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International investment portfolio diversification using cryptocurrencies

AUTHOR: KYRYLO KARTOZHNYKOV
STUDENT ID NUMBER: S1024203
SUPERVISOR: DR. JIANYING QIU
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ABSTRACT

One of the essential rules in investing claims: “Do not put all your eggs in one basket.” There is a broad acceptance among scholars and market practitioners that spreading available funds among traditional assets such as equities, bonds, commodities, cash, and cash equivalents allows you to achieve a lower level of risk-taking without sacrificing return. In this way, we can turn idiosyncratic risk into systematic risk. However, most financial advisors, such as SEC's Office of Investor Education and Advocacy (OIEA) and Northern Trust Corporation, neglect the relatively young market of cryptocurrencies. Reviewed literature supports that cryptocurrencies can be considered as conventional asset classes. In order to study diversification benefits from including cryptocurrencies into the portfolio, we are going to analyze six popular benchmarks over four traditional asset classes – equities, bonds, cash, and commodities, as well as Bloomberg Galaxy Crypto Index as a representative of the broad cryptocurrency market. Among the traditional assets, equities offer their investors the highest return. Although cryptocurrencies indicate higher risks, the risk-adjusted return, as measured by the Sharpe ratio, is much more favorable to investors. The inclusion of cryptocurrencies into the portfolio significantly increases the number of asset allocation options and, as a result, suits various risk-return profiles. Reviewed literature indicate of cryptocurrencies’ high diversification potential as found by Ankenbrand & Bieri (2018) that even 2% allocation into digital currencies can significantly improve portfolio’s Sharpe ratio. However, on the more extended period of observations and more mature cryptocurrency market, we can clearly see that the correlations in assets return remarkably increased. In our analysis, we found that portfolios consist only of traditional assets on average offer higher risk-adjusted returns than portfolios that include cryptocurrencies on the short time scale. While on the more extended investment horizon, portfolios with cryptocurrencies outperform traditional assets in terms of the mean return and risk-adjusted return. Thus, we can conclude - cryptocurrencies cannot serve as a safe-haven asset while there is undoubtedly diversification and risk spreading potential in asset allocation into cryptocurrencies.

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1. INTRODUCTION

Since the new cryptocurrency's (CC) bull run started after a massive correction in March 2020, the total cryptocurrency's market capitalization reaches its all-time high (ATH) at $ 1 755,6 bln.[1] (211% higher from its peak in 2018) on 20.02.2021. All the news feeds are full of stories like Bitcoin (BTC) price skyrocketed to $ 55 000, and the whole bitcoin market surpasses $ 1 trillion market cap.[2] Alternatively, another breaking news that Tesla Inc. buys bitcoin for $ 1,5 bln., which is the first and the biggest S&P500 company that hold BTC on its Balance Sheet.[3]

As there is a huge buzz on the market about bitcoin many other cryptocurrencies cannot get out of the shadow of Bitcoin. This could be clearly seen from the Google Trends analysis [Figure 1, Appendix 1]. While the attention to Bitcoin corresponds well to its growth and news representation, Ethereum, the second-largest cryptocurrency, is not any close in popularity by Google search users, although it brings its holders 175% year-to-date return almost double to that of 98% return from BTC.

Moreover, as the cryptocurrency market matures and more companies enter the market, we can see a downward trend in bitcoin dominance over the cryptocurrency market. [Figure 2, Appendix 1] As the first cryptocurrency, BTC absorbed almost 95% of the total CC market in 2013. Over the previous bull run, 2017-2018 Bitcoin market cap dominance volatiles dramatically from 87% at the beginning of 2017 to 33% at the beginning of 2018. As of today, the BTC market dominance is at 60%. So, there is a wide choice on the market of projects offering attractive returns and have relatively high liquidity.

Moreover, ordinary investors can benefit in terms of risk diversification from holding various cryptocurrencies in their portfolios. As the portfolio theory suggests, efficiently diversified asset allocation in an investment portfolio allows us to maximize return and minimize risk simultaneously for a given set of assets. Risk is determined by mean return, variance, and covariance of the equities' return. In such a way, we can transform idiosyncratic risk into systematic, which is lower portfolio drawdown potential (TOBIN, 1981). In Figure 1 Appendix 2 you can find a correlation matrix of some of the top 10 cryptocurrencies by market cap. As can be seen, some of them have a low or even negative correlation, with BTC which indicates that there is an opportunity for diversification by investing into crypto currencies.

The SEC's Office of Investor Education and Advocacy (OIEA) created an educational web portal that provides investors with unbiased information on investment decisions and protection

[1] According to the CoinMarketCap. https://coinmarketcap.com/
[2] Marktwaarde bitcoin door grens van 1 biljoen dollar na aanhoudende stijging | NU - Het laatste nieuws het eerst op NU.nl
[3] Tesla buys $1.5 billion in bitcoin, plans to accept it as payment (cnbc.com)

against fraud or securities abuse. They determine diversification as investing portfolio allocation into different asset classes such as stocks, bonds, and cash. As the financial market is a pretty complex construction with a vast range of available instruments investing in these three major asset classes is enough for achieving ordinary investor's financial goals. According to OIEA knowledgebase, the returns on these asset categories have not moved up and down simultaneously and their correlations are less than perfect (SEC's Office of Investor Education and Assistance, 2011). From the discussion above, we can clearly see that it is possible to earn a substantial return and spread-out risks by investing in cryptocurrencies, while these assets are missing in the OEIA recommendation.

This study broadens the existing literature by including various cryptocurrencies into an investment portfolio for diversification purposes. So, the research question could be stated as:

"Could the ordinary investor achieve higher risk-adjusted rate of return by including cryptocurrencies into his portfolio?"

The study's primary goal is to analyze diversification benefits by asset allocation into crypto currencies that can minimize risk for a particular return level. A mean-variance model is employed to determine the optimal portfolio in terms of risk-reward ratio. Portfolio's analysis is based on the Markowitz efficient frontier model. Moreover, the Sharpe ratio will be applied for optimal asset allocation for a potential portfolio.

As mentioned above, a diversified portfolio must be diversified at two levels: between asset categories and within asset categories. Because diversification can be so difficult, for the purpose of this research, we will diversify within each asset category by analyzing popular global benchmarks rather than cherry-picking from each asset category. As there are no cryptocurrencies ETF exists, several investment companies such as Grayscale Investments LLC operating on the market as a proxy-parties for ETF. However, they are quite young and does not have enough time-series data. In order to have robust research results from the study, we will make our analysis based on the Bloomberg Galaxy Crypto Index which was introduced in August 2017 and aims to provide proxies for the broader cryptocurrency market.

This paper is structured as follows. The second chapter reviews the most prominent theoretical analysis of the cryptocurrency market as an investment asset class. Moreover, based on these studies, our model will be stated, and data sources will be discussed in the third chapter. The fourth chapter provides the results of this analysis and inference for the hypothesis. Chapter five summarizes results and provides an overall conclusion as well as opens the door for the discussions. Finally, the critical evaluation of this thesis and its limitations will be provided in the 6-th chapter.

2. THEORETICAL FRAMEWORK

This section discusses in detail the benefits of diversifying traditional asset classes with cryptocurrencies. To achieve the goal of this study, a comprehensive study of the academic literature regarding cryptocurrencies as an asset class was conducted. Furthermore, this chapter dive into the theoretical framework on which the empirical research is based, consider the relationships between asset classes, the concept of diversification, the literature on international diversification, the benefits of diversification, and the optimization of portfolio risk and return.

2.1. Portfolio diversification theory

Portfolio selection is the capital allocating dilemma over various available assets, which perform two functions: maximizing "return" and minimizing "risk" of investment. Although the advantages of diversification in reducing risk have been cherished since the beginning of financial markets, the first portfolio selection arithmetical model was formed by Markowitz (1952). Markowitz's model measures "return" as a random expected return on the portfolio and accompanying "risk" is calculated as a variance in the portfolio's return. Markowitz proved that no matter what the investor’s reference point is: the highest level of acceptable risk or minimum desired return, answering a convex quadratic programming problem could lead to optimal asset allocation.

Moreover, Markowitz demonstrated that an investor could identify certain combinations of securities that maximize the expected return without taking too much risk on board, which in the experts' world got the name efficient frontier. Moreover, based on the Markowitz practice of expected variance of return (E-V), the investor should favor the portfolio, which is situated on an efficient frontier, because it accounts all risky investments and measures which combination might be an optimal fit.

The optimal portfolio framework is established on the two key components - risk and return. First, from the point of view of a certain level of risk, the investor must choose the one with the highest expected return. Second, regarding the desired level of the return, the portfolio with the lowest risk should be selected. Nevertheless, at a closer look, both statements are equivalent. Thus, the efficient frontier consists of points, each of which expresses a clear distribution of capital between the selected assets. This distinct allocation poses a particular risk for a particular level of return (Abidin et al., 2004).

