Normative dimensions of optimal income taxation using observable variables: a step from theory to practice

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Abstract

Traditional welfarist models of optimal income taxation have little room for normative input and are sensitive to changes in unobservable variables. This study explores normative dimensions of optimal income taxation while using completely observable variables. Specifically, a non-welfarist model is presented which constructs optimal income taxation schemes using the distribution of earnings, taxable income elasticity, and the normative tastes of the policymaker. The model is significantly easier to use than most traditional models due to the abandoning of the welfarist objective. Generalized social marginal welfare weights are used to describe the redistributive preferences of policymakers following seven unique principles of distributive justice. Additionally, different distributions of earnings with varying degrees of earnings inequality and different levels of taxable income elasticity are considered. Optimal marginal tax schemes are found to be sensitive to normative input and the shape of the earnings distribution, suggesting L-shaped, as opposed to U-shaped, patterns of optimal marginal tax rates under some circumstances. Policymakers could use the model for guidance on preferred tax reforms based on efficiency or normative grounds.

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1. Introduction

> "Philosophy-free tax theory or practice does not exist; there is only tax theory and practice conducted with insufficient attention to underlying philosophical assumptions. Moral philosophy fixes the ends to which taxation properly aims. . . . Philosophy, economics, or any other single field of study cannot have a monopoly on useful contributions to tax theory. A living, meaningful tax theory requires uniting philosophy and science."

Those are the beautiful words of LeFevre (2016, p. 768). Cowell (2018b) argues, in a broader sense, that any policy about redistribution captures a tradeoff between economic efficiency and distributive justice. This tradeoff, however, is above all relevant in the study of optimal income taxation (Auerbach, 2018; Cowell, 2018a). The trilogy of egalitarianism, utilitarianism, and libertarianism covers most of the principles of distributive justice. Broadly speaking, egalitarian principles are concerned with equality of outcome and redistribution to the poor, whereas libertarian principles prioritize self-ownership through the protection of liberty and property rights resulting in a weak concern for redistribution. Utilitarian principles are welfare based and consider redistribution to be just when it maximizes society's total welfare (Fleurbaey, 2004; Lamont & Favor, 2017). Literature on optimal income taxation is extensive, but limited by the fact that it has been dominated by a utilitarian approach (Mankiw, Weinzierl, & Yagan, 2009; Murray, 2017). Utilitarian principles of distributive justice are the only principles that inherently capture a concern about economic efficiency, making them attractive for models of optimal income taxation. In such utilitarian models, a social welfare function subject to several constraints is maximized. By introducing certain assumptions, the optimization problem captures a tradeoff between higher taxes for redistribution on the one hand and creating incentives for effort on the other hand. This resembles the tradeoff between distributive justice and economic efficiency (Mankiw *et al.*, 2009; Mirrlees, 1971; Myles, 2018). Significant contributions were made in the 1970s, most notably by James Mirrlees. In his utilitarian model in 1971, Mirrlees considers heterogeneity of skill and effort which both affect individuals' abilities to earn income. The policymaker can only observe earnings and sets marginal tax rates such that total welfare in society is maximized. Results suggest that optimal marginal income tax rates are characterized by the ratio of individuals affected marginally and inframarginally, declining marginal rates for high incomes, and the extent of inequality in the distribution of ability to earn income.

The approach presented by Mirrlees (1971), however, allows for only limited normative input and comes with numerous difficulties regarding high sensitivity towards unobservable distributions and variables. There is nothing wrong with the approach per se, but it remains very theoretical and not practical (Maniquet & Neumann, 2016; Mankiw *et al.*, 2009; Murray, 2017). This is not to say that it is not useful, since some of the results obtained by Mirrlees (1971) are still replicated today. Since the late-nineties, significant steps towards more applicable models of optimal income taxation with more room for normative input have been taken. Feldstein (1995) sparkled interest in the use of taxable income elasticities to capture the magnitude of behavioural responses towards income taxation. Studies by Diamond (1998) and Saez (2001) simplified the traditional model by Mirrlees (1971) by using more observable variables, and introduced social marginal welfare weights which reflect redistributive tastes of the government. Saez and Stantcheva (2016) are the first to present a non-welfarist model of optimal income taxation by applying social marginal welfare weights directly to observed earnings, rather than applying them to unobservable functions of utility. They present a model of optimal income taxation which relies on taxable income elasticities, the distribution of earnings, and generalized social marginal welfare weights. The generalized social marginal welfare weights used in Saez and Stantcheva (2016), however, are computed using functions of utility which are unobservable for the policymaker.

