Individual differences in infants' cognitive abilities underlying learning

Bachelor's Thesis in Artificial Intelligence by

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Abstract

Infants possess sophisticated learning strategies. Nevertheless, it is still unknown to what extent infants differ in these strategies and the cognitive factors underlying them. In this paper, we propose a Bayesian cognitive model to infer individual differences in infants' learning and attention from their looking behavior. Using this model, we replicated previous findings showing that infants tailor their attention depending on the learning progress offered by environmental stimuli. However, limitations in our data did not allow us to find individual differences in infants' cognitive abilities. We conclude that the current model holds promise to find individual differences if coupled with richer eye-tracking measures.

Keywords: individual differences, statistical learning, curiosity, learning progress, Bayesian cognitive modelling

Introduction

In the first year of life, infants must learn an incredible amount of information from the world around them. From birth, infants possess simple heuristics to guide their attention towards certain informative features in their environment. For instance, they are better able to detect objects and perceive their shapes because their gaze is pulled toward areas of high contrast (Salapatek & Kessen, 1966). Along similar lines, detecting animacy is helped by a bias towards motion onset (Aslin & Shea, 1990). Furthermore, infants are helped in picking up social information and cues that guide language learning from faces (Baldwin, 1993) by an innate bias to orient towards asymmetric, face-like stimuli (Farroni et al., 2005; Johnson, Dziurawiec, Ellis, & Morton, 1991). However, to go from these rudimentary biases to advanced and complex forms of social, linguistic and high-order cognitive abilities, infants require powerful learning mechanisms

Recent developmental work is starting to uncover these mechanisms (e.g., Pelz, Piantadosi, & Kidd, 2015; Kidd, Piantadosi, & Aslin, 2012; Addyman & Mareschal, 2013; Poli, Serino, Mars, & Hunnius, 2020). In a study by Poli, Serino, Mars, and Hunnius (2020), it was demonstrated that infants attend to stimuli in a way that maximizes their

learning. The authors found that infants were more likely to look away once the presented stimuli offered little to no learning progress. This can be an effective strategy which allows infants to avoid spending time on stimuli they already know or are (at that moment) too complicated to learn more about. Although learning progress was the strongest predictor of infant's look-away, stimulus surprise and predictability also played a role in determining infants' looking behaviour. These results support the idea that infants possess sophisticated learning strategies.

Yet, these findings might not tell the complete story. Like most of the studies on infants, these results were solely analyzed on the group-level. That is, statistical procedures were used to make inferences from the full group to generalize to the population but did not focus on infants individually. This approach is sensible because it is often difficult to obtain sufficient data from infants, as the data collection is greatly limited by their reduced attention span, occasionally interrupted by non-compliance or stopped altogether due to fuzziness. As a consequence, data is often very noisy which might hinder the detection of individual differences. In fact, under these conditions, even if individual differences might be present, statistical tests would be unlikely to find them.

However, detecting individual differences in infants' cognitive abilities would be highly beneficial, as they could be early predictors of later cognitive performance and possibly psychosocial wellbeing. For example, a vast number of studies found that it is possible to predict later IQ from measures of habituation in the first year of life. Relations have been found between the speed of habituation (Bornstein & Columbo, 2012) as well as attention to a novel stimulus and later cognitive development (Rose & Wallace, 1985). Correlations between habituation and IQ were spanning over 25 years (Fagan, Holland, & Wheeler, 2007) and explaining as much as 40% of the variance in cognitive performance (Fagan & McGrath, 1981). Moreover, studies on adults showed that depression and anxiety can be inferred from their performance in learning tasks (Mukherjee, Filipowicz, Vo, Satterthwaite, & Kable, 2020; Browning, Behrens, Jocham, O'Reilly, & Bishop, 2015). The authors were able to characterize learning differences related to psychosocial wellbeing, using a Bayesian model.

Related developmental work has already applied Bayesian modelling to infer individual cognitive abilities (Piantadosi, Kidd, & Aslin, 2014). In contrast to their work, we will focus on finding, rather than disproving, differences in these estimates. Thus, this paper will analyse the results from Poli et al. (2020) using Bayesian cognitive modelling to assess individual differences in infants' early attentional and learning skills .

First, we will elaborate on the experimental paradigm used to collect the data. Next, we describe our Bayesian model and inference. Then, we apply it to the acquired data to assess individual differences. Lastly, we will discuss our results and their implications.

Methods

Participants

Eighty-five 8-month-old infants (M = 7.97, 42 females) were recruited for the study from a database of volunteer families. These included all participants from Poli et al. (2020) and 35 more. Infants had to carry out at least 20 trials to be included in the analysis. Fifteen

infants failed to reach this threshold and were hence excluded from the analysis. The final sample consisted of 70 infants (M = 8.0, 34 females).

Experimental Paradigm

The experiment consisted of a visual statistical learning task in which all infants were shown in total sixteen sequences of cue-target trials. In each sequence, the cue consisted of a simple shape appearing in the middle of the screen. The target was the same shape appearing in one of four screen quadrants around the cue location. See figure C1 for an example of such a trial. The shape (e.g. a trefoil or a star) was the same across all trials of the sequence but changed across sequences. Four out of the sixteen sequences were deterministic, as the target always appeared in the same location. The other twelve sequences were probabilistic: the target could appear in any location, but one location was more likely than the others. Infants could, therefore, learn to predict the most likely target location of each sequence.

The sequences were shown one after the other. When the infant looked away from the screen for one second or more, the sequence was stopped. When the infant looked back to the screen, the following sequence was played. The experiment lasted until the infant had watched all sixteen sequences or became fussy. Parents were instructed not to interact with their child, unless infants sought their attention and, even in that case, not to try to bring infants' attention back to the screen. During the experiment, the looking behaviour of the infant was monitored using a Tobii X300 eye-tracker. For a more detailed description of the experimental paradigm, see the supplementary materials of Poli et al. (2020).

