050***	0.245***
	(0.013)	(0.025)	(0.013)	(0.026)	(0.015)	(0.027)	(0.015)	(0.027)
War_tD	-0.118*	-0.533***	-0.117*	-0.576***	-0.107	-0.465***	-0.107	-0.494***
	(0.061)	(0.114)	(0.061)	(0.118)	(0.065)	(0.125)	(0.065)	(0.127)
D	0.339	4.659***	0.309	4.633***	0.490	5.863***	0.495	5.849***
	(0.318)	(0.544)	(0.312)	(0.567)	(0.356)	(0.590)	(0.351)	(0.600)
$IVOL_{r,t-1}$	0.884***		0.890***		0.865***		0.864***	
	(0.033)		(0.032)		(0.032)		(0.031)	
$Return_{r,t-1}$	-4.939	-87.697***			1.262	-80.943***		
	(9.843)	(18.653)			(13.062)	(25.495)		
Constant	2.491***	20.575***	2.385***	20.595***	2.966***	21.474***	2.983***	21.415***
	(0.709)	(0.378)	(0.675)	(0.393)	(0.723)	(0.410)	(0.702)	(0.416)
Observations	238	238	238	239	238	238	238	239
R-squared	0.844	0.375	0.844	0.322	0.881	0.522	0.882	0.505

Notes: Standard errors are in parentheses. *** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level. *Source:* Author calculations, processed through R.

6.3 Day/night effects

Because US and EU stock markets close at night, information that gets publicized in the night gets captured by prices the next day, leading to large index price differences between trading days. Anticipation of these effects can also drive investors to sell stocks before markets close, driving prices up or down before the end of trading days. This can result in increased volatility within and between trading days. In the current study, the effects of war on volatility are studied. If day/night effects are not accounted for, this may bias results as day/night effects on volatility can be found instead of the effects of the war.

Recall that $HML_{r,t}$ is the daily difference between the highest and lowest index prices. If the highest or lowest daily index prices are equal to opening or closing prices, this suggests that there are day/night effects. This would cause $HML_{r,t}$ to be overestimated. The variable $DVOL_{r,t}$ captures volatility between trading days as it is calculated using returns. These returns are calculated using daily closing prices. If closing prices are often the highest or lowest prices on a day, suggests day/night effects to also bias $DVOL_{r,t}$.

Analysis shows that for the US, 16.7 percent of opening prices are the highest daily prices and 0.4 percent of closing prices are the highest daily prices. 17.5 percent of the opening prices are the lowest daily prices and 0.8 percent of closing prices are the lowest daily prices.

For EU, 14.2 percent of opening prices are the highest daily prices and 4.9 percent of closing prices are the highest daily prices. 18.6 percent of the opening prices are the lowest daily prices and 3.2 percent of closing prices are the lowest daily prices.

The amount of times that opening prices are the highest or lowest daily prices is not insignificant. While day/night effects do likely bias estimates, they do not do it excessively. Future studies may more actively try to account for day/night effects to offer more accurate results.

7 Conclusion

There is a lack of literature on the relationship between the Ukraine war and stock market volatility. Nevertheless, this relationship has pronounced implications for risk managers, policymakers, and investors. The current paper studies the relationship between the recent Ukraine war and three distinct volatility measures: intra-day volatility, between-day volatility, and implied volatility over the period September 3, 2021, to August 18, 2022.

In a brief detour, we find no immediate and lasting effect of the Ukraine war on US and EU returns. Likely effects of the war on returns have been brief and are hence not captured by the relatively long time horizon used. This explains the differing findings from the works of Boungou and Yatié (2022), Lo et al. (2022), Boubaker et al. (2022), Umar et al. (2022) and Ahmed et al. (2022). There is some evidence for a delayed negative effect of the war on US returns, similar to what was found by Kaplanski and Levy (2010), Hudson and Urquhart (2022), and Boungou and Yatié (2022). Additional research is required to further develop our understanding of the relationship between US and EU returns and the Ukraine war. Specifically, long-term and delayed effects should be studied more extensively.

Base level intra-day stock market volatility has increased in the US after the start of the war on 24 February 2022. Before the war broke out, the effect of the war's attention intensity on intra-day stock market volatility was substantial. After the war's start, this effect is greatly diminished, becoming negligible. This can be explained by uncertainty about whether the Ukraine war would break out before the 24th of February 2022; expectations regarding whether there would be a war – and therefore investor behavior – are heavily driven by news regarding the Ukraine war, before the war's start.

Opposite to the US, base-level intra-day stock market volatility remained equal in EU after the start of the Ukraine war. EU intra-day volatility remained strongly connected to the level of war attention intensity after the war began. Possibly, higher base intra-day volatility was already captured by the European stock market before the start of the war. Future literature could more thoroughly study such anticipation effects. With regards to the unchanged significant effect of the intensity of attention for the Ukraine war, this could be explained by Europe's closer proximity to Ukraine; Europe is more directly affected by events during the war. Hence, news about the war continues to strongly drive volatility after the start of the war. This hypothesis is in line with the findings of Boubaker et al. (2022).

Base US between-day volatility increased substantially after the war began. US between-day volatility remains unaffected by the Ukraine war's attention intensity, even after the start of the war. This is remarkably different from the effect of war attention intensity on US intra-day volatility. This suggests that the effects of Ukraine war news are brief enough to only affect US index price fluctuations during trading days. Before the end of those trading days, prices stabilize, causing between-day volatility to be unaffected.

Base EU between-day volatility was unaffected by the start of the war. The effect of war attention intensity remained stable after the start of the war. This finding is consistent with the findings regarding EU intra-day volatility. As for within-day volatility, higher base EU between-day volatility may have entered the market before the war started. Given that both EU intra-day and between-day volatility are significantly affected by the war, this implies that the effects of the Ukraine war on EU volatility are longer lasting compared to the effects on US volatility.