Suppose an investor was able to derive an optimally allocated portfolio based on the collected data about securities' historical return and risk. After some periods of time, it can be discovered that some assets' prices have grown more than others. Consequently, the structure of the portfolio has changed. Moreover, investor's own return expectations have likely changed; and

the statistically derived risk evaluation was revised. If a computer program with reviewed data was used, he would probably find that the current portfolio bears little similarity to any member of a recently calculated set of "efficient" portfolios.

Notwithstanding the mean-variance model accomplishments in the academic world, practitioners are hesitant to use this model. The above-mentioned problem can be best described by the quote: “Although Markowitz efficiency is a convenient and useful theoretical framework for portfolio optimality, in practice it is an error-prone procedure that often results in error-maximized and investment-irrelevant portfolios” (Michaud, 1998). This behavior reflects the fact that the solutions of optimization problems are usually very responsive to deviations in the parameters of the problem; As calculations of market parameters are prone to statistical errors, the results of further optimization are not very feasible (Goldfarb & Iyengar, 2003).

In order to be determined as an "efficient" portfolio, there are two constraints imposed. The first is that the portfolio fully invested, meaning that the sum of the weights of the components (assets) equal to one. The second is a restriction on selling short; all asset weights in the portfolio must be larger or equal to zero. With the help of these two constraints, all points of the efficient frontier are within the minimum risk and the maximum return portfolio that can be obtained, and thus the efficient frontier can be determined.

Given the constraints mentioned above, following Markowitz’s Mean-Variance approach, we will construct and examine portfolios in a range from Minimum variance and Maximum return. These complicated calculations can be done using modern statistical tools like Microsoft Excel. In the beginning, based on the historical data, we need to identify the mean return over the observed period of time and the Variance-Covariance matrix, which helps spot co-movements in the assets return. Using Excel’s built-in functions and formulas described in the data and research design section, we calculated potential portfolio return, standard deviation, and variance. However, there is a question that arises – how to assign weights for an efficiently diversified portfolio? Given the whole universe of assets, such computation done in manual order might be time-consuming. However, Excel has sufficient functions to help in automatic calculations. In this situation Solver function is a suitable instrument that, based on the iterative calculations, can do all the jobs for us with specified goals. For example, we can ask Excel: “for a given return, variance and variance-covariance matrix by changing weights parameters, what would be the portfolio that minimizes portfolio’s variance.” The same approach with corresponding goals is used to discover weights of the Maximum return portfolio.

In our research, we will analyze both the US and EU equity and bond market as well as such traditional assets as Commodities and foreign exchange. In 20201, US stock markets accounted for almost 56 percent of world stocks. The next largest share of the stock market

belongs to Japan, followed by China, representing 7.4% and 5.4% of the total world stock market, which is not any close to the US share. The New York Stock Exchange (NYSE) and NASDAQ are the largest stock exchange operators in the world.[4]

The risk of US equities has a large idiosyncratic component, much of which cannot be mitigated by portfolio diversification. A relatively high degree of positive correlation in the economy implies that diversification of portfolios at the international level can help reduce risk. (Levy et al., 1970) The most rational portfolio selection models assume that investors own diversified portfolios to reduce or eliminate uncompensated risk, and virtually all asset pricing models claim that securities are priced by a diversified, marginal investor who requires little or no compensation for the maintaining of idiosyncratic risk (Goetzmann & Kumar, 2008).

The benefits of international portfolio diversification are well documented by scholars (Coeurdacier & Guibaud, 2011; Driessen & Laeven, 2007; Gilmore & McManus, 2002). Even though some authors arguing about the degree of advantage from international portfolio diversification sometimes are overestimated (Jorion, 1985) there is no evidence when the investor cannot benefit in terms of risk and return from the global asset allocation.

2.2. Cryptocurrencies as an asset class

The first known cryptocurrency - Bitcoin, developed and implemented by an anonymous Japanese programmer (or possibly a group of programmers) under the pseudonym Satoshi Nakamoto in 2009, as a decentralized peer-to-peer (P2P) network. Thereby, there are neither central clearinghouses nor financial or other institutions involved in the transactions, unlike the current financial system. Bitcoins are not tied to any real currency. The exchange rate is determined by supply and demand in the market. It works all over the world and can be used as a currency for all types of transactions (for both virtual and real goods and services). These products and services range from internet services and online products to tangible goods (such as clothing and accessories, electronics, books, etc.) and professional or travel services, thus competing with official currencies such as the euro or dollar. Although Bitcoin is a virtual currency scheme, there are certain innovations that share some characteristics with regular money (European Central Bank, 2012).

Twelve years ago, the publication of Nakamoto's white paper laid the foundation for a new asset class known as the cryptocurrency market. To date, inspired by Nakamoto, developers worldwide have created thousands of altcoins (cryptocurrencies other than bitcoins). In recent years, the colossal expansion of the cryptocurrency market has attracted a large number of both retail and institutional traders and significant attention from regulators and academics.

[4] As of January 1st, 2021, according to Statista.com (c). https://www.statista.com/statistics/710680/global-stock-markets-by-country/

There is a broad unanimity among market practitioners and scholars about three different asset classes, stocks, bonds, and cash equivalents exist. In addition, forex, real estate, and commodities are often treated as separate asset classes. In connection with the active adoption of cryptocurrencies in the past few years, a discussion has emerged about whether cryptocurrencies can be considered a separate class of assets. Even though the market is still relatively young, the authors found evidence that cryptocurrencies fulfill most of the requirements as an independent asset class (Ankenbrand & Bieri, 2018).

Kinlaw et al. (2017) define asset class as "a stable set of investment units that are internally homogeneous and externally heterogeneous, which, when added to a portfolio, increase its expected utility without the use of selection skills and which can be accessed at a minimal cost". Based on this determination, there are 7 requirements that cryptocurrencies must meet in order to be considered a separate asset class.

- **Stable aggregation.** The cryptocurrency market should not significantly fluctuate in its composition. Else, it would take constant effort to determine its real components.
  Even though there a broad variety of cryptocurrencies on the market designed in a different manner, its mechanisms are based on algorithms for reaching consensus on the current state of the system, not on a centralized transaction verification authority. Consequently, all tokens differ from the established asset classes by its purpose and/or cryptographic framework and decentralized management. Thus, the composition of cryptocurrencies as an asset class is considered relatively stable (Ankenbrand & Bieri, 2018).
- **Investable.** To be considered an asset class, cryptocurrencies should be directly investable for a wide range of investors.
  According to CoinMarketCap, there are 65 exchanges with a daily trading volume over $1 billion with different business models offering tokens to retail and institutional investors.[5] Generally, cryptocurrencies are available for purchase through specialized burses, although not all of them allow exchange for fiat money, we can conclude that CC partially fulfill this characteristic. The current adoption trend indicates that the situation can change soon due to mass acceptance.
- **Internally homogenous**. As an asset class, cryptocurrencies should maintain some homogeneity among tokens and its drivers.
  There are three different categories of cryptographic tokens can be distinguished: tokenized securities, utility tokens, and cryptocurrencies. Due to their different frameworks, not all cryptographic tokens carry out the key functions of money in the same

[5] Top Cryptocurrency Exchanges Ranked By Volume | CoinMarketCap. (https://coinmarketcap.com/rankings/exchanges/)

manner. Despite different approaches to regulation, cryptocurrencies are still regulated similarly in different jurisdictions. Based on a class approach from a legal point of view, as well as their decentralized and cryptographic basis, cryptocurrencies satisfy the homogeneity requirement (Ankenbrand & Bieri, 2018).

- **Externally heterogenous.** Cryptocurrencies must be substantially distinct from traditional asset classes (i.e., stocks and bonds).

  Ankenbrand & Bieri (2018) show that cryptocurrencies weakly correlated with traditional asset classes, with correlation coefficients varying from – 0.02 (bonds) to 0.05 (stocks). Mainly, cryptocurrencies show a weaker correlation with stocks, bonds, commodities, and forex than these asset classes between themselves. These outcomes show some potential for diversification by adding tokens into a portfolio of classical investment instruments. This point will be discussed in detail in following subsections.
- **Expected utility.** To affect expected utility cryptocurrencies should either enhance the portfolio's expected return or hedge its risk.

  Ankenbrand & Bieri (2018) finds that the composition of cryptocurrencies and the conventional asset increases optimal investments' the average excess return, adjusted for risk, as represented by the Sharpe ratio. An investment portfolio that includes cryptocurrencies achieves a higher Sharpe ratio of 2.10 relatively to 1.58 for an asset pool.
- **Selection skills.** Because the asset class is required to be internally homogeneous, investors do not need to be proactive in selecting specific components to increase the expected utility of their portfolio.

  Ankenbrand & Bieri (2018) shows that even a 2.1% portfolio's amount allocated into cryptocurrencies can significantly improve the Sharpe ratio of your holdings. Moreover, accepting in some jurisdictions of cryptocurrency's ETFs by authorities and developing crypto trusts like Grayscale open opportunities for diversified passive investing.
- **Cost-effective access.** As an asset class, it must be available at a moderate transaction price. The asset class should not significantly impair the liquidity of the portfolio so that the portfolio can be regularly rebalanced without incurring high costs.