From the data obtained in this setup, we aim to infer individual differences in infants' early attentional and learning skills using a Bayesian cognitive model.

Model

Our model aims at detecting whether infants use different strategies to allocate their attention during learning. In this model we assumed that they could either use surprise or learning progress or a combination of both. In addition, the model allows for individual variation in attentional and learning skills. First, infants might differ in their baseline attention. Say infant one is more easily distracted than infant two and thus consistently looks away from the screen sooner. Second, they might vary in the extent to which they integrate new information. Infant one might have a high learning rate and rely heavily on new information, whereas infant two might have a low learning rate and prioritize older information instead.

A graphical depiction of the model, following guidelines of Lee and Wagenmakers (2013), can be found in figure 1. The shaded node A denotes the look away data, i.e. for each sequence of every participant at what trial they looked away. The model can predict this using a categorical distribution, with an estimate of the probability of looking away for every trial of that sequence (depicted by PA). These probabilities are obtained by transforming the result of a linear regression to a value between 0 and 1 using an inverse logit function. Adhering to the linear effects reported in Poli et al. (2020), this regression uses Shannon Information (I) quantifying surprise and Kullback-Leibner Divergence (Dkl) quantifying learning progress with predictor weights β^I and β^D for I and Dkl and intercept

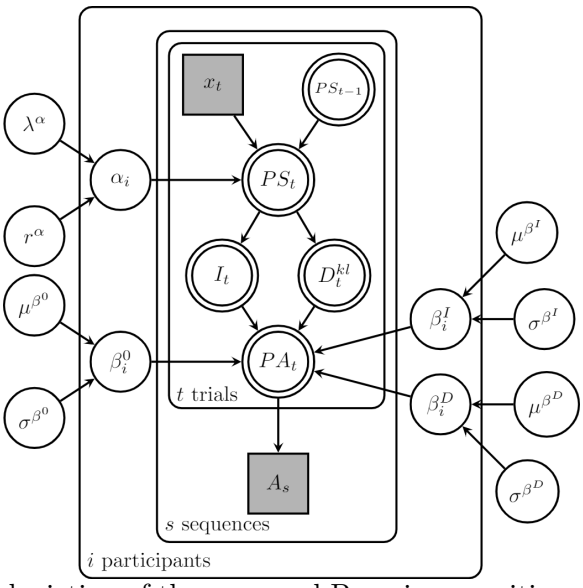

Figure 1. A graphical depiction of the proposed Bayesian cognitive model, following guidelines of Lee and Wagenmakers (2013).

β^0. We argue that β^0 can be interpreted as a measure of baseline attention as it influences the trial of look away, regardless of the stimuli.

To calculate surprise and learning progress, the model first predicts the probability of the stimulus appearing in each of the locations (PS). Using a learning rate (α), these are updated every trial depending on the location x in which the next target appears. A table summarizing the parameters and their meanings can be found in table 1.

Table 1
Parameters

parameter	meaning
α	Learning rate
β^0	Baseline attention
β^I	Linear predictor weight of surprise
β^D	Linear predictor weight of learning progress

Inference

To find individual differences, we have to fit our data to the model and learn about the latent parameters (α, β^0, β^I and β^D). This process is referred to as Bayesian inference. Unlike classical approaches, this will provide us with a probability distribution (called the posterior distribution) over these parameter values. The more these distributions differ between participants, the more (cognitively) different the infants are shown to be.

However, obtaining these posterior distributions is not an easy task. For many models there is no simple analytical solution and therefore some sort of approximation has to be

Table 2
Group Results

	α	β^0	β^I	β^D
HDI min	0.00	-2.39	-0.45	-26.74
HDI max	6.48	-0.87	0.96	0.57
BF	6.14	280.18	0.17	17.44

used. A common approximation method in Bayesian inference is called Markov Chain Monte Carlo (MCMC). MCMC encompasses drawing many samples from a posterior distribution to approximate it. By taking a biased random walk across the parameter space, it uncovers the range of likely parameter values. It favors spaces of high probability, but also visits areas of low probability such that, in the limit, it correctly samples from the full distribution.

For our analysis we use JAGS (Plummer, 2003), a program that implements slice samplers, a subtype of MCMC. We ran the model for 300.000 iterations of which 150.000 burn-in, drawing a sample every ten steps on eight chains. We assessed convergence visually and using the Rhat statistic.

In our analysis we will focus on determining the presence of individual differences. First, we will determine the group effect of surprise and learning progress using highest density intervals and Bayes Factors using the Savage-Dickey method. Following that, we will show the individual differences for all relevant variables using the same methods.

Results

Figures 2 to 5 show the 95% Highest Density Intervals (HDI) and Bayes factors for the inferred parameters visually. Note that here the Bayes factors are shown on a log scale, to aid visual inspection. Furthermore, the group and individual level estimates can be found numerically in tables 2 as well as C2 and C3, respectively. Specifically, we obtain HDI [-0.45, 0.96] and Bayes factor 0.17 for surprise and HDI [-26.74, 0.57] and Bayes factor 17.44 for learning progress on the group level.

Judging from the individual estimates visualized in sub figures B and D, we do not seem to find any individual differences in the cognitive capacities of infants. With the biggest difference in HDI of [0.01, 5.16] and [0.04, 7.27] for α we find no clear distinction. The same also applies for β^0 with [-2.58, -1.28] and [-2.00, -0.68] and β^I with [-0.57, 0.64] and [-0.11, 1.01] as well as β^D with [-27.34, -0.16] and [-25.96, 0.78]. Interestingly, the plots show wide distributions which we address in the discussion.

Discussion

In this paper, we developed a Bayesian Cognitive model to detect individual differences in infants' learning and attention. We allowed for differences in the use of surprise and learning progress as well as the learning rate and sustained attention.