Volatility expectations in both US and EU are found to be sticky; investors' volatility expectations between days are highly correlated with each other. The Ukraine war is found to have had no significant effect on US volatility expectations. Izzeldin et al. (2023) suggest this is because expectations were that the war would not be prolonged. A significantly positive effect of the war on volatility expectations is found for EU, although this effect is rather limited. The effect did not change after the start of the war. This finding is consistent with the findings regarding the war's effects on intra-day and between-day volatility. As with the other volatility measures, it is likely that volatility expectations in EU are driven more by the war compared to the US because of Europe's closer proximity to Ukraine (Boubaker et al., 2022).

The effects of delayed and expedited effects of war attention intensity on returns and the volatility measures are tested. Tests are conducted for the effects of omitting control variables. And the possible presence of day/night effects is studied. Overall, it is demonstrated that the findings presented in this paper are robust.

This study is not without its limits. First, the estimation of the variable that accounts for the intensity of attention for the war uses worldwide Wikipedia page views. It is feasible that page views differ between EU and US. Normalizing the data likely has solved most of the issue, but some slight differences may remain and are not accounted for in the models. Second, the used models only allow the effect of the war on returns and the volatility measures to change once at the start of the Ukraine war on the 24th of February 2022. It is however likely that such effects can change more often. Further studies could more narrowly study the changes in returns and volatility over time. Third, the intra-day and between-day volatility variables used in this study are rather simple and likely inaccurate (Patton, 2011). More sophisticated volatility models could be applied to get more precise estimates of the relation between the Ukraine war and volatility. For instance, the GARCH model (Bollerslev, 1986; Berkes et al., 2003) – which is widely used in literature – could be used to account for the presence of periods of low and high volatility. Appendices 9.1.9 and 9.1.10 show the possibility of the presence of such periods. Finally, the models used assume linear relationships between the attention to the Ukraine war and the explanatory variables. This may not necessarily be true; future studies can test for model specifications that yield more accurate results.

Regardless of its limits, this study's results help in understanding financial market sentiment and volatility behavior during the Ukraine war, as well as differences between Europe and the United States in this regard. Findings may be useful when forming expectations regarding future conflicts. Although it must be noted that no wars and their consequences are equal. The method used to derive the proxy for attention for the war can be used similarly for a broad range of cases. Researchers can use any selection of related search terms to create a proxy that allows for the study of for example other wars or events.

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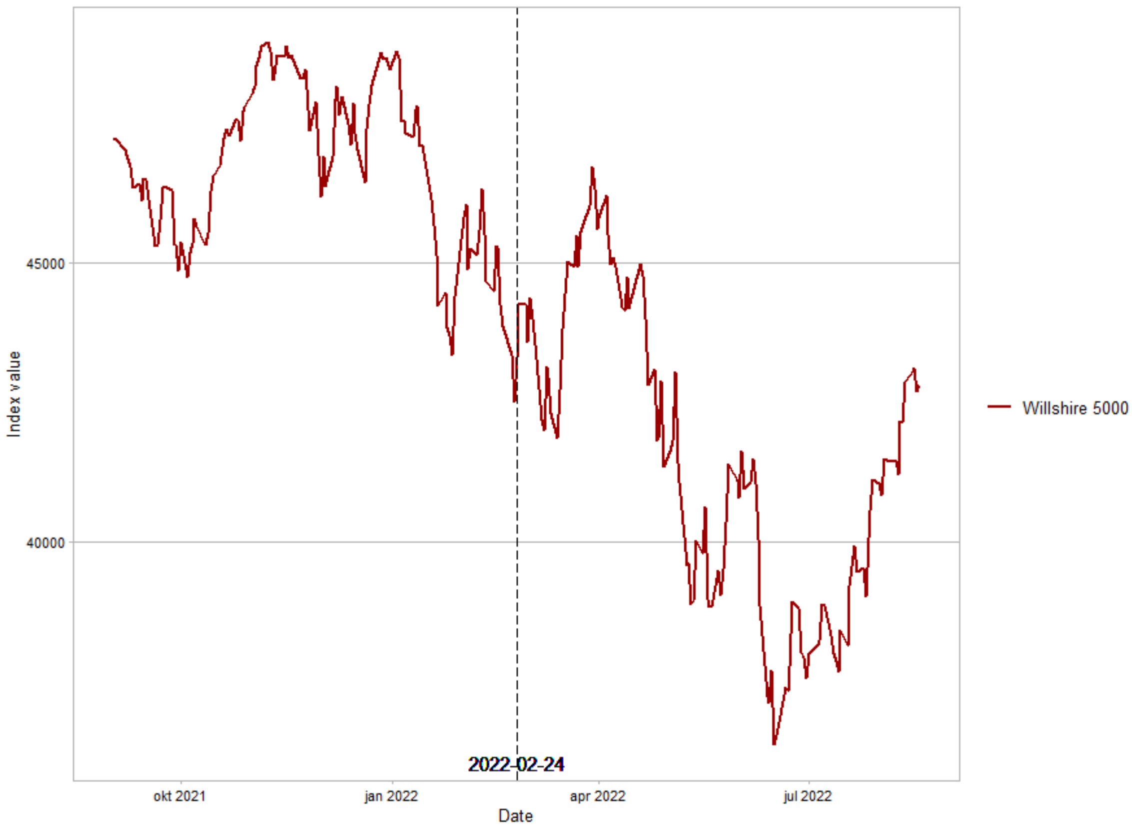
9 Appendix