  Transaction costs and bid-ask spreads are expected to decrease as the cryptocurrency market becomes more accessible, competitive, and mature, indicating a positive assessment of this requirement.

2.3. Crypto as a safe-haven and Connections with traditional asset classes

As mentioned above, in order to adjust the portfolio's utility, cryptocurrencies should either increase the investment portfolio's risk-adjusted return or hedge the portfolio's risk. Hereby, this part of the discussion would be focused on the safe-haven effect of the cryptocurrency market and other traditional asset classes.

The safe-haven concept emerged in response to the investors' loss-aversion (Tversky & Kahneman, 1991), where investors are more inclined to avoid losses than any risky related assumed profits (Hwang & Satchell, 2010). This loss aversion prompts investors to search for safe-haven assets that do not correlate or negatively correlate with traditional assets, especially during market shocks (Baur & Lucey, 2010). For the different time horizons, various assets can be accommodated as a safe haven, i.e., forex (Ranaldo & Söderlind, 2010), gold (Baur & Lucey, 2010), long-term government bonds (Flavin et al., 2014), and newly cryptocurrencies.[6]

As cryptocurrencies become trendy, they attract more researchers to analyze their investment characteristics, together with their safe-haven opportunities. Urquhart & Zhang (2019) research the hedging and safe haven properties of Bitcoin using hourly trading data and discover that it behaves as a diversifier relatively for various international currencies. Guesmi et al. (2019) detected that portfolio risk is significantly lower by adding bitcoin into a portfolio of gold, oil, and emerging market stocks. Shahzad et al. (2019) find evidence of Bitcoin's safe-haven properties, but this attitude varies over time.

In contrast, Smales (2019) argues that bitcoin should not be considered as a safe-haven asset as it is less liquid, more volatile, and more expensive in transactions than other assets. Another point of skepticism about cryptocurrencies as a right asset for a portfolio is lack of regulation and mass adoption, as well as high risk of the illicit transaction and money laundering, and generally market immaturity. Klein et al. (2018) doubted the safe-haven properties of cryptocurrencies as evidence point out that bitcoin is positively correlated with traditional assets in adverse market conditions. Last but not least, there is evidence of speculative behavior on the cryptocurrency market and a high risk of bubble formation (Cheah & Fry, 2015).

Most unsophisticated investors fall into naive diversification bias that is a portfolio selection that looks well-diversified but consists of underlying assets that are highly correlated, especially during times of market stress. In other words, the total investment portfolio’s risk comes from the correlation among the asset’s risk exposures.

Kuo Chuen et al. (2017) analyzed correlations between the mainstream assets such as S&P500, government bonds, gold, oil, commodities, REITs, private equity on the one hand and top-10 cryptocurrencies standalone and CRIX index as a whole on the other hand.[7] They

[6] Conlon, T., Corbet, S., & McGee, R. J. (2020). Are cryptocurrencies a safe haven for equity markets? An international perspective from the COVID-19 pandemic. Research in International Business and Finance, 54, 101248. https://doi.org/10.1016/j.ribaf.2020.101248

[7] CRIX is a market capitalization-weighted index. The weight of a cryptocurrency in CRIX calculate on its market capitalization compared to other cryptocurrencies in CRIX. A joint project of Humboldt University in Berlin, Germany, SKBI Singapore University of Management, and CoinGecko. CRIX - VCRIX (thecrix.de)

found that almost all correlations are less than 0.1. For example, the correlation between S&P500 and six tokens plus CRIX index is less than 0.05. Very low correlations support the claim that cryptocurrencies can be a promising investment class in hedging the risks of underlying assets.

Krueckeberg & Scholz (2018), in their analysis of the correlation between asset classes and cryptocurrencies, find that cryptocurrencies in general move independently of all traditional asset classes. In their research only 5 out of 520 analyzed pairs (CC, indices, equities, currencies, bonds, commodities, real estates) show a statistically significant correlation for all 3 metrics: Spearman ρ, Kendall τ and Pearson r. Whereas Bitcoin as a main driver of the crypto-market shows only case of weak positive correlation with real estate.

One of the most recent research made on the topic was published by the global investment manager Van Eck Securities Corporation. They analyzed the correlation between traditional assets and bitcoin as a representative of the crypto market with over 60% market share on the publication date. Historically, Bitcoin has kept a poor correlation with traditional asset classes and commodities. 2020 was a unique year for investors in many ways, and bitcoin was no exception as bitcoin's price hit new all-time highs. In addition, the correlation of the Bitcoin calendar year with traditional asset classes has also reached record highs, but the correlation remains low compared to correlations between traditional assets. Bitcoin's year return correlation with S&P500, oil, currencies, and gold vary in a range of 0.23-0.34, which is much higher than we observed ever before (Zinoviev, 2021).

3. DATA AND RESEARCH DESIGN

This section provides a detailed description of all data used in this study, as well as the methodology underlying our analysis of global portfolio diversification effects.

The aim of this research – to construct a diversified portfolio that allows reducing risk exposure for a certain level of return. The mean-variance model is used to find the optimal asset allocation within the portfolio. Efficient diversification of set of assets is achieved when it maximizes expected return simultaneously minimizing portfolio's variance which is represented by Markowitz efficient frontier (Markowitz, 1952).

Portfolio's return is calculated as the sum of the weights of individual assets multiplied by their expected return over time, which is calculated as follows:

$$E(R_p) = \sum_{t=1}^{\infty} \sum_{i=1}^{N} w_i E(R_i)$$

We know from Finance 101 that an investor must diversify and maximize expected returns. The rule states that an investor should diversify his funds among all those securities that bring him the maximum expected return. However, the performance of securities is highly correlated. The portfolio with the highest expected return is not necessarily the portfolio with the least variance. There is a rate at which an investor can get the expected return by allowing deviation or reduce variance by giving up the expected return (Markowitz, 1952). Thus, in order to build a diversified portfolio, we need to incorporate variance-covariance matrix to the equation. Portfolio variance is calculated using the standard deviation of each security in the portfolio and the correlation between the securities in the portfolio.

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2\rho_{ij}w_i w_j \sigma_i \sigma_j$$

Important to note that the model imposed 2 constraints, as discussed in the previous section:

1. $\sum w = 1$, meaning full deposit is invested;
2. $w_n \geq 0$, meaning short selling is not considered.

The normative economic theory suggests that an individual investor should behave in line with the concept of "Homo Economicus". In other words, he is a rational profit maximizer led by self-interest with defined preferences (Vriend, 1996). Moreover, the CAPM model advocates that all the participants would hold the same market portfolio in an efficient market.

In our research, we will follow the analysis of Ankenbrand & Bieri (2018) to see whether their findings still hold true as it was already pointed that the correlation between cryptocurrencies and traditional assets is changing. As discussed above, the mean-variance approach assumes the inverse relationship between higher correlation and diversification benefits. Authors analyzed the performance of cryptocurrencies till August 2017, and we are going to increase the time horizon up to date. Moreover, we will increase the traditional asset bucket with two benchmarks for the European market: STOXX Europe 600 (as a European S&P500 alternative) and Bloomberg Barclays Euro Aggregate Index (as a broad benchmark for the European bond market). We made this extension due to a well-documented home bias (Coval & Moskowitz, 1999) most of them cannot be sufficiently mitigated by investing only in the domestic companies.[8] Our dataset consists of the following benchmarks.

Table 1. Asset classes and their substitutes

Asset class	*Benchmark*	*Details*
Equity	S&P500 Index	The index includes the 500 largest companies and covers about 80% of the available market capitalization in United States of America.

[8] Home bias – investor's tendency to have higher sentiments for local companies' equity.

	STOXX Europe 600	Index consists of 600 components representing large, medium and small companies from 17 European countries: AT, BE, DK, FI, FR, DE, IE, IT, LU, NL, NO, PL, PT, ES, SE, CH and UK.
Bond	Bloomberg Barclays U.S. Aggregate Bond index	Universal flagship benchmark that measures the investment-grade US dollar fixed-rate bond market.
	Bloomberg Barclays Euro Aggregate Index	A benchmark that measures investment-grade denominated in the euro fixed-rate bond market, including treasury, government, corporate and securitized issues.
Foreign exchange	Bloomberg Dollar Spot Index	Index monitors the dynamics of a basket of 10 leading world currencies against the US dollar.
Commodities	Bloomberg Commodity Index	Widely diversified index tracking all major commodity markets
Cryptocurrency	Bloomberg Galaxy Crypto Index	Benchmark designed to measure the performance of the largest cryptocurrencies traded in US dollar.

As it said before the bull cycle in the crypto market attracts many talented developers with bright ideas and projects. Since Ankenbrand & Bieri (2018) performed their research in 2017 and had to construct crypto index themselves, the existing literature and institutional involvement in cryptocurrency market involvement increased significantly. For our analysis, the Bloomberg Galaxy Crypto Index was chosen as a proxy for the digital currency market as one of the most reputable sources of investing information. Index select tokens based on their liquidity and reliability as well as fulfill minimum thresholds for daily traded USD-value where no single constituent can exceed 40%.[9] As of 30th June, the benchmark consists out of the following tokens:

- Bitcoin – 40.00%;
- Ethereum – 40.00%;
- Litecoin – 8.39%;
- Bitcoin Cash – 8.37%;
- EOS – 3.24%.