The results found on the group level are similar to those found in Poli et al. (2020). The posteriors show a negative relation between the probability of look away and learning progress. The linear effect of learning progress is supported with strong evidence by the

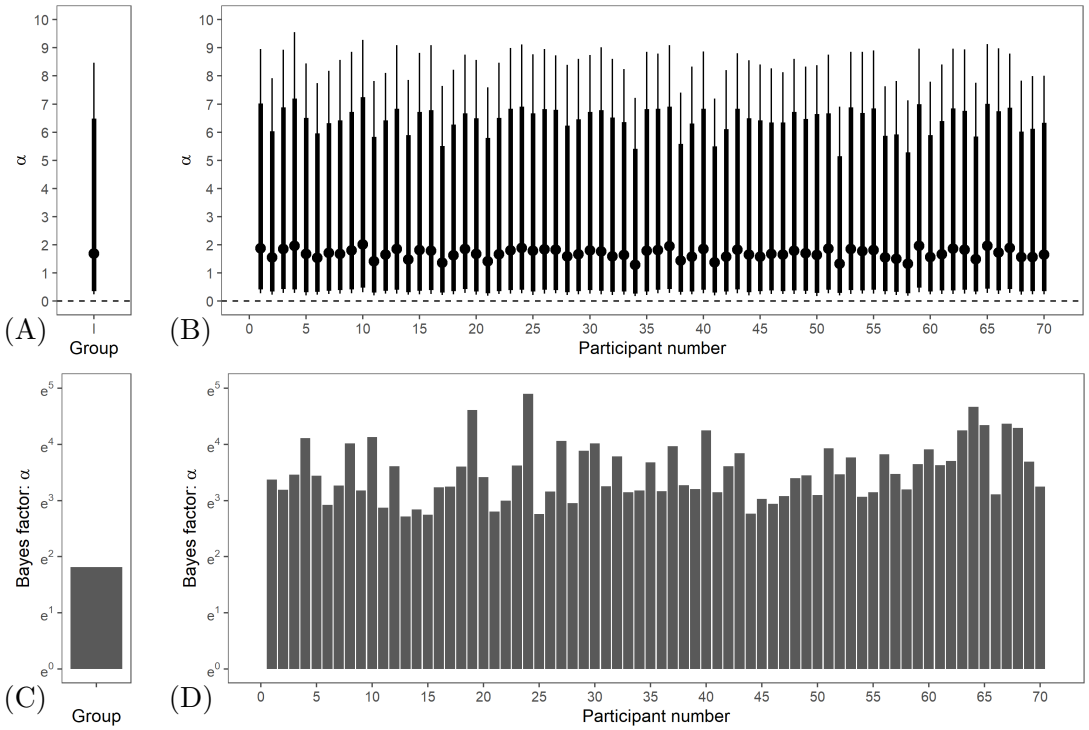

Figure 2. Estimates for *alpha*, the learning rate parameter.

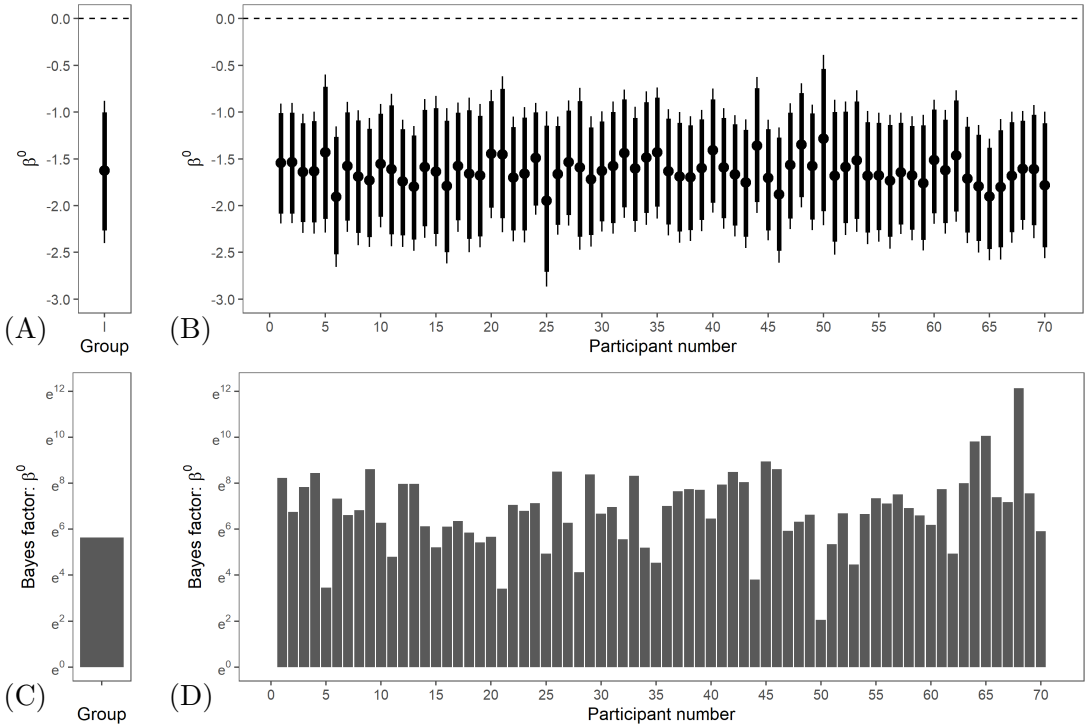

Figure 3. Estimates for β^0, the intercept or sustained attention.