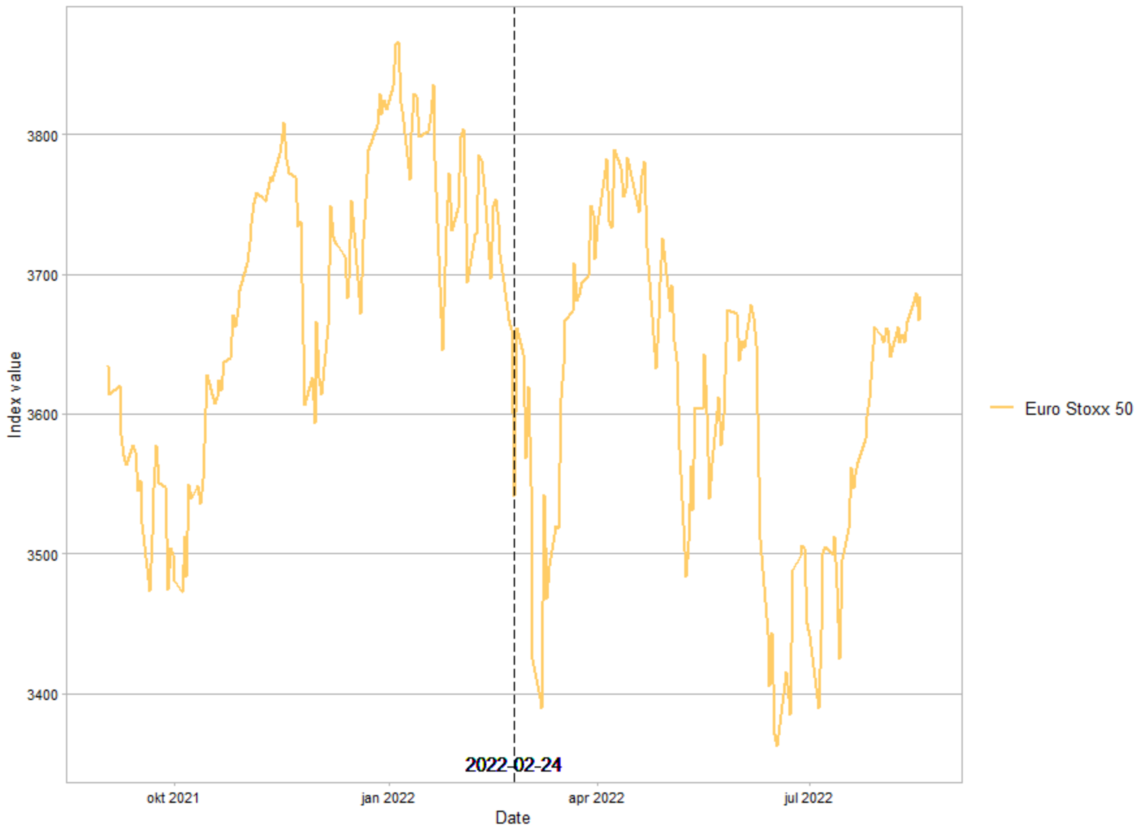
9.1 Plots of main variables

9.1.1 Willshire 5000 index



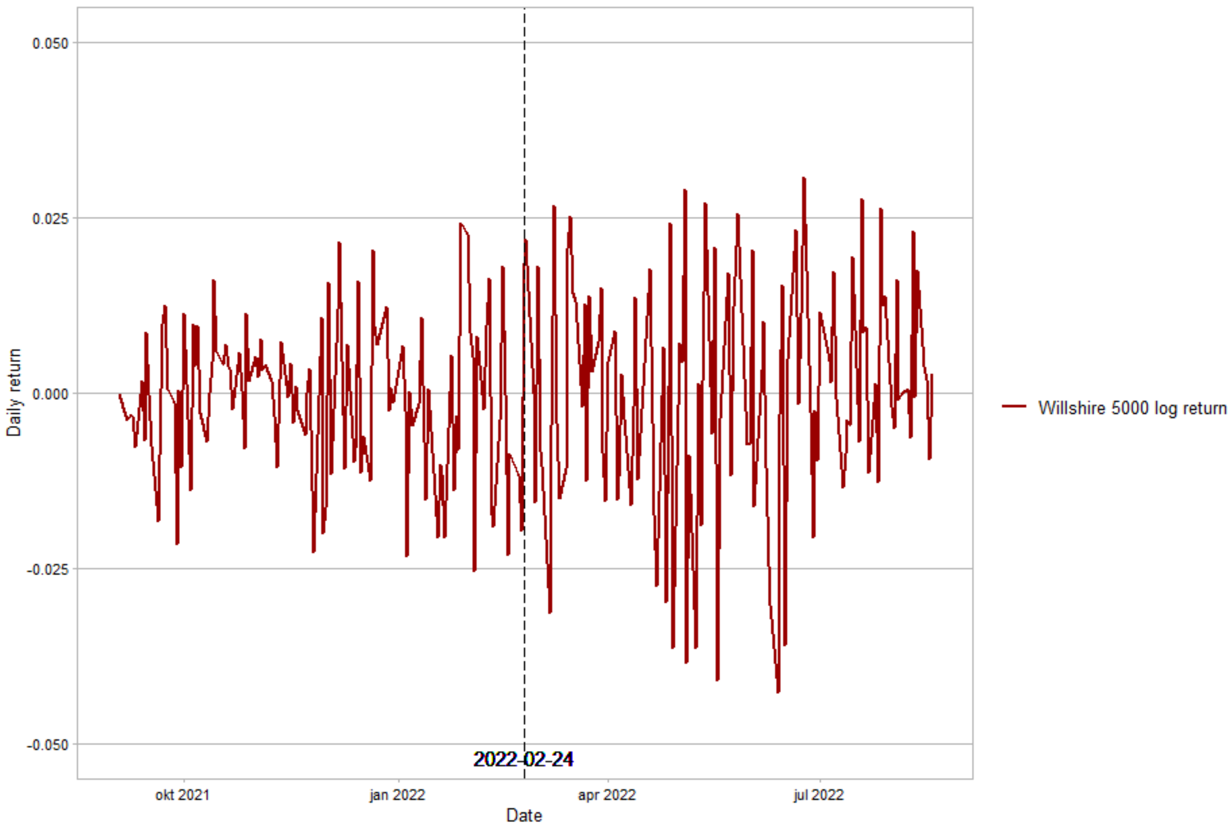
Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