This study aims to improve on previous literature by exploring normative dimensions of optimal income taxation while continuing to work on the objective of using completely observable variables. Specifically, this study would be the first to construct optimal income taxation schemes which rely solely on the distribution of earnings, taxable income elasticities, and the normative tastes of the government. This is an advancement in literature for four reasons. First, using generalized social marginal welfare weights to represent different principles of distributive justice "brings back social preferences as a critical element for optimal tax theory analysis" (Piketty & Saez, 2013; Saez & Stantcheva, 2016, p. 43). Moving beyond the traditional welfarist framework broadens the ethical scope of optimal income taxation theory and lifts the debate to a higher level, as it allows for discussion amongst a more diverse range of normative perspectives (Saez & Stantcheva, 2016). Second, as optimal income taxation schemes rely on the distribution of earnings in this study, schemes based on different principles of distributive justice can be compared across different distributions of earnings. This can help to answer questions about the relationship between the shape of optimal marginal tax rate patterns and the distribution of earnings, which are currently unanswered in literature (Saez, 2001). Third, my study contributes to the efforts made since Mirrlees' model in 1971 to close the gap between optimal income taxation in theory and practice (Mankiw *et al.*, 2009). Murray (2017) argues that it is of great importance that policymakers and analysts are able to understand the underlying mechanisms leading to results found in literature on optimal income taxation. This understanding is vital for the design of practical income taxation policies, which makes practical models with room for normative input highly relevant. The fourth and final area of knowledge to which my study can contribute is a more general one: the interplay between justice and economics. Economic demands of taxation are naturally intertwined with justice demands of taxation. Income taxation and distributive justice are inherently connected and welfarism alone cannot embody this connection. This calls for the development of competing theories of optimal taxation based on different principles of justice (LeFevre, 2016).

To construct optimal income taxation schemes – which rely solely on the distribution of earnings, taxable income elasticities, and the normative tastes of the government – the model by Saez and Stantcheva (2016) serves as the foundation. Different to the model by Saez and Stantcheva (2016) is that in this study, taxable income elasticities are always assumed to be constant for simplicity reasons. Moreover, the Pareto parameter is computed in a different way to correct for volatility and divergence towards infinity for high earners. Most importantly, generalized social marginal welfare weights are estimated according to a novel function to match different principles of distributive justice. Specifically, this novel function – inspired by the functions for utilitarian generalized social marginal welfare weights in Saez (2001) and Madden and Savage (2020) – uses only redistributive tastes and the distribution of earnings as its inputs, without relying on unobservable utility functions. This results in a revised model of optimal marginal tax rates which relies solely on the distribution of earnings, taxable income elasticities, and the normative tastes of the government. By definition, the generalized social marginal welfare weights indicate how total tax revenue should be redistributed in society. When optimal marginal income tax rates are computed using different principles of distributive justice and tax revenue is redistributed accordingly, the different tax schemes and their effects on the distribution of income can be compared visually and quantitatively. Graphs of optimal marginal tax rates and disposable (after-tax) income over earnings are created, as is commonly done in literature. Moreover, Lorenz curves are graphed to visualize effects on inequality reduction, a common objective related to taxation and redistributive policy. Lorenz curves are directly related to Gini coefficients, which are used as a quantitative measure of inequality. These graphical and quantitative results can be compared using different principles of distributive justice and different distributions of earnings to examine their role in optimal income taxation schemes and effects on inequality. Both Lorenz curves and Gini coefficients are widely used and accessible measures of inequality which should benefit interpretation of results even by non-economists (Cowell, 2018a).

The distributions of earnings in this study are simulated using a generalized beta of the second kind (GB2) distribution where the very right tail is replaced by a Pareto distribution with desired parameters. Configurations of GB2 parameters fitting empirical earnings distributions are reported by Bandourian, Turley, and McDonald (2002). The use of simulated distributions of earnings yields large benefits in flexibility, as opposed to using empirical distributions, so that schemes can be compared across distributions with a varying degree of inequality. Data on taxable income elasticities are based on empirical estimates and normative tastes are estimated according to different principles of distributive justice.