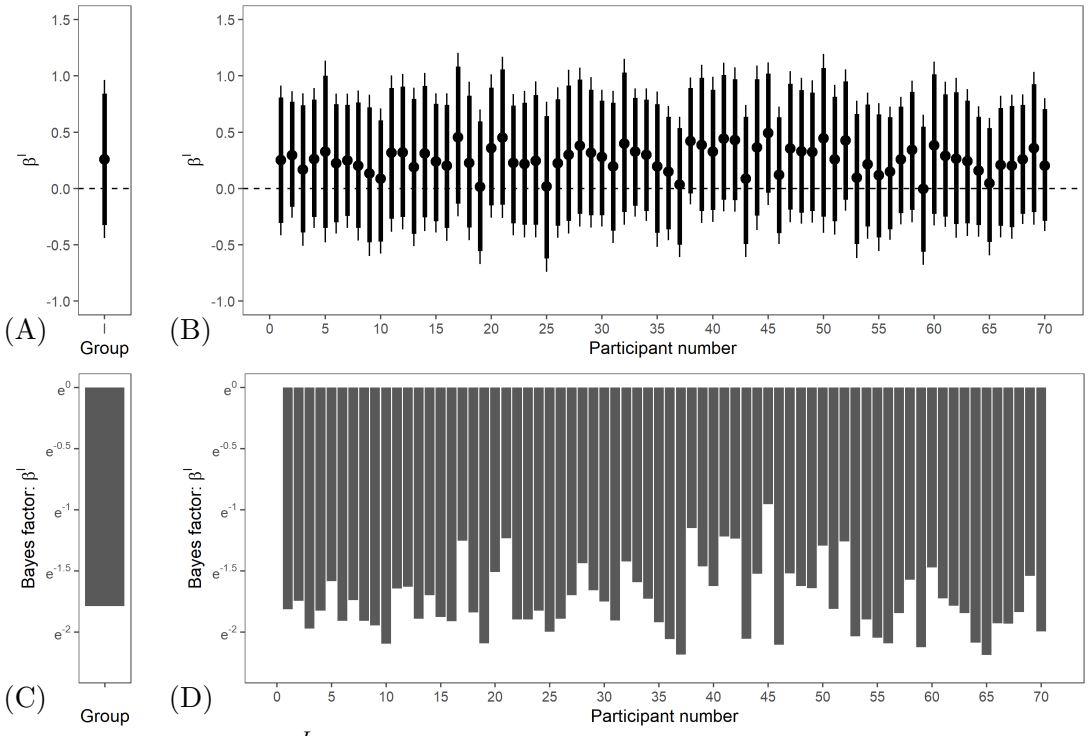

Figure 4. Estimates for β^I, the predictor weight of surprise.

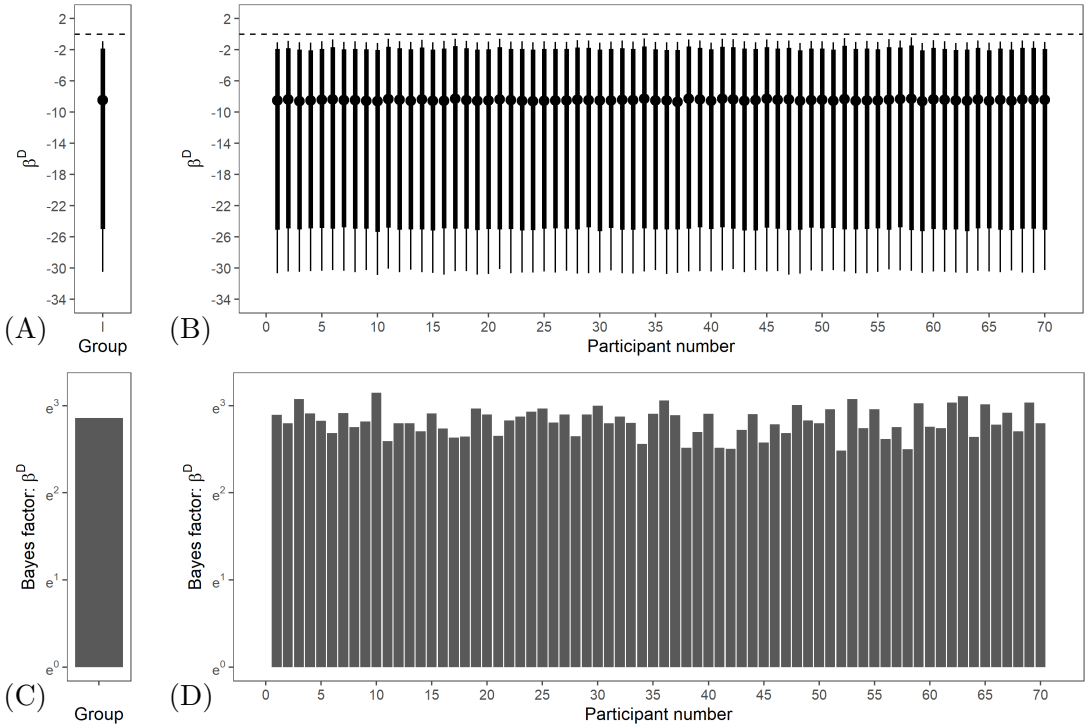

Figure 5. Estimates for β^D, the predictor weight of learning progress.

Bayes factor (see table C1). However, there is also moderate evidence against the relation between the probability of look away and surprise.

Despite our efforts, the task and the analysis used did not allow us to detect individual differences.

A simple explanation would be that infants do not differ cognitively. However, that would contradict much literature (e.g. Rankin et al., 2009; Colombo, 1993) and there is good evidence that the cognitive differences do exists between infants but that we were not able to find them. Here we will address several potential causes regarding data collection and our methods. Whenever possible, we will also supply improvements for future research.

As stated in the introduction, infant data is greatly limited. Furthermore, our data might show a lack of individual differences because of a biased sample. We have, for example, no high-risk infants in this sample. Additional data collection, specifically from high-risk infants, would test this alternative hypothesis.