9.1.2 Euro Stoxx 50 index

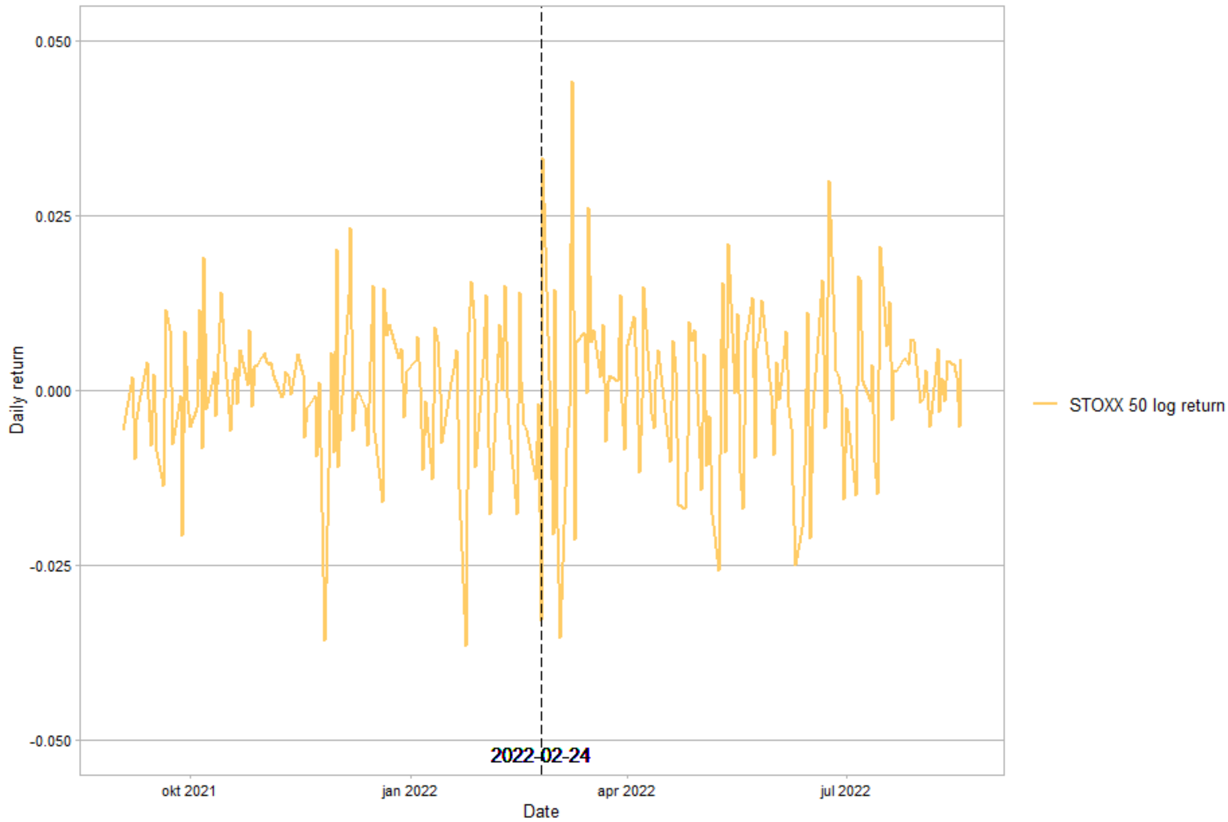


Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

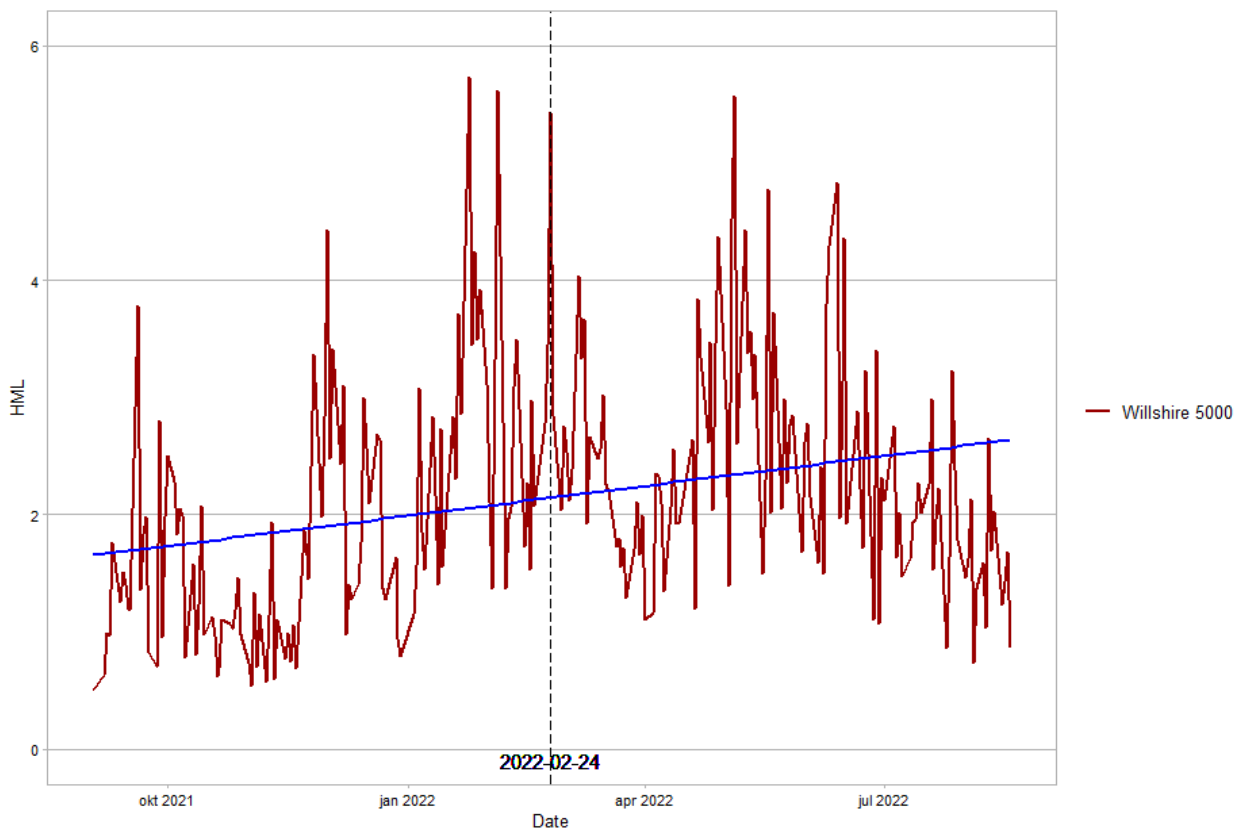
9.1.3 Stock market index return US