The information presented in the introduction is discussed in detail according to the following structure. Section 2 reviews the relevant literature on both distributive justice and optimal income taxation. Section 3 guides the reader through the methodological construction of the model of optimal income taxation schemes. Section 4 covers the data used for numerical simulations in this study. Section 5 presents an analysis of the results and underlying mechanisms using varying principles of distributive justice, taxable income elasticities, and earnings distributions. Section 6 concludes and proposes directions for future research.

2. Literature review

Cowell (2018b) distinguishes two primary objectives regarding redistribution: one concerns equity, while the other is about economic efficiency. Any decision about what redistributive policy is desired therefore depends on considerations of both justice and economic efficiency (Cowell, 2018b). Most policymakers pursue both justice and economic efficiency, but these two seldom coincide, since distributions which are considered just are not necessarily economically efficient (LeFevre, 2016; Weinzierl, 2014). In fact, the only time distributive justice and economic efficiency perfectly align is when the most just distribution is considered to be the most efficient distribution. Weinzierl (2014), however, shows that people’s perceptions of just distributions are not rooted in efficiency. Hence, policymakers are left with what can be described as a tradeoff between pursuing distributive justice and economic efficiency (Kaplow, 2007; Mirrlees, 1971; Murray, 2017). Policies related to redistribution are different forms of taxation policies touching mostly on income, wealth, and commodities (Kaplow, 2018). The debate on distributive justice is most alive in the world of optimal income taxation though. The distribution of income is the primary device in economics through which inequality between individuals is studied (Cowell, 2018a). Using income brings an important advantage that both economists and policymakers are interested in: the very explicit connection between income and fiscal policy. Income taxation is a large source of government revenue and also a political instrument used for redistributive purposes (Auerbach, 2018). The literature on optimal income taxation thus captures the tradeoff between distributive justice and economic efficiency very well, and thereby provides the best knowledge on how to efficiently apply certain principles of distributive justice.

2.1. Distributive justice

The field of distributive justice is concerned with questions about the moral preference of economic, political, and social frameworks related to a society's distribution of resources (Lamont & Favor, 2017). Since the second half of the twentieth century, however, economic distribution among individuals has been the narrow focus, especially the distribution of income through taxation (Shorrocks, 2018). As Fleurbaey (2004) points out, the study of distributive justice is not only relevant for political philosophers, but also for economists since they are involved in the construction of policies which affect well-being in society. Though positive economics can be used to examine the effects of different economic policies towards the distribution of economic benefits and burdens, on its own it cannot inform us which policy to pursue. This requires moral guidance, which is precisely what the principles of distributive justice provide. Combined, positive economics and distributive justice could yield optimal policies, but this process is not clear-cut as most societies are divided between different principles of distributive justice. Generally speaking, when left and right agree on what is economically efficient, they still differ in their views on freedom and justice and therefore do not desire the same policy (Kornhauser, 1996; LeFevre, 2016). A more detailed analysis on theories of philosophy and justice and their connection to the political landscape is written by Kymlicka (2002). To provide a clear structure in this study, Fleurbaey (2004) and Lamont and Favor (2017) are followed to broadly distinguish egalitarian, utilitarian, and libertarian principles of distributive justice.