As the cryptocurrencies are heavily volatile assets it could be considered as a risky investment for a risk-averse investor and thus be left out of the consideration. However, as the Mean-Variance approach is based not only on risk and return but also on the variance-covariance between the various asset types – thus in case of low or negative correlation even the conservative investor could benefit from allocation of some portion of its wealth into cryptocurrencies.

[9] The Bloomberg Galaxy Crypto Index (BGCI) Factset: BGCI-Factsheet-6-30-21.pdf (bbhub.io)

Out of the whole sample we will select best diversified portfolios with highest historic Sharpe ratio while limiting riskiness of the portfolio as a simple tool to assess ex post funds effectiveness. (Sharpe, 1994) The ratio can be calculated as follows:

$$S_h = \frac{\bar{R}}{\sigma} = \frac{\frac{1}{T}\sum_{t=1}^{T} R_t}{\sqrt{\frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T-1}}}$$

The main statistical tool applied in our analysis is the Microsoft Excel spreadsheets. Using its Analysis function, we will construct variance-covariance matrix for a given set of assets based on its historical return. Further, Solver function is going to be used in order to assign weights for an efficient portfolio that maximizes expected return for a particular level of risk.

Our study also aims to construct Efficient frontiers with and without cryptocurrencies to graphically represent the set of efficient portfolios for various risk-return characteristics as well as diversification benefits from including cryptocurrencies. Following the previous approaches, Excel offers all sufficient tools to reach the goal of this research. Given the short-selling constraints, the Efficient frontier is nothing more than all feasible portfolios in the range of Minimum Variance and Maximum Return portfolios. As soon as we have both portfolios' weights and risk-return metrics, we can construct covariance between those two. When all the data is calculated, those portfolios can be regarded as an allocation between two assets and using Excel's What-if Analysis, we can calculate the possible portfolios between Minimum Variance and Maximum return.

4. RESULTS

In this section we are going to discuss the results of our analysis and take a closer look at the achieved asset allocation. As mentioned above the analyzed data consists of the eight popular indexes serving as a proxy for: USA equity (S&P 500), European equity market (STOXX Europe 600), USA bonds (Bloomberg Barclays U.S. Aggregate Bond index), European bonds market (Bloomberg Barclays Euro Aggregate Index), forex market (Bloomberg Dollar Spot Index), diverse commodities (Bloomberg Commodity Index) and cryptocurrency market (Bloomberg Galaxy Crypto Index).

Before we are going to analyze the diversification benefits from adding cryptocurrencies into the portfolio, we need to take a closer look at summary statistics and overview returns and risks associated with various types of assets.

Table 2. Weekly summary statistics and risk/return profile of the selected market proxies for the period from 1st January 2012 to mid-June 2021

Weekly data	*US Stock*	*EU Stock*	*US Bond*	*Euro Bond*	*Com-modity*	*Forex*	*Crypto*
Mean	0.253%	0.127%	0.006%	0.013%	-0.047%	-0.008%	1.849%
StD	2.24%	2.35%	0.49%	0.45%	1.88%	0.85%	12.49%
Variance	0.05%	0.06%	0.00%	0.00%	0.04%	0.01%	1.56%
Kurtosis	7.086	8.327	6.045	3.246	2.030	5.449	2.222
Skewness	-0.725	-1.274	-0.777	-0.514	-0.553	0.528	0.186
Range	27.08%	27.10%	5.96%	4.48%	14.71%	8.81%	98.11%
Min	-14.98%	-18.4%	-3.28%	-2.46%	-9.14%	-4.06%	-42.78%
Max	12.10%	8.66%	2.68%	2.02%	5.57%	4.75%	55.33%
Sum	152.26%	76.53%	3.60%	7.87%	-28.58%	-2.30%	380.89%
proportion of negative weeks (%)	40.20%	42.36%	46.51%	43.02%	50.33%	48.36%	44.17%
Sharpe ratio	0.81	0.39	0.09	0.21	-0.18	-0.07	1.07
Observations	602	602	602	602	602	275	206

Noteworthy to mention, the table illustrates some extreme values for different measures (standard deviation, variance, kurtosis) caused by the March 2020 sell-off due to the Covid-19 pandemic. However, we should not exclude them from the analysis as history shows that such Black Swan events are not negligible and occur every +/- 10 years. Moreover, this point is also the main point of criticism of the Mean-Variance asset allocation theory. Markowitz's portfolio theory is built on the belief that history repeat itself and the past fund performance will continue in the future (Goldfarb & Iyengar, 2003).

As can be seen, the table clearly illustrates the risks involved in investing as 2 out of 7 indexes deliver a negative return for its investors both in terms of the mean return and over the observed period. These findings contradict the idea of the "homo economicus" as it is supposed to be a profit-maximizer. However, in addition to historical return, the Mean-variance model relies also on co-movements between assets, so we need to explore forex and commodity hedging characteristics further.

The table also sheds light on the European and American equity and bond market differences in risk and return. The mean and total return on S&P 500 is twice as high as STOXX Europe 600, 0.253% and 0.127%, respectively, while bearing identical risk in terms of standard deviation and variance. As a result, we can observe a 2 times higher Sharpe ratio on the American benchmark relatively European equity. While on the bond market, we can see the reverse effect. The return on the European benchmark is 117% higher than on the American

equivalent. Similar to the equity market, bond's risk measures are identical. Hereby, the risk-adjusted return on Bloomberg Barclays Euro Aggregate Index is 130% higher.

What is really stands out from the dataset is the Bloomberg Galaxy Crypto Index. The first thing that capture the attention is that over 3 times shorter observer period it delivers 380% return. The index delivers on average 1.85% weekly to its investors, which is 7 times higher than mean return on S&P 500 index. Although the risk associated with investing in the crypto-currency index appears to be significantly higher than any other traditional assets, the risk-adjusted basis has a 130% higher Sharpe ratio than S&P 500. This finding reveal that the results of Ankenbrand & Bieri (2018) are still holds true, even taken into account higher risks of digital currencies market the Sharpe ratio is significantly higher than the other assets clas-ses. The second place in the ranking would take equity indexes with S&P 500 – 0.81 and STOXX Europe 600 – 0.39.

Table 3. Correlation matrix. Index proxies: S&P 500 index (US Stock), STOXX Europe 600 index (EU Stock), Bloomberg Barclays U.S. Aggregate Bond index (US Bond), Bloomberg Barclays Euro Aggregate index (EU Bond), Bloomberg Dollar Spot Index (Foreign Exchange), Bloomberg Commodity Index (Commodity), and Bloomberg Galaxy Crypto Index (Cryptocurrencies)

	US Stock	***EU Stock***	***US Bond***	***Euro Bond***	***Commodity***	***Forex***	***Crypto***
US Stock	1.0000						
EU Stock	0.8025	1.0000					
US Bond	0.0591	-0.0103	1.0000				
Euro Bond	0.0519	0.1058	0.0095	1.0000			
Commodity	0.4991	0.4267	-0.021	-0.0417	1.0000		
Forex	-0.415	-0.2593	-0.007	-0.2321	-0.4724	1.0000	
Crypto	0.2334	0.3032	-0.067	0.0226	0.3316	-0.258	1.0000

As the modern portfolio theory prescribe, the key indicator for assessing the diversification potential of the cryptocurrency market is the correlation of its profitability with other asset clas-ses, as shown in Table 3.

As was found by Ankenbrand & Bieri (2018) and our analysis correlation between traditional assets does not differ significantly. Although the authors found a negative correlation between S&P 500 index and the Bloomberg Barclays U.S. Aggregate Bond index, it is still low enough to allow investors to hedge some portion of the risk in our analysis. As can be seen from the table Commodity market has a degree of correlation with equity indexes both US and EU while has a negative correlation with a bond index (-0.021 and -0.0417). The forex market appears to have diversification potential with all types of assets due to a negative correlation in returns.

As was already pointed by the Van Eck Securities Corporation experts correlation between cryptocurrencies and traditional assets has changed dramatically since 2017. Ankenbrand & Bieri (2018) found a correlation between their crypto index and S&P 500 equal to 0.05. In contrast, we can see that co-movement in return is equal to almost 25% with the US equity and even higher 30% with the EU equity market. We can also observe a high degree of correlation in return between the crypto index and commodities. Surprisingly, correlation with the bond market had not experienced such adjustments, as it is still negative with US bonds and slightly positive with EU bonds. Furthermore, the table reveals a significantly negative correlation between cryptocurrencies and foreign exchange, which creates an opportunity for hedging risks.