9.1.4 Stock market index return EU



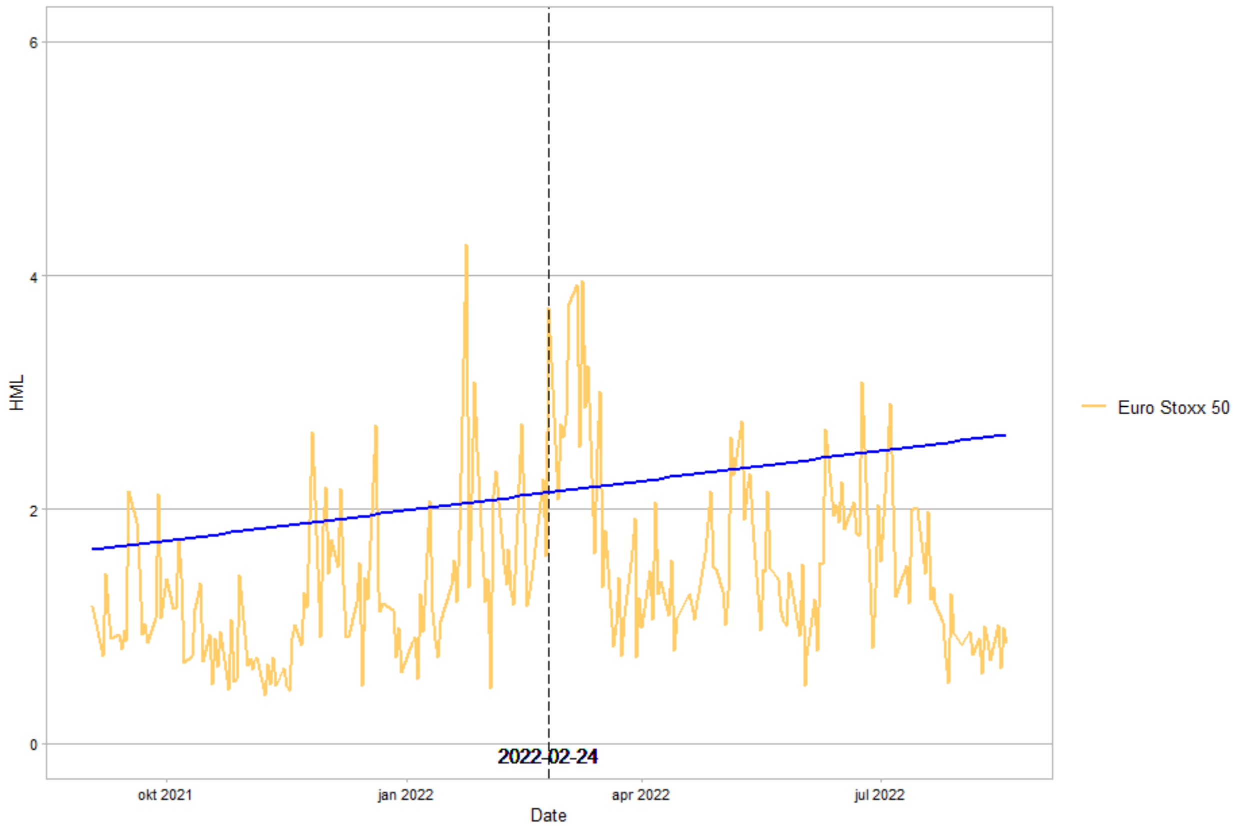
9.1.5 HML US



Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. The blue line gives the overall trend.

Source: Author database, processed through R.