Regarding our Bayesian model, we have used look away as the sole dependent variable. Reducing rich eye-tracking data to this single numerical measure per sequence might greatly reduce our explanatory power. This exemplification is also supported by the large Highest Density Intervals found in the estimates. To overcome this limitation, more and richer measures of looking behaviour might be added to the model. Following Poli et al. (2020) for example dwell time and saccadic latencies could be used.

Lastly, one might expect individual differences to be shrouded by a pooling effect. This seems sensible because by specifying group level priors on the parameters, the estimates for these parameters are pulled towards a group mean (Gelman & Hill, 2006). Even so, Stein's paradox illustrates the flaw in this reasoning. Stein's paradox states that the best estimate of a person's true ability is not their own performance, but an adjusted measure that brings an individual's performance estimate more in line with the observations for all other individuals (Pharrell & Lewandowski, 2018). Thus, rather than pooling, limitations in our data are likely to conceal individual differences.

Conclusion

Using Bayesian Cognitive modelling, we have replicated the effect of learning progress on look-away and found moderate evidence against a linear effect of surprise. Furthermore, we have been unable to identify individual differences. We have identified limitations in our data and our model as possible causes. Most notably, our conclusions were limited by having look away as the single dependent variable. Our proposed improvements thus do not only limit themselves to additional data collection but focus on including more and richer measures of looking behaviour.

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References

Addyman, C., & Mareschal, D. (2013). Local Redundancy Governs Infants' Spontaneous Orienting to Visual-Temporal Sequences. *Child Development*, *84*(4), 1137–1144. Retrieved from http://doi.wiley.com/10.1111/cdev.12060 doi: 10.1111/cdev.12060

Aslin, R. N., & Shea, S. L. (1990). Velocity Thresholds in Human Infants: Implications for the Perception of Motion. *Developmental Psychology*. doi: 10.1037/0012-1649.26.4.589

Baldwin, D. A. (1993). Infants' ability to consult the speaker for clues to word reference. *Journal of Child Language*. doi: 10.1017/S0305000900008345

Bornstein, M. H., & Columbo, J. (2012). Infant cognitive function and mental developoment. In *The jackbs foundation series on adloescence. early childhood developmetn and later outcomes.*

Browning, M., Behrens, T. E., Jocham, G., O'Reilly, J. X., & Bishop, S. J. (2015, 4). Anxious individuals have difficulty learning the causal statistics of aversive environments. *Nature Neuroscience*, *18*(4), 590–596. Retrieved from http://www.nature.com/articles/nn.3961 doi: 10.1038/nn.3961

Colombo, J. (1993). *Infant cognition* (Vol. 5). Sage.

Dijkstra, N., Hinne, M., Bosch, S. E., & van Gerven, M. A. J. (2019, 12). Between-subject variability in the influence of mental imagery on conscious perception. *Scientific Reports*, *9*(1), 15658. Retrieved from http://www.nature.com/articles/s41598-019-52072-1 doi: 10.1038/s41598-019-52072-1

Fagan, J. F., Holland, C. R., & Wheeler, K. (2007). The prediction, from infancy, of adult IQ and achievement. *Intelligence*. doi: 10.1016/j.intell.2006.07.007

Fagan, J. F., & McGrath, S. K. (1981). Infant recognition memory and later intelligence. *Intelligence*. doi: 10.1016/0160-2896(81)90002-7

Farroni, T., Johnson, M. H., Menon, E., Zulian, L., Faraguna, D., & Csibra, G. (2005). Newborns' preference for face-relevant stimuli: Effects of contrast polarity. *Proceedings of the National Academy of Sciences of the United States of America*. doi: 10.1073/pnas.0502205102

Gelman, A., & Hill, J. (2006). *Data Analysis Using Regression and Multilevel/Hierarchical Models*. doi: 10.1017/cbo9780511790942

Jeffreys, H. (1961). *The Theory of Probability*. OUP Oxford.

Johnson, M. H., Dziurawiec, S., Ellis, H., & Morton, J. (1991). Newborns' preferential tracking of face-like stimuli and its subsequent decline. *Cognition*. doi: 10.1016/0010-0277(91)90045-6

Kidd, C., Piantadosi, S. T., & Aslin, R. N. (2012). The Goldilocks effect: Human infants allocate attention to visual sequences that are neither too simple nor too complex. *PLoS ONE*, *7*(5), 1–8. doi: 10.1371/journal.pone.0036399

Lee, M. D., & Wagenmakers, E.-J. (2013). *Bayesian Cognitive Modeling*. Cambridge: Cambridge University Press. Retrieved from http://ebooks.cambridge.org/ref/id/CBO9781139087759 doi: 10.1017/CBO9781139087759

Mukherjee, D., Filipowicz, A. L. S., Vo, K., Satterthwaite, T. D., & Kable, J. W. (2020). Reward and punishment reversal learning in major depressive disorder. doi: 10.31234/osf.io/aqgx3

Pelz, M., Piantadosi, S. T., & Kidd, C. (2015). The dynamics of idealized attention in complex learning environments. *5th Joint International Conference on Development and Learning and Epigenetic Robotics, ICDL-EpiRob 2015*, 236–241. doi: 10.1109/DEVLRN.2015.7346147

Piantadosi, S. T., Kidd, C., & Aslin, R. (2014). Rich analysis and rational models: Inferring individual behavior from infant looking data. *Developmental Science*, *17*(3), 321–337. doi: 10.1111/desc.12083

Plummer, M. (2003). *JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling.*

Poli, F., Serino, G., Mars, R. B., & Hunnius, S. (2020). Infants tailor their attention to maximize learning. *under review*.