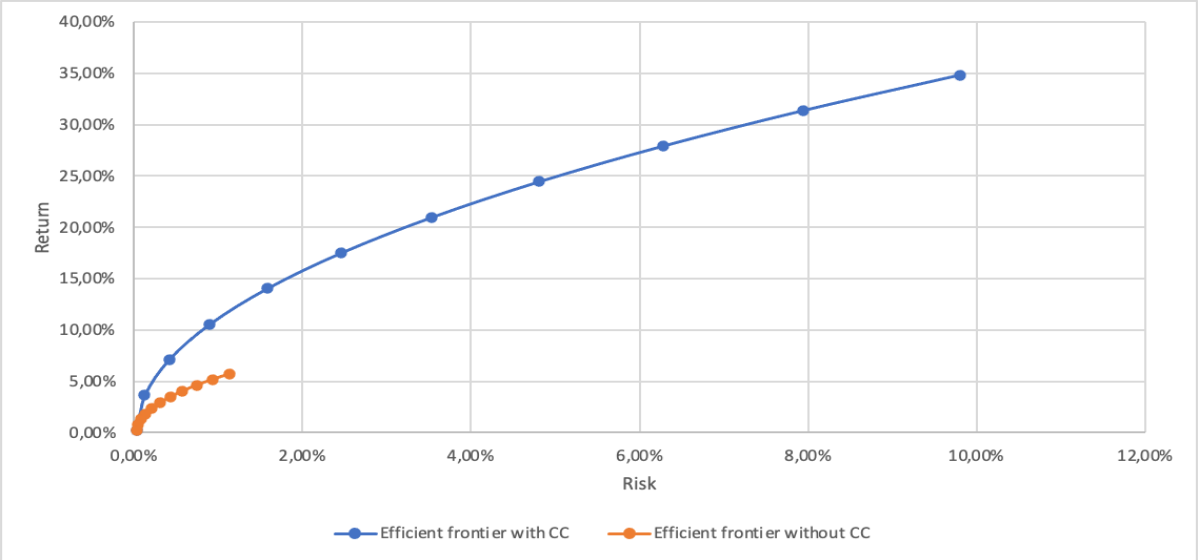


Figure 1. Efficient frontiers with and without cryptocurrency index. Return measured as an annual expected return. Risk calculated as an expected annual variance.

As described in the literature review, in order to be considered an investable asset class, cryptocurrencies should deliver particular utility to their investors in terms of either increased expected return of a portfolio or decreased risk. This characteristic can be assessed by Figure 1, where two efficient frontiers have been constructed. The first (blue curve) created based on the entire investment pool that includes all proxies of certain asset classes: stocks, bonds, commodities, foreign exchange, and cryptocurrencies. The second (orange curve), though, is based only on the traditional asset classes.

The two efficiency frontiers emphasize the fact that investing in cryptocurrencies along with traditional assets increases the set of optimal portfolios and, as a result, expands the spread of risk and return of portfolios. Investors can get higher average returns at the same level of risk, either higher risk-adjusted return for taking more risk on board by including cryptocurrency in their portfolio. As we have already seen from Figure 1, including cryptocurrencies into the

portfolio significantly expands the variety of asset allocation options that can deliver much higher risk-return prospects.

Table 4. Comparison of annual performance of tangency portfolios

	Portfolio 1	***Portfolio 2***	***Portfolio 3***	***Portfolio 4***
US Stock	0,00%	30,00%	30,00%	30,00%
EU Stock	20,56%	30,00%	30,00%	30,00%
US Bond	30,00%	9,22%	4,60%	0,00%
Euro Bond	19,44%	0,00%	0,00%	10,00%
Commodity	30,00%	30,00%	30,00%	0,00%
Forex	0,00%	0,00%	0,00%	0,00%
Crypto	0,00%	0,78%	5,40%	30,00%
Risk-Return characteristics				
Return	0,84%	5,97%	10,39%	34,84%
StD	2,42%	4,55%	5,37%	11,66%
Variance	0,06%	0,21%	0,29%	1,36%
Sharpe ratio	***34,88%***	***131,19%***	***193,59%***	***298,79%***

Table 4 illustrates a degree of improvement in risk-return characteristics for tangent portfolios with higher level of investment in cryptocurrencies.[10] Portfolio 1, in our case, represents a minimum-variance portfolio when most of the deposit are allocated in the least risky assets: bonds and commodities. As a rule of thumb, by giving away all of the risks, we also reduce the return prospects of our portfolio, which leads to a Sharpe ratio of 35%. As the Mean-Variance model considers in addition to risk and returns covariance between assets return for a different profile, we need a change in our asset allocation. As can be seen, Portfolios 2 and 3 are similar in their composition as 60% of the assets are spread equally between US and EU equities, another 30% allocated into commodity the only difference in allocation in US bonds and Crypto. Such changes in asset allocation permit to increase Sharpe ratio by 150%. Thus, investors can significantly improve their performance by investing only a small part of their total investment in cryptocurrency. Portfolio 4 composition is the Max return allocation, which does not suit every investor as it bears a high degree of risk. However, the Sharpe ratio indicates a significantly favourable risk-adjusted profile to the investor. This observation suggests that cryptocurrency as an asset class is a good diversifier in a traditional portfolio that can deliver a lot of usefulness to its investors.

[10] All of the portfolios provided on the table are efficient and diversified. Risk preferences are individual, while the rationality model prescribes - the investor should choose the portfolio with a higher risk-adjusted return. The table is aimed illustrate how asset allocation affects the portfolio's mean return, risk characteristics, and Sharpe ratio.

Ankenbrand & Bieri (2018) found that the allocation of 2% of funds into cryptocurrencies could significantly improve the portfolio's risk-adjusted return. Our analysis found the opposite effect due to the changed degree of the correlation between digital currencies and traditional assets. For example, adding 2% of cryptocurrencies while equally reduce assets weights among traditional assets reduce the Sharpe ratio of the portfolio ten folds. Hereby, in order to achieve efficient asset allocation, we need to recalculate weights based on the new given parameters.

Despite the general rule that the longer investing horizon reduces the risk of adverse return, a well-documented myopic loss aversion bias found that the investors have a relatively short evaluation period and are prone to take less risk (Gneezy & Potters, 1997; Langer & Weber, 2005). Moreover, investors' expectations over time are subject to changes. As pointed above, the fundamentals of the asset and as well as the correlations in assets return are not stable and can vary significantly over the years. All these taken into account, we have to look at the diversified annual portfolio performance over the observed years.

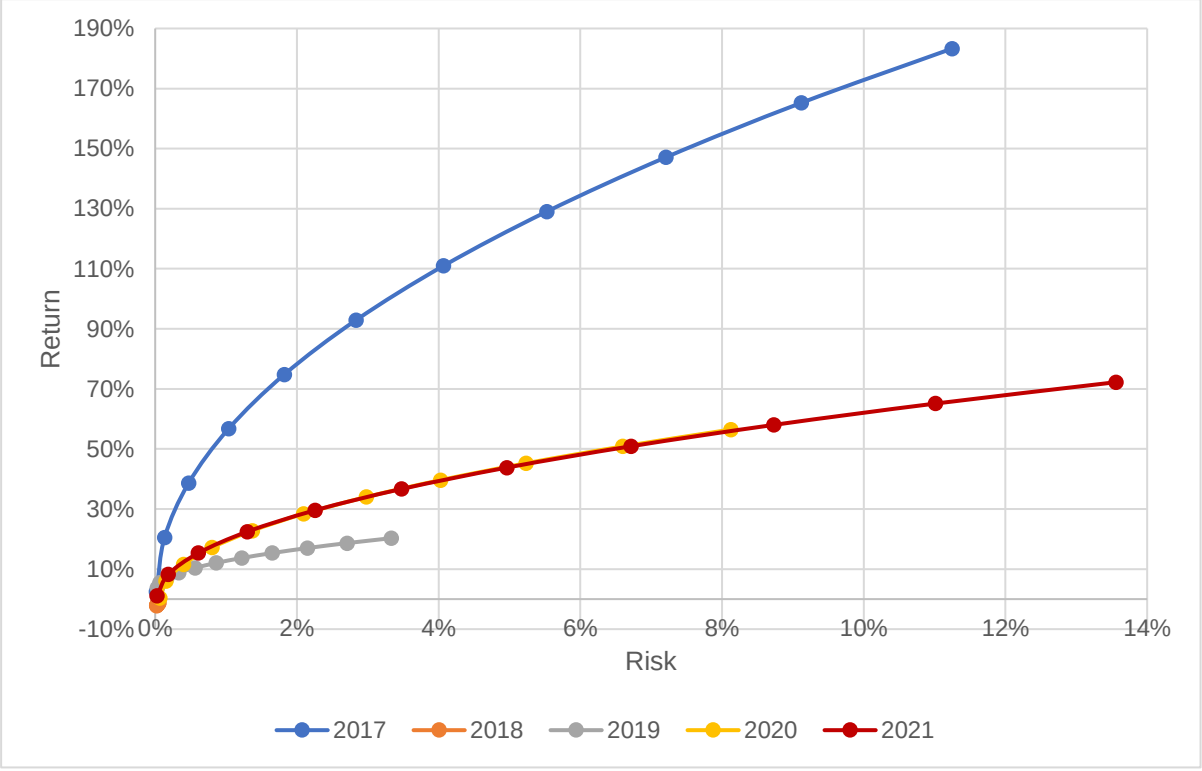


Figure 2. Efficient frontiers with cryptocurrency index per year. Return measured as an annual expected return. Risk calculated as an expected annual variance.