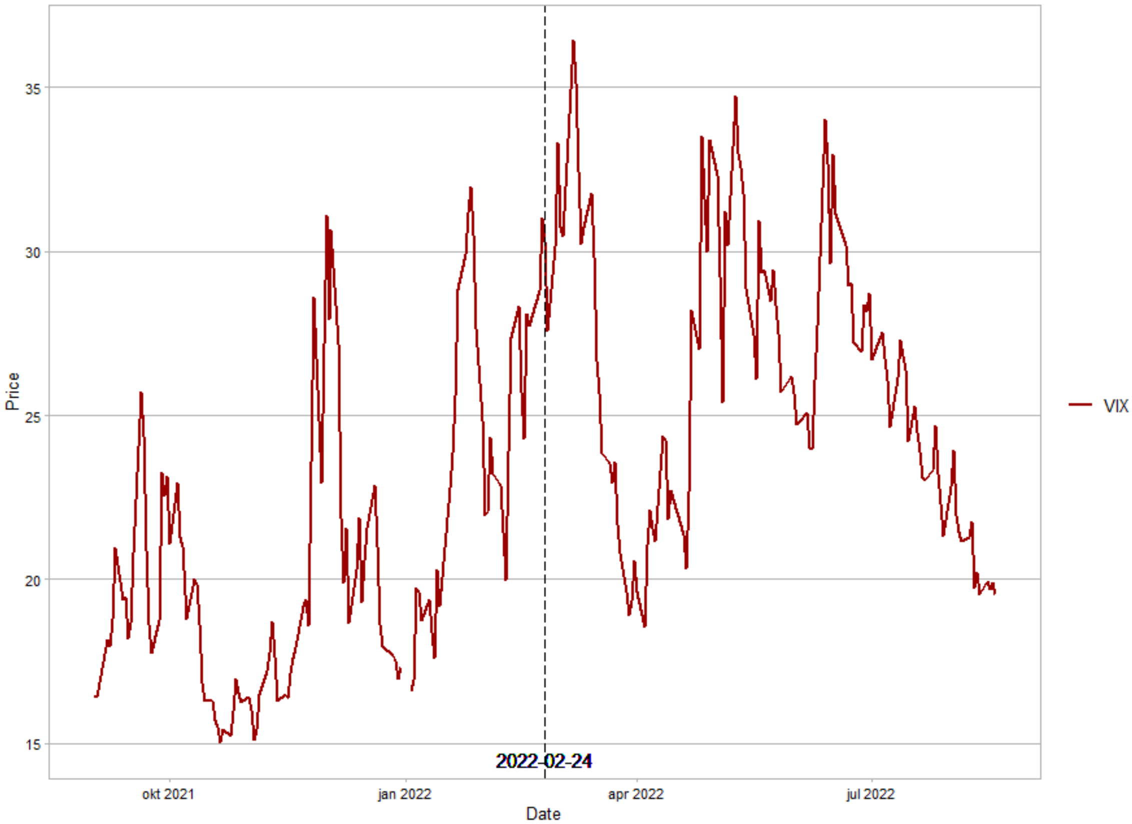
9.1.6 HML EU



Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. The blue line gives the overall trend.

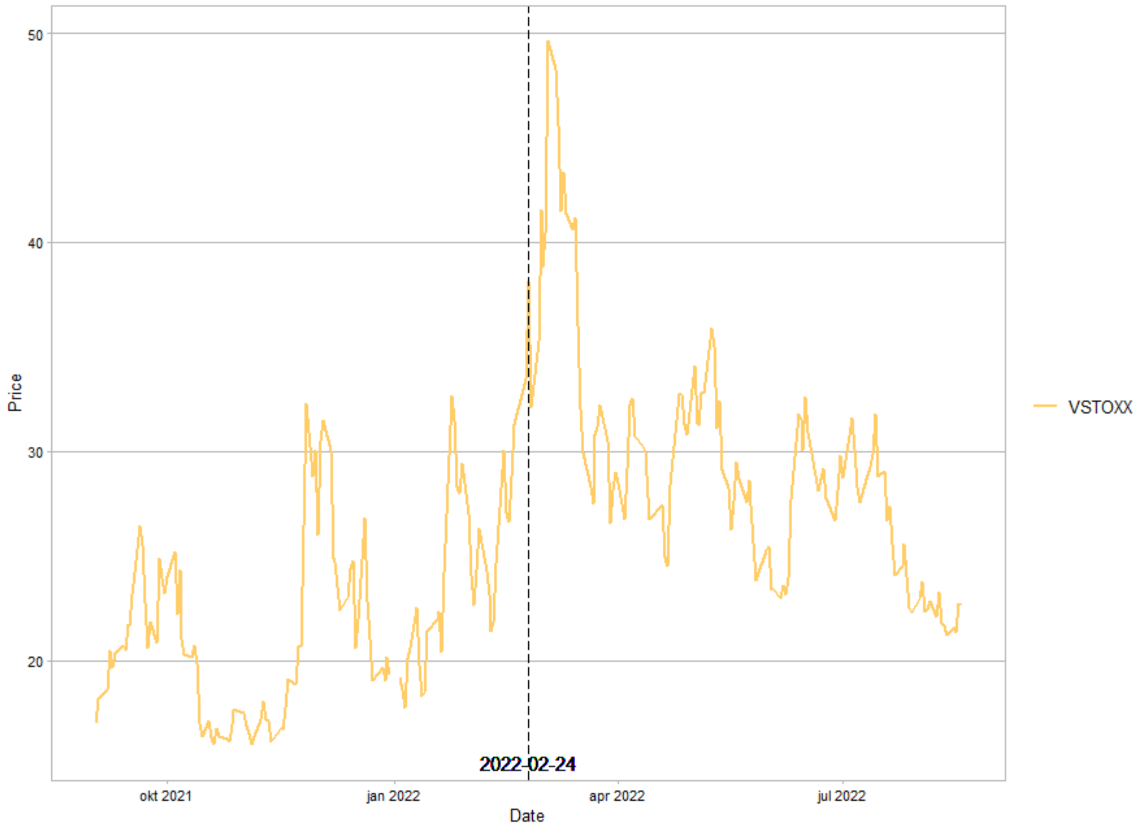
Source: Author database, processed through R.

9.1.7 VIX price



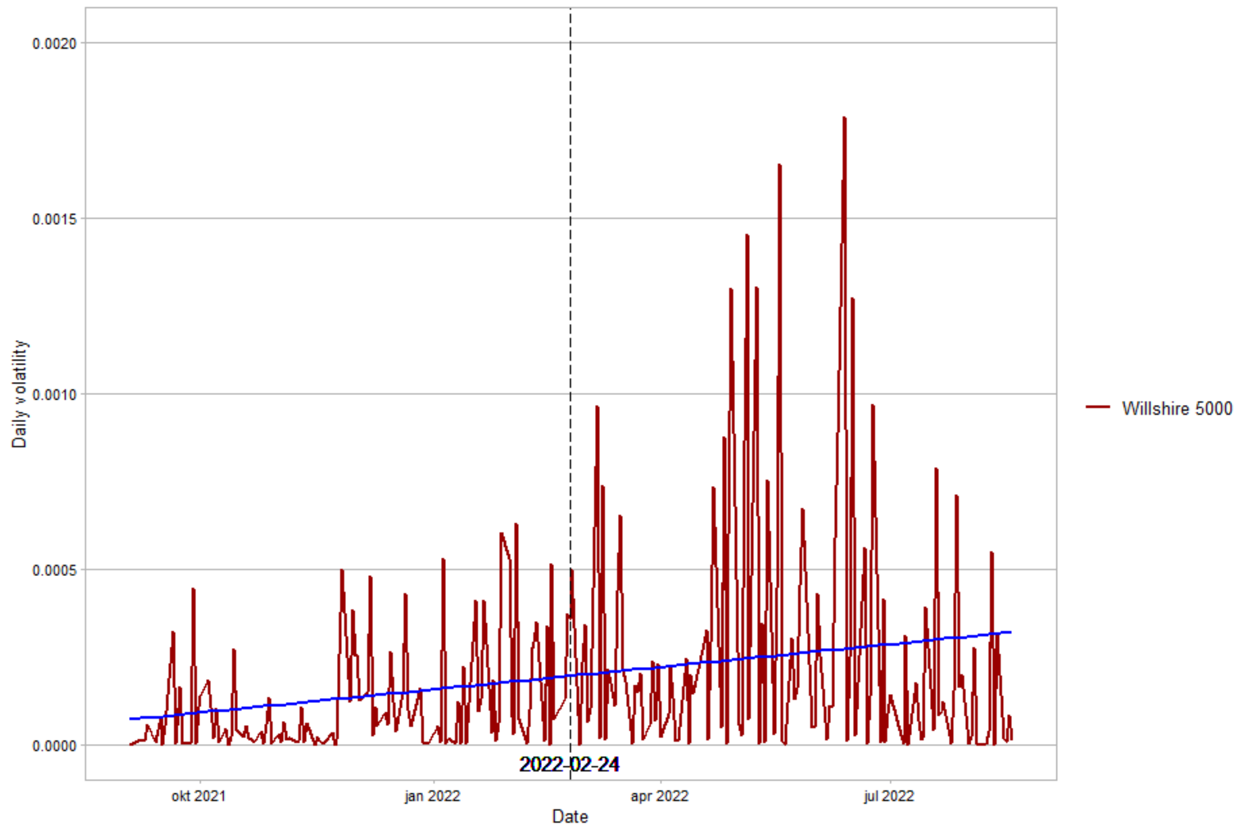
Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