Rankin, C. H., Abrams, T., Barry, R. J., Bhatnagar, S., Clayton, D. F., Colombo, J., ... Thompson, R. F. (2009, 9). Habituation revisited: An updated and revised description of the behavioral characteristics of habituation. *Neurobiology of Learning and Memory*, *92*(2), 135–138. Retrieved from `https://linkinghub.elsevier.com/retrieve/pii/S1074742708001792` doi: 10.1016/j.nlm.2008.09.012

Rose, S. A., & Wallace, I. F. (1985). Visual Recognition Memory: A Predictor of Later Cognitive Functioning in Preterms. *Child Development*. doi: 10.2307/1130096

Salapatek, P., & Kessen, W. (1966). Visual scanning of triangles by the human newborn. *Journal of Experimental Child Psychology*. doi: 10.1016/0022-0965(66)90090-7

Wagenmakers, E. J., Lodewyckx, T., Kuriyal, H., & Grasman, R. (2010). Bayesian hypothesis testing for psychologists: A tutorial on the Savage-Dickey method. *Cognitive Psychology*, *60*(3), 158–189. Retrieved from `http://dx.doi.org/10.1016/j.cogpsych.2009.12.001` doi: 10.1016/j.cogpsych.2009.12.001

Appendix A
Mathematical model definition

Following the example of Dijkstra, Hinne, Bosch, and van Gerven (2019), we construct a Bayesian hierarchical model as follows. For i participants, s sequences with t trials we define the likelihood

$$\begin{aligned}
counts_0 &= [1,1,1,1] \\
counts_t &= counts_{t-1}[x_t] + \alpha_i \\
PS_t &\sim Dirichlet(counts_t) \\
I_t &= -\log_2 PS_{t,x_t} \\
D_t^{kl} &= PS_t * \log_2\left(\frac{PS_t}{PS_{t-1}}\right) \\
\theta_t &= ilogit(\beta_i^0 + \beta_i^I * I_t + \beta_i^D * D_t^{kl}) \\
PA_t &= \begin{cases} \theta_t * \prod_{x=1}^{t-1} 1-\theta_x & \text{if WATCHED}(t-3,\ldots,t) \\ 1 - \sum_{x=1}^{t-1} \theta_x & \text{if } t = 16 \\ 0 & \text{otherwise} \end{cases} \\
A_s &\sim Categorical(PA)
\end{aligned} \tag{1}$$

From WATCHED$(t-3,\ldots,t)$ it can be seen that only trials after the third one that the infant has observed can be predicted by the model. Furthermore $t = 16$ exemplifies no look away. For the exact details of the implementation, the code is available from the first author.

As is common practice, we center the prior distribution for each participant-level parameter on their population level equivalent to introduce hierarchy. In mathematical terms,

$$\begin{aligned}
\alpha_i &\sim Gamma(r^\alpha, \lambda^\alpha) \\
\beta_i^0 &\sim Gaussian(\mu^{\beta^0}, \sigma^{\beta^0}) \\
\beta_i^I &\sim Gaussian(\mu^{\beta^I}, \sigma^{\beta^I}) \\
\beta_i^D &\sim Gaussian(\mu^{\beta^D}, \sigma^{\beta^D})
\end{aligned} \tag{2}$$

For the population-level parameters we formalize the subsequent (hyper-)priors. We formalize the subsequent (hyper-)priors for the population-level parameters.

$$\begin{aligned} r^{\alpha} &\sim Gamma(3,1) \\ \lambda^{\alpha} &\sim Gamma(2,1) \\ \mu^{\beta^0} &\sim Gaussian(0,0.0001) \\ \sigma^{\beta^0} &\sim Gamma(1,1) \\ \mu^{\beta^I} &\sim Gaussian(0,0.0001) \\ \sigma^{\beta^I} &\sim Gamma(1,1) \\ \mu^{\beta^D} &\sim Gaussian(0,0.0001) \\ \sigma^{\beta^D} &\sim Gamma(1,1) \end{aligned} \tag{3}$$

All being vague, possibly with the exception of r^{α} and λ^{α}.

Appendix B
Bayes factor calculation

Bayes factors were calculated using the Savage-Dickey method (Wagenmakers, Lodewyckx, Kuriyal, & Grasman, 2010). For this method, we approximated both the posterior as well as the prior model probabilities and computed the ratio of these densities at the null value, i.e.:

$$BF_{10} = \frac{p_1(D)}{p_0(D)} = \frac{p_1(\phi = \phi_0)}{p_1(\phi = \phi_0 \mid D)} \tag{4}$$

Appendix C
Supplementary figures and tables

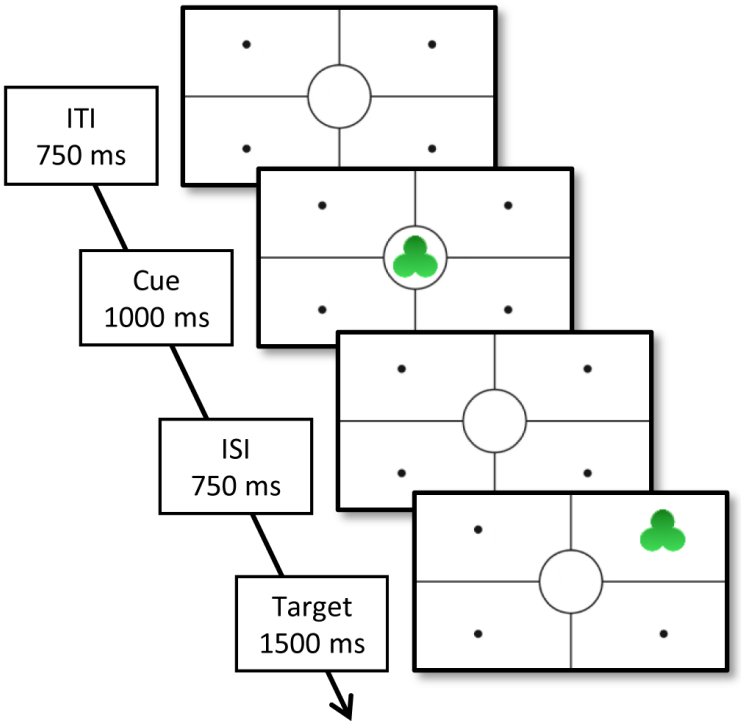

Figure C1. A trial of the learning task. The cue looms in the center of the screen and the target rotates in one of the four quadrants. ITI=Inter-Trial-Interval; ISI=Inter-Stimulus-Interval.