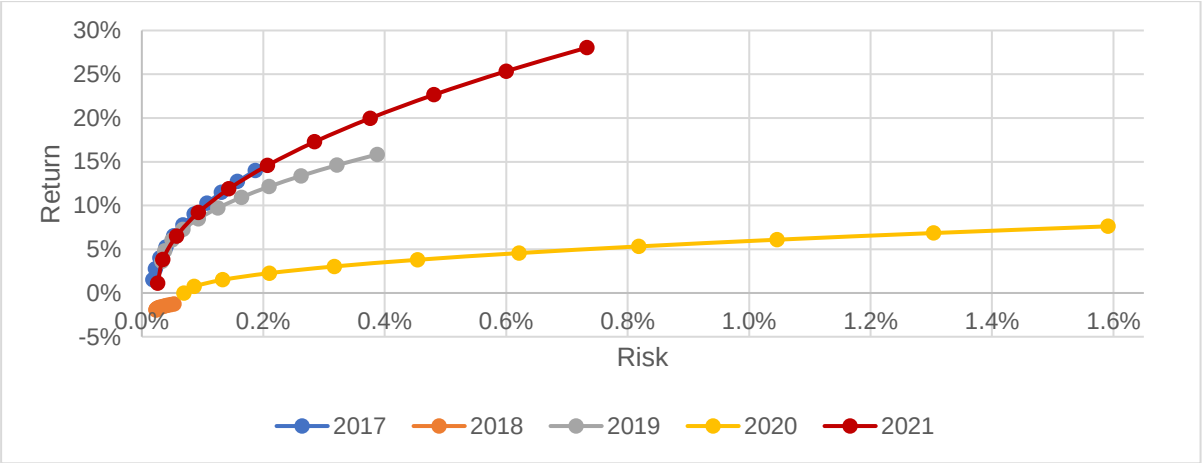


Figure 3. Efficient frontiers without cryptocurrency index per year. Return measured as an annual expected return. Risk calculated as an expected annual variance.

Figure 2 illustrates efficient frontiers of annual portfolios' performance, including cryptocurrencies. While figure 3 demonstrates the annual performance of the portfolios without cryptocurrencies. As can be seen from the charts, the inclusion of cryptocurrencies into an investment portfolio can significantly increase the amount of asset allocation options that can be suitable for investors with a different risk-return profile. The scale of the risk-return increases 20-folds by including cryptocurrencies. While the traditional assets portfolio's annual return is bounded to 30% for 2021 and up to 15% for all other years, most of the portfolios with cryptocurrencies go far beyond that.

Over the observed five months in 2017, by including up to 30% in cryptocurrencies, investors could achieve more than 180% return. While the portfolio without digital assets displays much modest results up to 15%. On the risk-adjusted basis, portfolios with cryptos have as twice as high a Sharpe ratio as portfolios consists only traditional assets, as displayed in Appendix 3. 2018 year was an adverse period for all types of assets as they have a negative mean return; thus, diversification aimed to decrease the investment risk. If the rational investor were to re-balance its portfolio annually – it was efficient to sell all the crypto tokens and enter into more defensive assets. Each crypto investor should know that the digital assets market develops in cycles of hype and bubbles. Once the bubble bursts, the so-called crypto-winter is coming. Thus, 2019 is also not a good year for investment in cryptocurrencies due to high volatility and low return, as investing in traditional assets would bring a much higher risk-adjusted return. 2020 was the historic year when the COVID-2019 pandemic hits hard financial markets, and all the economies around the world experienced closures and supply-chain disruption. The cryptocurrency market experienced a relatively low shock in comparison with traditional assets an814-d single index components. Due to the ambiguous economic prospects, money floods alternative investments such as cryptocurrencies, and they recover from the March crash immediately, while for traditional assets, it took the whole year to recover to pre-March levels.

Hereby, investors with cryptocurrencies achieve over the year higher risk adjusted return. One of the possible reasons that the cryptocurrencies were on the rise in 2020 is Bitcoin 3rd halving which reduces miners' reward for solving blocks and decreases digital coin supply. The end of 2020 was dominated by the investors' optimism about the market perspectives due to development and rolling-out vaccines worldwide. Together with massive quantitative easing efforts introduced by central banks globally and historically low-interest rates, investors flow to the traditional markets. As a result, portfolios without cryptocurrencies achieve a higher Sharpe ratio of 45% against 27% with cryptocurrencies.

To sum up, for most of the years, traditional assets offer their investors favorable risk-adjusted return relatively portfolios with cryptocurrencies. However, over a more extended period of time, portfolios with digital assets outperform traditional assets both in terms of the mean return and risk-adjusted return.

5. CONCLUSION

This paper aims to research the diversification benefits by allocating a certain amount of funds into cryptocurrencies in addition to other traditional assets. Most of the reviewed literature studied the diversification utility of Bitcoin as a dominant representative of cryptocurrencies. Recent studies found a low historical correlation between conventional assets and cryptocurrencies. We contribute to the existing literature by analyzing the efficient portfolio allocation between the US and EU equity and bond markets, commodities and foreign exchange, and Bloomberg Galaxy Crypto Index as a representative of the broad cryptocurrency market. In order to cover this gap, the research question was stated as follows:

"Could the ordinary investor achieve higher risk-adjusted rate of return by including cryptocurrencies into his portfolio?"

Weekly return data from popular market indices were used as proxies for traditional asset classes for the period between January 2010 (or the most recent available) and July 2021. The equity market is represented by the S&P 500 and STOXX Europe 600, covering most of the equities in the US and EU. The bond market proxies are expressed by Bloomberg Barclays US Aggregate Bond Index and Bloomberg Barclays Euro Aggregate Index as the popular benchmarks for the American and European bond market. Other asset classes such as foreign exchanges and commodities are portrayed by Bloomberg Dollar Spot Index and Bloomberg Commodity Index. And the most recent Bloomberg Galaxy Crypto Index serves as a market benchmark for the cryptocurrency market. Summary statistics of the described indexes indicate that cryptocurrencies provide the highest risk-adjusted return over the observed period. Digital currencies are followed by equities and then bonds with the highest Sharpe ratio. Over

the observed period of time, Commodities and Forex deliver a negative return to their investors and, as a result, have a negative Sharpe ratio.

Our research is originated on the Mean-Variance approach described in Modern portfolio theory developed by Harry Markowitz. As described in the literature review, the model is based on the three key components: mean return, variance, and covariance among the assets return. The model imposes two essential restrictions: the full deposit is invested, and short-selling is forbidden. The primary statistical tool applied is Excel Spreadsheets; moreover, the Solver function was used to assign assets weights for an efficient portfolio asset allocation.

The analysis sheds light that among all the indexes considered a proxy for various asset classes, cryptocurrencies deliver the highest risk-adjusted return as measured by the Sharpe ratio. The digital tokens in this metric are followed by the equity class, even though the United States has as much as twice higher Sharpe ratio than the European market. With Sharpe ratio 0,21 for European bonds and 0,09 for American bonds debt market take third place. As the result of the negative mean return, commodities and the foreign exchange market have a negative risk-adjusted return.

Investors can achieve a higher utility from the inclusion of cryptocurrencies into their portfolios as the number of options in asset allocation increases. We illustrated that an efficient portfolio with cryptocurrencies delivers a higher mean return with the price of risk exposure. However, the diversification benefits are limited compared to those that used to be in the past, as described in the literature review section. Due to the high level of correlation of cryptocurrencies with the equity market and commodities, it cannot serve as a safe haven rather than a diversifier. Meanwhile, the higher risk associated with digital currencies can be hedged with bonds and foreign exchanges. Alternatively, due to the different nature of the risks related to cryptocurrencies, they can be used as a risk-spreading instrument for an diversified portfolio.

As we found in our analysis, long-term investors, by investing in cryptocurrencies, can achieve higher risk-adjusted returns as measured by Sharpe ratio and mean return overall. However, in the shorter period (annually), we can clearly see that cryptocurrencies are not always an efficient investment asset. If an investor needs to rebalance his portfolio yearly due to significant changes in prices or different expectations about the economy over the future periods, he might incur significant transaction costs, which reduces diversification benefits.

Due to still development stages on the digital assets market more studies needed for diversification utility of cryptocurrencies on the longer time scale. Another implication – halving rounds and predictable market cycles offer investors an opportunity to exploit market inefficiencies. Moreover, transaction costs and various risk techniques, and predictability of the market cycles might be handy to incorporate for deeper analysis of the risk-return benefits.