9.1.8 VSTOXX price



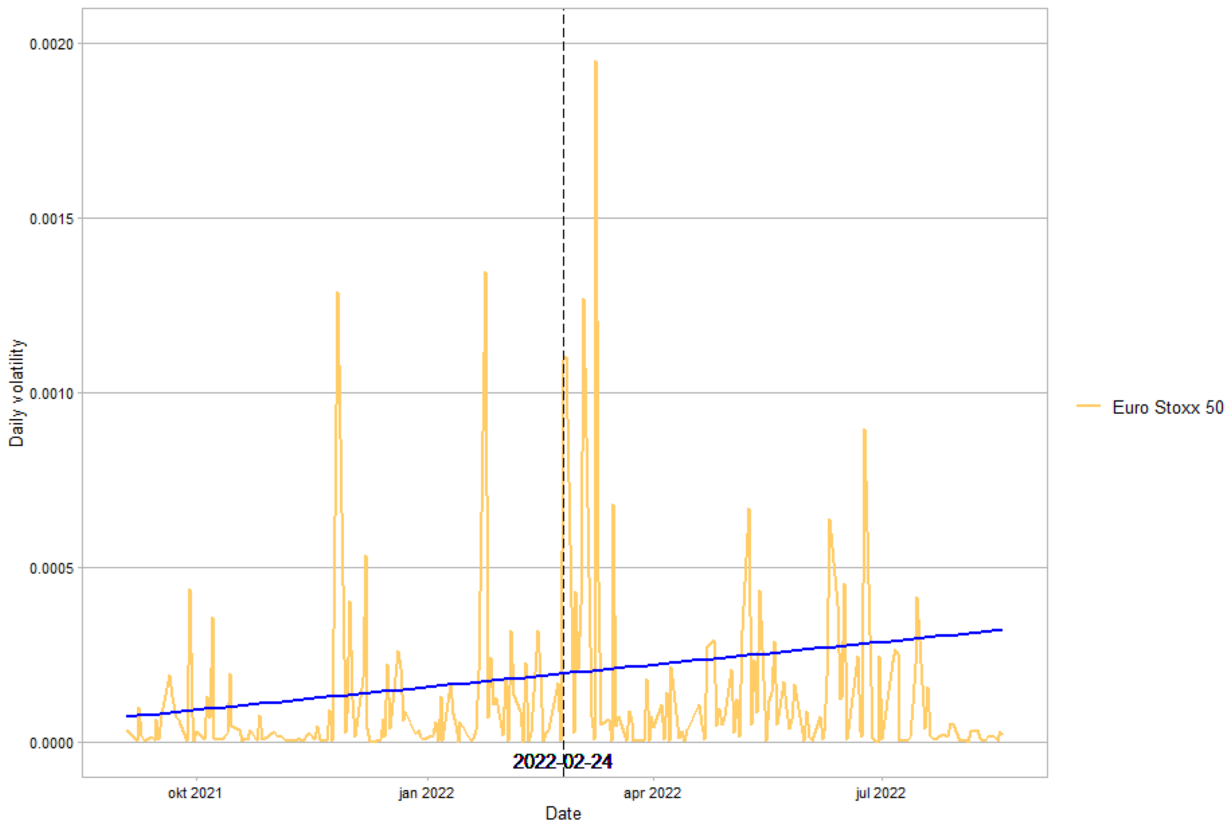
Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

9.1.9 Daily volatility US



Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. The blue line gives the overall trend.
Source: Author database, processed through R.