The trilogy of egalitarianism, utilitarianism, and libertarianism goes back centuries and has its roots in theories by Jean-Jacques Rousseau and Karl Marx, Jeremy Bentham, and John Locke respectively (Fleurbaey, 2004). Mostly over the last decades many principles categorized somewhere within this trilogy have emerged. As the spectrum of egalitarian, utilitarian, and libertarian principles is large we will first define its boundaries by looking at two extremes. The term strict is used for illustrative purposes to denote a radical devotion to a principle, without room for any exceptions. The two principles of redistributive justice within the trilogy that define the boundaries of redistribution would be those of strict egalitarianism and strict libertarianism. Strict egalitarian principles are based on the idea that individuals are morally equal and that economic distributions should align with this equality. In more economic terms, this means that income should be equal for every individual at every point in time. This is usually paired with strict principles of intergenerational justice in order to prevent inequalities in wealth due to differences in savings. The very opposite of strict egalitarian principles are strict libertarian principles. Libertarian principles are rooted in the concept of self-ownership and are characterized by the concern for liberty and property rights, which would be violated when pursuing redistributive ideals. As such, according to a strict libertarian, the market should not be used for redistribution. Instead, the market provides a just distribution of income when it functions according to the ideas of liberty and property rights, as this is the only way to respect the concept of self-ownership (Lamont & Favor, 2017; Nozick, 1974). The principles of strict egalitarianism and strict libertarianism can be used to define the boundaries on the spectrum of principles of distributive justice as they result in two distributive extremes: complete redistribution of income and no redistribution of income. These extremes could also be interpreted as: complete desire for equity and no desire for equity. Utilitarian principles of distributive justice fall somewhere between these boundaries, and are based on the maximization of total welfare in society. This is usually done by maximizing a social welfare function based on utility levels of individuals in that society (Fleurbaey & Maniquet, 2018). Utilitarianism is therefore a welfarist principle, though these two terms are often used interchangeably in literature. What such maximization would imply for the final distribution of income is hard to say, as the possibilities theoretically range from strict egalitarian to *laissez faire* (no redistribution) distributions. The reason for this wide range is the many differences in underlying assumptions about utility and welfare, and the theory's sensitivity to such differences (Lamont & Favor, 2017). In the literature on optimal income taxation, though, the range of possible outcomes is narrower and mostly suggests degressive tax schemes with redistribution to the poor (Fleurbaey & Maniquet, 2018). According to strict utilitarian principles, any form of redistribution is deemed just if and only if it has a positive effect on society's total welfare. In contrast to strict egalitarians and libertarians, strict utilitarians share no intrinsic concerns about the degree of equity, as any desire for redistribution is simply used as a means to improving total welfare (Lamont & Favor, 2017). Interestingly, a strict utilitarian follows a strictly equal distribution of individual weights in the contribution to society's total welfare (Fleurbaey, 2004; LeFevre, 2016). Alternatively, strict libertarian principles follow a strictly equal distribution of liberties and rights (Fleurbaey, 2004). In essence, each of the three principles in the trilogy has roots in equal distribution, but they differ in what they prioritize to distribute equally.

Most prominent principles of distributive justice used in the world of optimal income taxation are based on some combination of egalitarian, utilitarian, and libertarian principles and fall somewhere within the boundaries specified in the previous paragraph (Fleurbaey & Maniquet, 2018). The main criticism of strict egalitarianism, for instance, is one based on welfare. It is argued that economies grow over time and that this growth can make everyone better off, but that this growth is hampered when incomes are strictly equal. Rawls (1971; 1993) takes this into account and proposes the difference principle. This can be seen as a Pareto efficiency requirement only for those worst off in society. In other words: inequalities are justified and desired when they are beneficial for the least advantaged group, but not for any other group in society. Where strict egalitarianism is concerned with the relative position of the poor, Rawls is concerned with the absolute position of the poor (Lamont & Favor, 2017). A notable criticism of strict utilitarianism is that it completely disregards individuals' deservedness of economic benefits and burdens as a consequence of their actions. Advocates of desert-based principles of distributive justice argue that utilitarians do not consider what individuals deserve in an economy, based on contribution, effort, or compensation. The key thought in desert-based principles is that the extent to which individuals give to the collective social product differs, and that this difference should be reflected in the distribution of outcomes (Lamont & Favor, 2017). In literature, desert-based principles are usually combined with welfarist objectives, but levels of effort are especially relevant for redistribution (Fleurbaey & Maniquet, 2018; Saez & Stantcheva, 2016). Though many more principles of distributive justice could be defined, the ones defined so far are the most relevant to this study. The literature on optimal income taxation is mostly centered around utilitarian principles (Mankiw *et al.*, 2009). Whenever studies deviate from pure utilitarianism, however, we see egalitarian, Rawlsian, desert-based, and libertarian ideas attempted to be incorporated (Fleurbaey & Maniquet, 2018).