BIBLIOGRAPHY

Abidin, S. Z., Ariff, M., Nassir, A. M., & Mohamad, S. (2004). International Portfolio Diversification: A Malaysian Perspective. In *Investment Management and Financial Innovations* (Vol. 3, Issues 1, Iss. 3). Business Perspectives, Publishing Company . http://www.irbis-nbuv.gov.ua/cgi-bin/irbis_nbuv/cgiirbis_64.exe?I21DBN=LINK&P21DBN=UJRN&Z21ID=&S21REF=10&S21CNR=20&S21STN=1&S21FMT=ASP_meta&C21COM=S&2_S21P03=FILA=&2_S21STR=imfi_2004_1_3_6

Ankenbrand, T., & Bieri, D. (2018). Assessment of cryptocurrencies as an asset class by their characteristics. *Investment Management and Financial Innovations*, *15*(3), 169–181. https://doi.org/10.21511/imfi.15(3).2018.14

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financial Review*, *45*(2), 217–229. https://doi.org/10.1111/j.1540-6288.2010.00244.x

Cheah, E. T., & Fry, J. (2015). Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economics Letters*, *130*, 32–36. https://doi.org/10.1016/j.econlet.2015.02.029

Coeurdacier, N., & Guibaud, S. (2011). International portfolio diversification is better than you think. *Journal of International Money and Finance*, *30*(2), 289–308. https://doi.org/10.1016/j.jimonfin.2010.10.003

Coval, J. D., & Moskowitz, T. J. (1999). Home bias at home: Local equity preference in domestic portfolios. *Journal of Finance*, *54*(6), 2045–2073. https://doi.org/10.1111/0022-1082.00181

Driessen, J., & Laeven, L. (2007). International portfolio diversification benefits: Cross-country evidence from a local perspective. *Journal of Banking and Finance*, *31*(6), 1693–1712. https://doi.org/10.1016/j.jbankfin.2006.11.006

European Central Bank. (2012). *VIRTUAL CURRENCY SCHEMES*. https://www.ecb.europa.eu/pub/pdf/other/virtualcurrencyschemes201210en.pdf

Flavin, T. J., Morley, C. E., & Panopoulou, E. (2014). Identifying safe haven assets for equity investors through an analysis of the stability of shock transmission. *Journal of International Financial Markets, Institutions and Money*, *33*, 137–154. https://doi.org/10.1016/j.intfin.2014.08.001

Gilmore, C. G., & McManus, G. M. (2002). International portfolio diversification: US and Central

European equity markets. *Emerging Markets Review*, *3*(1), 69–83. https://doi.org/10.1016/S1566-0141(01)00031-0

Gneezy, U., & Potters, J. (1997). An Experiment on Risk Taking and Evaluation Periods. *The Quarterly Journal of Economics*, *112*(2), 631–645. https://doi.org/10.1162/003355397555217

Goetzmann, W. N., & Kumar, A. (2008). Equity portfolio diversification. *Review of Finance*, *12*(3), 433–463. https://doi.org/10.1093/rof/rfn005

Goldfarb, D., & Iyengar, G. (2003). Robust portfolio selection problems. *Mathematics of Operations Research*, *28*(1), 1–38. https://doi.org/10.1287/moor.28.1.1.14260

Guesmi, K., Saadi, S., Abid, I., & Ftiti, Z. (2019). Portfolio diversification with virtual currency: Evidence from bitcoin. *International Review of Financial Analysis*, *63*, 431–437. https://doi.org/10.1016/j.irfa.2018.03.004

Hwang, S., & Satchell, S. E. (2010). How loss averse are investors in financial markets? *Journal of Banking and Finance*, *34*(10), 2425–2438. https://doi.org/10.1016/j.jbankfin.2010.03.018

Jorion, P. (1985). International Portfolio Diversification with Estimation Risk. In *Source: The Journal of Business* (Vol. 58, Issue 3). https://about.jstor.org/terms

Kinlaw, W., Kritzman, M., Turkington, D., & Markowitz, H. (2017). *A Practitioner's Guide to Asset Allocation*. John Wiley & Sons, Inc. https://www.wiley.com/en-us/A+Practitioner%27s+Guide+to+Asset+Allocation-p-9781119397809

Klein, T., Pham Thu, H., & Walther, T. (2018). Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance. *International Review of Financial Analysis*, *59*, 105–116. https://doi.org/10.1016/j.irfa.2018.07.010

Krueckeberg, S., & Scholz, P. (2018). Cryptocurrencies as an Asset Class? *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3162800

Kuo Chuen, D. L., Guo, L., & Wang, Y. (2017). Cryptocurrency: A New Investment Opportunity? *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.2994097

Langer, T., & Weber, M. (2005). Myopic prospect theory vs. myopic loss aversion: how general is the phenomenon? *Journal of Economic Behavior & Organization*, *56*(1), 25–38. https://doi.org/10.1016/J.JEBO.2003.01.005

Levy, H., Sarnat, M., Levy, H., & Sarnat, M. (1970). International Diversification of Investment Portfolios. *American Economic Review*, *60*(4), 668–675. https://econpapers.repec.org/RePEc:aea:aecrev:v:60:y:1970:i:4:p:668-75

Markowitz, H. (1952). Portfolio Selection. In *Source: The Journal of Finance* (Vol. 7, Issue 1).

Michaud, R. O. (1998). *Efficient asset management : a practical guide to stock portfolio optimization and asset allocation*. Boston, Mass. : Harvard Business School Press.

Ranaldo, A., & Söderlind, P. (2010). Safe haven currencies. *Review of Finance*, *14*(3), 385–407. https://doi.org/10.1093/rof/rfq007

SEC's Office of Investor Education and Assistance. (2011). *Beginners' Guide to Asset Allocation, Diversification, and Rebalancing | Investor.gov*. https://www.investor.gov/additional-resources/general-resources/publications-research/info-sheets/beginners-guide-asset

Shahzad, S. J. H., Bouri, E., Roubaud, D., Kristoufek, L., & Lucey, B. (2019). Is Bitcoin a better safe-haven investment than gold and commodities? *International Review of Financial Analysis*, *63*, 322–330. https://doi.org/10.1016/j.irfa.2019.01.002

Sharpe, W. F. (1994). *The Sharpe Ratio*. The Journal of Portfolio Management. https://web.stanford.edu/~wfsharpe/art/sr/SR.htm#Sharpe66

Smales, L. A. (2019). Bitcoin as a safe haven: Is it even worth considering? *Finance Research Letters*, *30*, 385–393. https://doi.org/10.1016/j.frl.2018.11.002

TOBIN, J. (1981). Portfolio Theory. *Science*, 974–974. https://doi.org/10.1126/science.974-b

Tversky, A., & Kahneman, D. (1991). Loss Aversion in Riskless Choice: A Reference-Dependent Model. *The Quarterly Journal of Economics*, *106*(4), 1039–1061. https://doi.org/10.2307/2937956

Urquhart, A., & Zhang, H. (2019). Is Bitcoin a hedge or safe haven for currencies? An intraday analysis. *International Review of Financial Analysis*, *63*, 49–57. https://doi.org/10.1016/j.irfa.2019.02.009

Vriend, N. J. (1996). Rational behavior and economic theory. *Journal of Economic Behavior and Organization*, *29*(2), 263–285. https://doi.org/10.1016/0167-2681(95)00063-1

Zinoviev, D. (2021, February 8). *Bitcoin's Correlation to Markets Hits a Record in 2020 | VanEck*. https://www.vaneck.com/us/en/blogs/digital-assets/bitcoins-correlation-to-markets-hits-a-record-in-2020/?country=us&audience=retail