Table C1
Interpretation of Bayes factors (BF) according to Jeffreys (1961) and Lee and Wagenmakers (2013)
Adapted from (Dijkstra et al., 2019)

BF	log(BF)	Strength of evidence
> 100	4.6	Extreme evidence for H_+
$30 - 100$	$3.4 - 4.6$	Very strong evidence for H_+
$10 - 30$	$2.3 - 3.4$	Strong evidence for H_+
$3 - 10$	$1.1 - 2.34$	Moderate evidence for H_+
$1 - 3$	$0 - 1.1$	Anecdotal evidence for H_+
1	0	No evidence for H_0 or H_+
$1/3 - 1$	$-1.1 - 0$	Anecdotal evidence for H_0
$1/3 - 1/10$	$-2.3 - -1.1$	Moderate evidence for H_0
$1/10 - 1/30$	$-3.4 - -2.3$	Strong evidence for H_0
$1/30 - 1/100$	$-4.6 - -3.4$	Very strong evidence for H_0
$< 1/100$	< -4.6	Extreme evidence for H_0

Table C2
Individual Results

	α			β^0			β^I			β^D		
n	HDI min	HDI max	BF	HDI min	HDI max	BF	HDI min	HDI max	BF	HDI min	HDI max	BF
1	0.01	7.02	29.16	-2.17	-0.90	3,760.09	-0.42	0.90	0.16	-26.48	0.59	18.10
2	0.00	6.04	24.33	-2.17	-0.89	858.18	-0.26	0.87	0.17	-26.48	0.92	16.43
3	0.00	6.88	31.87	-2.25	-0.99	2,515.01	-0.49	0.86	0.14	-26.59	0.30	21.69
4	0.00	7.20	61.06	-2.26	-0.97	4,638.75	-0.35	0.89	0.16	-26.80	0.29	18.35
5	0.00	6.51	31.29	-2.30	-0.62	31.69	-0.49	1.12	0.21	-26.75	0.71	16.92
6	0.01	5.96	18.62	-2.64	-1.14	1,526.73	-0.39	0.85	0.15	-26.56	0.64	14.67
7	0.01	6.32	26.25	-2.27	-0.88	742.84	-0.34	0.85	0.18	-26.40	0.58	18.44
8	0.03	6.45	55.66	-2.39	-0.95	928.19	-0.47	0.86	0.15	-26.71	0.52	15.72
9	0.00	6.72	24.08	-2.44	-1.06	5,521.17	-0.62	0.81	0.14	-26.49	0.67	16.72
10	0.04	7.27	62.09	-2.25	-0.93	531.05	-0.57	0.71	0.12	-27.03	0.15	23.31
11	0.00	5.84	17.66	-2.44	-0.81	122.27	-0.36	1.03	0.19	-26.87	0.52	13.36
12	0.01	6.43	37.03	-2.45	-1.09	2,884.24	-0.35	1.02	0.20	-26.79	0.35	16.43
13	0.00	6.84	15.09	-2.47	-1.14	2,895.25	-0.51	0.90	0.15	-26.28	0.68	16.44
14	0.00	5.90	17.08	-2.33	-0.85	459.45	-0.37	1.03	0.18	-26.64	0.69	14.99
15	0.00	6.72	15.59	-2.44	-0.84	184.00	-0.40	0.83	0.15	-26.75	0.63	18.40
16	0.00	6.78	25.37	-2.63	-0.97	450.77	-0.42	0.88	0.15	-26.62	0.64	15.43
17	0.00	5.51	25.70	-2.26	-0.88	577.50	-0.25	1.19	0.29	-26.63	0.92	13.91
18	0.01	6.27	36.82	-2.49	-0.84	345.56	-0.47	0.93	0.16	-26.48	0.64	14.11
19	0.02	6.67	100.71	-2.44	-0.91	228.74	-0.67	0.70	0.12	-26.57	0.41	19.41
20	0.01	6.49	30.49	-2.15	-0.78	291.23	-0.22	1.04	0.22	-26.92	0.30	18.18
21	0.00	5.79	16.55	-2.25	-0.59	30.50	-0.27	1.16	0.29	-26.91	0.69	14.20
22	0.00	6.53	20.08	-2.36	-1.03	1,157.47	-0.40	0.86	0.15	-26.53	0.72	16.93
23	0.01	6.84	37.39	-2.36	-0.92	901.56	-0.45	0.84	0.15	-26.77	0.44	17.73
24	0.03	6.92	134.10	-2.10	-0.91	1,257.87	-0.44	0.95	0.16	-26.58	0.41	18.77
25	0.01	6.68	15.79	-2.84	-0.97	140.06	-0.73	0.78	0.14	-26.51	0.77	19.43
26	0.01	6.84	23.61	-2.28	-1.03	4,977.32	-0.44	0.89	0.15	-26.76	0.30	16.53
27	0.02	6.83	58.06	-2.19	-0.85	531.63	-0.42	1.03	0.18	-26.67	0.40	18.12
28	0.00	6.24	19.19	-2.43	-0.70	62.10	-0.33	1.08	0.24	-27.08	0.48	14.13
29	0.03	6.48	48.73	-2.44	-1.05	4,407.52	-0.36	0.98	0.19	-26.77	0.43	18.12
30	0.02	6.72	55.53	-2.28	-1.01	792.65	-0.34	0.87	0.17	-27.00	0.18	20.05
31	0.02	6.80	25.95	-2.30	-0.89	1,067.38	-0.46	0.89	0.15	-26.50	0.41	16.43
32	0.01	6.53	44.08	-2.12	-0.75	259.57	-0.34	1.12	0.24	-27.10	0.34	17.71
33	0.00	6.36	23.23	-2.23	-0.90	4,096.59	-0.25	0.89	0.20	-26.43	0.73	16.51
34	0.01	5.42	24.09	-2.20	-0.77	180.79	-0.29	0.89	0.18	-26.76	0.78	12.96
35	0.01	6.83	39.52	-2.11	-0.72	93.35	-0.50	0.88	0.15	-26.95	0.14	18.31