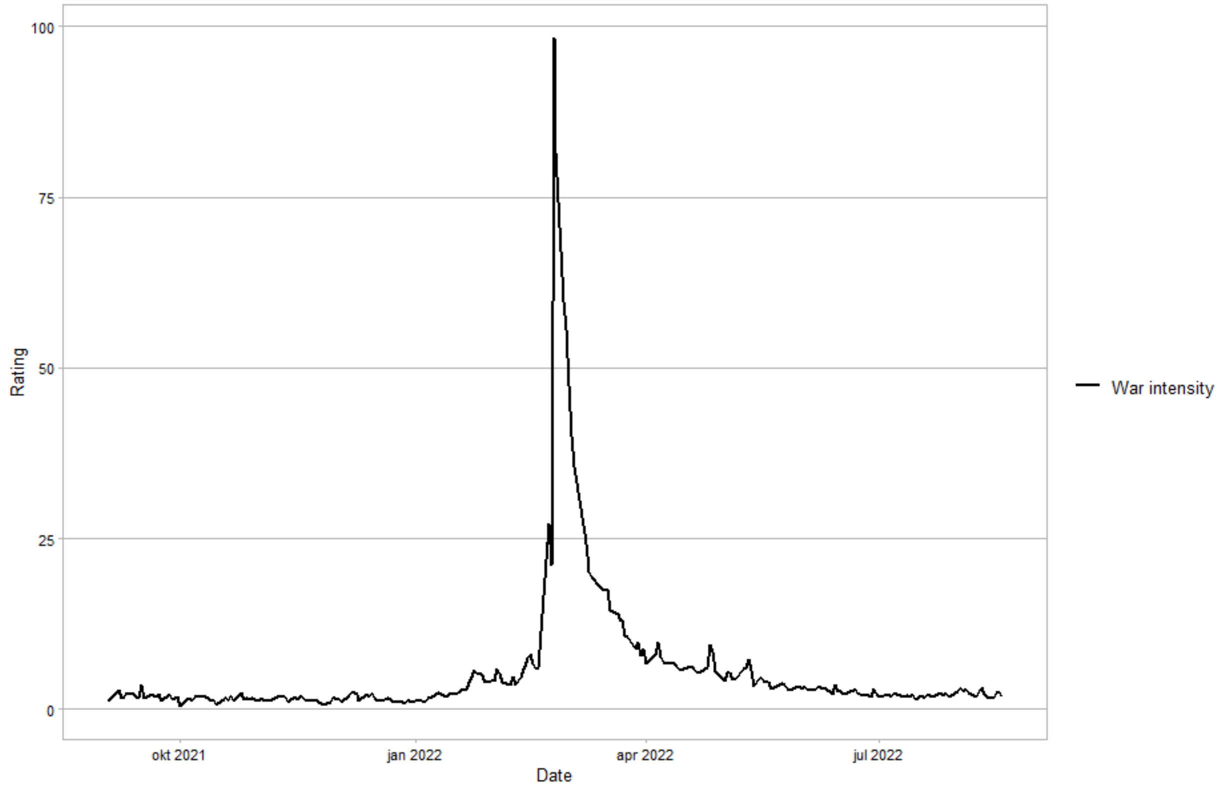
9.1.10 Daily volatility EU



Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. The blue line gives the overall trend.

Source: Author database, processed through R.

9.1.11 War attention intensity rating



Notes: 2022-02-24 marks the date of the Russian invasion of Ukraine. *Source:* Author database, processed through R.

9.2 American stock market index

The S&P 500 index is most often used to represent the American stock market. Due to constraints on daily volatility data, this study uses the Willshire 5000 index instead. Consequently, it is appropriate to test whether these two indices are similar, and thus whether the Willshire 5000 is a proper substitute. S&P500 data from Yahoo Finance is used for this comparison. This data is of insufficient quality to conduct this paper's main analysis but serves in this case. Figure 3. shows a comparison of both indices. Analysis shows the indices to have a pairwise correlation coefficient of 0.986. Given the very strong correlation between the two indices, it is likely that the volatility of both indices behaves similarly as well. The Willshire 5000 can thus be used to substitute for the S&P500 index.

Figure 3. S&P500 and Willshire 5000 indices

Notes: Both indices are normalized to allow for comparison. *Source:* Author database, processed through R.

9.3 Implied volatility derivation

Implied volatility is derived from option pricing. This is done using option pricing models. Commonly, the Black-Scholes model is used:

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Where:

$C(S,t) =$	Call option price,
$N(\dots) =$	Cumulative standard normal distribution function,
$T =$	Time to maturity,
$S =$	Stock price,
$K =$	Strike price,
$r =$	risk-free rate,
$\sigma =$	(implied) volatility,

The Black-Scholes model is solved for volatility by inputting all other variables. For this paper, it is not important to understand each variable. The main takeaway is that volatility can be inferred from the known properties of options.

Endnotes.

[i] The actual score for the war intensity variable ranges from 0.52 to 98.25. This difference is negligible, and rounding makes interpretation more straightforward.

[ii] In literature, the S&P 500 index is more often used to represent US financial market. Instead, the Willshire 5000 index has been used as S&P 500 index data of sufficient quality is not readily available. Refer to Appendix 9.2 for justification.

[iii] Page view statistics can be downloaded at https://pageviews.wmcloud.org/.

[iv] The United States' mean stock market index return has historically been positive. The negative mean log return is the consequence of the relatively small time horizon used.

[v] When log returns are nearly zero, they approximate normal or simple returns.
e.g. $e^{log\ return} = e^0 - 1 = 0 = simple\ return$

[vi] This calculation highlights the usefulness of the additive nature of logarithmic returns.

[vii] Multiplying the value of $HML_{r,t}$ by the average price of other US or EU indices that are strongly correlated – such as the S&P 500 for the US – gives the absolute daily differences between the highest and lowest prices for those indices.

[viii] Because $DVOL_{r,t} = Return_{r,t}^2$: $\sqrt{DVOL_{r,t}} = Return_{r,t} = \ln\left(1 + \frac{Index_{r,t} - Index_{r,t-1}}{Index_{r,t-1}}\right)$.

[ix] This has implications for the distribution of War_t. Section 6 of this paper discusses and accounts for the issue.

[x] Boungou and Yatié (2022) also used the term "conflict". Yet, analysis shows that the correlation between this term and the other terms is very low. Therefore, the term has been dropped from the analysis.

[xi] Studies commonly use index returns instead of prices, given that returns tend to be stationary, while prices often have an upward trend.

[xii] Boungou and Yatié in their work recommend replication of their study using a longer time horizon.

[xiii] Section 6.1 of this paper demonstrates that this may be because of the delayed effects of the Ukraine war on volatility.

[xiv] Negative effects on returns were found by Boungou and Yatié (2022) to be significant for at least four weeks after the Russian invasion. These negative effects were found to decline over time.