APPENDIX 1

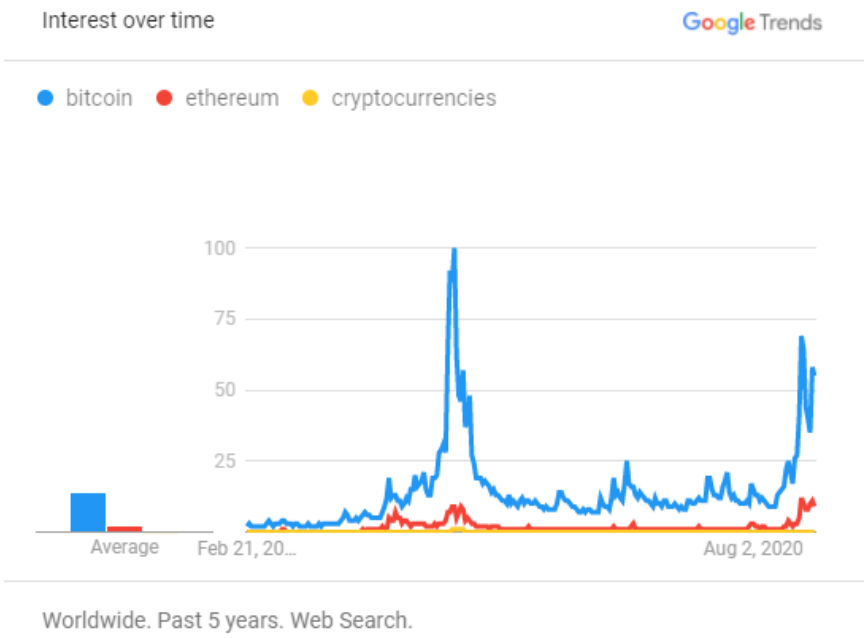


Figure 4. INTEREST TO BITCOIN, ETHEREUM AND CRYPTOCURRENCIES IN GOOGLE TRENDS. RETRIEVED FROM TRENDS.GOOGLE.COM

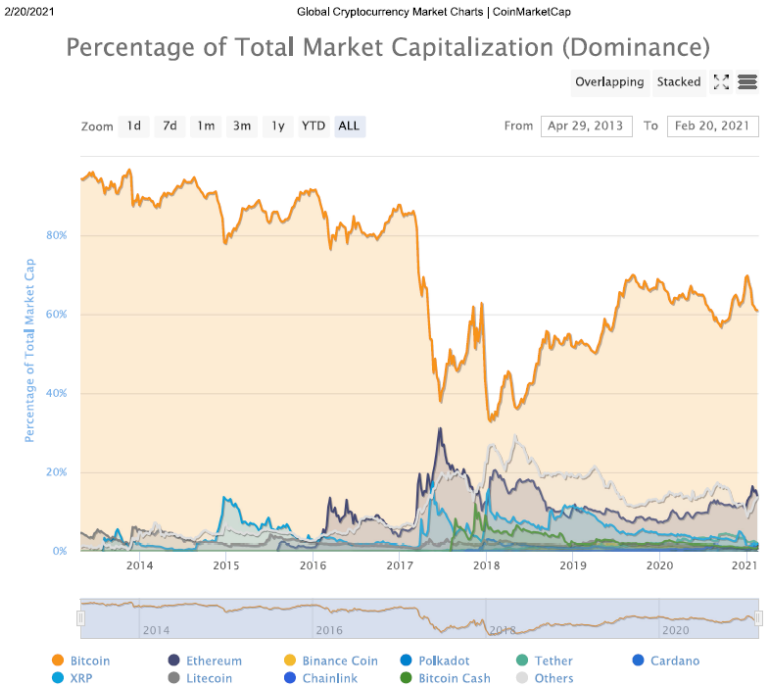
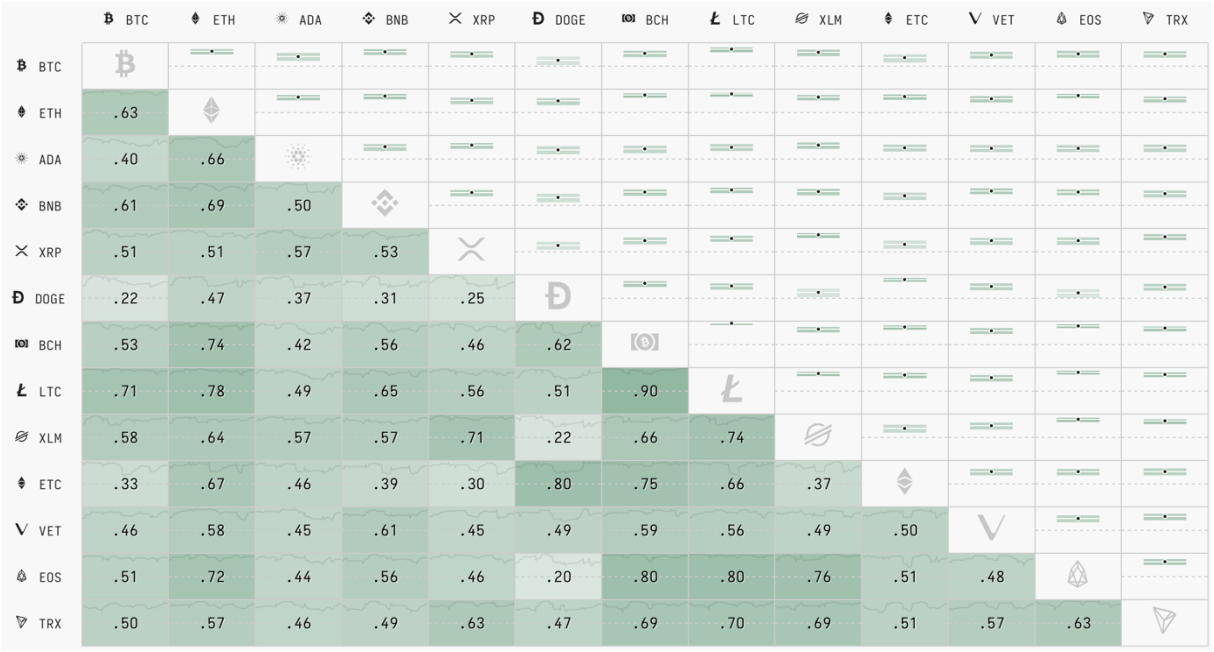


Figure 5. PERCENTAGE OF TOTAL MARKET CAPITALIZATION. RETRIEVED FROM COIMARKETCAP.COM.

APPENDIX 2



	BTC	ETH	ADA	BNB	XRP	DOGE	BCH	LTC	XLM	ETC	VET	EOS	TRX
BTC													
ETH	.63												
ADA	.40	.66											
BNB	.61	.69	.50										
XRP	.51	.51	.57	.53									
DOGE	.22	.47	.37	.31	.25								
BCH	.53	.74	.42	.56	.46	.62							
LTC	.71	.78	.49	.65	.56	.51	.90						
XLM	.58	.64	.57	.57	.71	.22	.66	.74					
ETC	.33	.67	.46	.39	.30	.80	.75	.66	.37				
VET	.46	.58	.45	.61	.45	.49	.59	.56	.49	.50			
EOS	.51	.72	.44	.56	.46	.20	.80	.80	.76	.51	.48		
TRX	.50	.57	.46	.49	.63	.47	.69	.70	.69	.51	.57	.63	

Figure 6. FIGURE 4. DETRANDED 1-YEAR CORRELATION MATRIX FOR THE GIVEN CRYPTOCURRENCIES: BITCOIN, ETHEREUM, CARDANO, BINANCE COIN, RIPPLE, DOGECOIN, BITCOIN CASH, LITECOIN, STELLAR, ETHEREUM CLASSIC, VECHAIN, EOS AND TRON.

APPENDIX 3

Table 5. Comparison of annual performance of tangency portfolios with cryptocurrencies

	2017		*2018*		*2019*		*2020*		*2021*	
Portfolio	***minvar***	***maxret***	***minvar***	***maxret***	***minvar***	***maxret***	***minvar***	***maxret***	***minvar***	***maxret***
Mean	2,47%	183,32%	-2,22%	-1,27%	3,80%	20,29%	0,36%	56,45%	1,13%	72,23%
StD	1,33%	33,54%	1,51%	2,30%	1,80%	18,26%	2,63%	28,51%	1,62%	36,83%
Var	0,02%	11,25%	0,02%	0,05%	0,03%	3,33%	0,07%	8,13%	0,03%	13,56%
Sharpe ratio	***186,06%***	***546,59%***	***-146,89%***	***-55,17%***	***211,16%***	***111,13%***	***13,48%***	***198,03%***	***69,84%***	***196,14%***
					Portfolio composition					
US Stock	2,06%	30%	0%	10%	0,68%	30%	1,26%	30%	0,00%	30%
EU Stock	1,95%	10%	0%	0%	0%	30%	1,26%	0%	5,37%	10%
US Bond	30%	0%	30%	30%	29,25%	10%	29,79%	30%	30%	0%
EU Bond	30%	0%	30%	30%	30%	0%	30%	10%	30%	0%
Commodity	11,95%	30%	11,06%	0%	12,27%	0%	7,48%	0%	5,36%	30%
Forex	23,85%	0%	28,69%	30%	27,30%	0%	30%	0%	29,27%	0%
Crypto	0,20%	30%	0,25%	0%	0,51%	30%	0,22%	30%	0%	30%

Table 6. Comparison of annual performance of tangency portfolios without cryptocurrencies

	2017		*2018*		*2019*		*2020*		*2021*	
Portfolio	***minvar***	***maxret***	***minvar***	***maxret***	***minvar***	***maxret***	***minvar***	***maxret***	***minvar***	***maxret***
Mean	1,53%	14,00%	-1,95%	-1,27%	3,59%	15,83%	-0,03%	7,62%	1,13%	28,04%
StD	1,34%	4,32%	1,53%	2,30%	1,82%	6,23%	2,64%	12,61%	1,62%	8,56%
Var	0,02%	0,19%	0,02%	0,05%	0,03%	0,39%	0,07%	1,59%	0,03%	0,73%
Sharpe ratio	***113,67%***	***323,66%***	***-127,60%***	***-55,17%***	***196,99%***	***254,30%***	***-1,06%***	***60,37%***	***69,80%***	***327,52%***

					Portfolio composition					
US Stock	2,94%	30%	0%	10%	0%	30%	1,20%	30%	0%	30%
EU Stock	1,30%	30%	0%	0%	0%	30%	1,57%	10%	5,40%	30%
US Bond	30%	0%	30%	30%	29,43%	30%	29,61%	30%	30%	0%
EU Bond	30%	10%	30%	30%	30%	10%	30%	30%	30%	0%
Commodity	12,38%	30%	11,55%	0%	13,47%	0%	7,61%	0%	5,34%	30%
Forex	23,37%	0%	28,45%	30%	27,10%	0%	30%	0%	29,26%	10%