Table C3
Individual Results

	α			β^0			β^I			β^D		
n	HDI min	HDI max	BF	HDI min	HDI max	BF	HDI min	HDI max	BF	HDI min	HDI max	BF
36	0.05	6.63	0.70	-2.21	-0.89	340.96	-0.49	0.67	0.11	-27.50	1.52	9.97
36	0.00	6.84	23.78	-2.29	-0.95	1,106.05	-0.49	0.71	0.13	-26.96	0.23	21.30
37	0.02	6.91	52.91	-2.38	-0.98	2,109.13	-0.60	0.65	0.11	-26.63	0.39	18.03
38	0.00	5.59	26.39	-2.34	-1.01	2,321.52	-0.11	1.01	0.32	-26.55	0.76	12.38
39	0.00	6.31	24.60	-2.28	-0.98	2,226.38	-0.32	1.08	0.23	-26.48	0.53	14.86
40	0.02	6.84	70.09	-2.09	-0.76	638.15	-0.32	0.96	0.20	-25.96	0.78	18.32
41	0.00	5.50	23.31	-2.23	-0.95	2,813.01	-0.21	1.11	0.30	-26.21	1.40	12.37
42	0.01	6.11	36.97	-2.31	-1.01	4,867.40	-0.22	1.06	0.29	-26.06	1.31	12.22
43	0.01	6.83	46.70	-2.42	-1.06	3,148.84	-0.59	0.76	0.13	-26.86	0.54	15.18
44	0.00	6.49	15.85	-2.09	-0.64	44.93	-0.37	1.09	0.22	-26.88	0.22	18.21
45	0.00	6.42	20.70	-2.37	-1.08	7,750.02	-0.14	1.13	0.39	-26.31	1.12	13.15
46	0.00	6.35	18.95	-2.60	-1.16	5,472.95	-0.50	0.72	0.12	-26.45	0.65	16.22
47	0.00	6.35	21.69	-2.24	-0.90	376.47	-0.30	1.05	0.22	-26.72	0.75	14.67
48	0.01	6.73	29.94	-2.00	-0.68	558.99	-0.32	0.97	0.20	-26.37	0.54	20.23
49	0.01	6.47	31.42	-2.25	-0.91	752.04	-0.30	0.96	0.19	-26.53	0.50	16.93
50	0.00	6.64	22.20	-2.24	-0.42	7.81	-0.37	1.21	0.27	-26.68	0.50	16.41
51	0.01	6.68	50.93	-2.52	-0.87	211.83	-0.38	0.94	0.16	-26.68	0.30	19.30
52	0.01	5.16	32.07	-2.30	-0.88	809.65	-0.20	1.06	0.28	-26.78	0.74	12.00
53	0.02	6.89	43.37	-2.28	-0.77	86.79	-0.59	0.80	0.13	-26.90	0.29	21.65
54	0.00	6.68	21.40	-2.42	-1.00	785.23	-0.46	0.83	0.15	-26.97	0.24	15.49
55	0.00	6.85	23.35	-2.37	-0.99	1,547.93	-0.53	0.78	0.13	-27.34	-0.16	19.30
56	0.01	5.87	45.76	-2.45	-1.02	1,233.75	-0.47	0.71	0.12	-26.52	0.91	13.69
57	0.00	5.92	32.24	-2.28	-0.98	1,842.88	-0.33	0.80	0.16	-26.85	0.57	15.71
58	0.00	5.29	24.49	-2.37	-1.04	1,017.15	-0.30	0.96	0.21	-26.83	0.88	12.17
59	0.00	6.99	38.45	-2.47	-1.02	729.34	-0.67	0.66	0.12	-27.12	0.11	20.65
60	0.02	5.91	50.02	-2.18	-0.85	486.60	-0.33	1.12	0.23	-26.64	0.89	15.80
61	0.03	6.41	37.81	-2.19	-0.78	138.68	-0.45	0.97	0.17	-27.05	-0.06	20.83
63	0.01	6.76	70.28	-2.38	-1.03	3,005.55	-0.44	0.87	0.16	-27.13	0.15	22.35
64	0.02	5.85	106.32	-2.47	-1.12	18,437.53	-0.42	0.73	0.12	-26.76	0.58	14.04
65	0.01	7.02	77.12	-2.58	-1.28	23,360.43	-0.57	0.64	0.11	-26.36	0.71	20.42
66	0.00	6.75	22.44	-2.54	-1.05	1,632.31	-0.44	0.82	0.15	-26.97	0.37	16.13
67	0.02	6.88	78.96	-2.39	-0.99	1,318.16	-0.44	0.84	0.14	-26.04	1.04	18.54
68	0.01	6.03	73.27	-2.25	-0.99	187,491.48	-0.31	0.84	0.16	-26.50	0.72	14.97
69	0.01	6.12	40.01	-2.32	-0.91	1,918.58	-0.33	1.02	0.21	-27.20	0.44	20.81
70	0.01	6.33	25.72	-2.56	-0.99	369.01	-0.38	0.80	0.14	-26.81	0.21	16.40