Principles based on welfarism, such as utilitarianism, are the only principles of distributive justice which capture an inherent concern about economic efficiency. This is true under the safe assumption that economic benefits and burdens are part of what is considered welfare or utility. The maximization of total welfare in society therefore translates into an optimization problem of economic efficiency in society. This explains why models of optimal income taxation have historically been dominated by the maximization of social welfare functions based on utility functions of individuals (Mankiw *et al.*, 2009; Myles, 2018). Knowing this and the fact that the study of optimal income taxation captures the tradeoff between economic efficiency and moral attitudes to equality raises questions though. One might wonder what use there is to this range of literature for an egalitarian, who does not believe in the utilitarian principles of distributive justice? Murray (2017) perfectly illustrates why this does not make the findings related to optimal income taxation using a utilitarian framework any less relevant. Irrespective of where you are on the distributive justice spectrum, almost everyone agrees that, ceteris paribus, we should aim for the highest possible level of social welfare. Thus, even when welfarism is not considered a priority, it can still support the process of finding the most efficient state for whatever the priority is. If, say, an egalitarian prioritizes redistribution to a basic minimum income of 30% of the median, then the utilitarian framework can still be of help to find the most efficient tax rates given this constraint. Only strict libertarians are an exception to this story, as they would completely reject the importance of social welfare in favour of self-ownership (Murphy & Nagel, 2002).

2.2. Optimal income taxation

The most significant contributions in the study of optimal income taxation were made in the 1970s. Mirrlees (1971) maximizes a utilitarian social welfare function characterized by unobserved heterogeneity of individuals. This means that the individuals in the model have different levels of skill, which affect their ability of earning income. Moreover, individuals have different levels of effort which also affects their income. Only before tax income, however, can be observed by the policymaker. Taxation does not affect ability, but it does affect effort. Additionally, marginal utility of consumption is diminishing which creates an incentive for the policymaker to redistribute to the poor. This approach captures a very important problem for the policymaker: the tradeoff between high taxes to raise tax revenue for redistribution on the one hand versus low taxes to create incentives for effort, which would lead to higher incomes and thereby higher tax revenue in the future, on the other hand (Mankiw *et al.*, 2009; Myles, 2018). Many studies followed this approach to optimal income taxation (Seade, 1977; Seade, 1982; Stiglitz, 1982). The tradeoff is in line with the aforementioned idea that the desired redistributive policy is dependent on both economic efficiency and principles of distributive justice (Cowell, 2018b). However, keep in mind that this is a utilitarian approach with efficiency, not redistribution, as the end goal. A difficulty with Mirrlees' model is that 'lazy' individuals with a high level of skill can be incentivized to lower their effort, thereby earning less and avoid paying more taxes. The problem is that, through the eyes of the policymaker, this could look identical to a lower skilled 'hard-working' individual with high effort if they have the same level of earnings. Though these two individuals might look similar for the policymaker, their behavioural response to a change in marginal tax rates is not similar due to their difference in skill and effort. Despite this challenge, it is possible to solve the maximization problem of the social welfare function and its respective assumptions proposed by Mirrlees. However, the process of doing so is very complex and sensitive to changes in arguably undeterminable variables such as the distribution of ability and properties of the utility functions (Fleurbaey & Maniquet, 2018; Mankiw *et al.*, 2009; Mirrlees, 1971; Murray, 2017). Mirrlees (1971) concludes that efficient taxation based on the distribution of ability to earn income (both skill and effort) is fairly similar to simply using the observed distribution of earnings (before-tax incomes). Ideally, marginal tax rates are high for individuals with low levels of skill, and marginal tax rates are low for individuals with high levels of skill. This is because the behavioural response of lowering effort is most costly to total welfare for individuals with high levels of skill. It would therefore not be optimal to use the distribution of observed earnings, as this results in more costly losses of effort compared to using the distribution of ability to earn income.[1] However, it would

[1] The disadvantage of using the observed distribution of earnings rather than the distribution of ability to earn income is the difference in heterogeneity. The heterogeneity of observed earnings does not perfectly match the heterogeneity of ability to earn income, as the ratio of skill to effort is not reflected in an observation of earnings.

be a very convenient technique to solve the issue of the distribution of skill (and therefore also the distribution of ability to earn income) being unobservable in the real world.

Though Mirrlees' welfarist approach and the many studies that followed and improved on it are complex and therefore not easily applicable, they do yield some useful insights about efficient income taxation. The first important result from Mirrlees (1971) is that a change in marginal tax rates has the smallest effect on efficiency when it affects few individuals at the margin and many inframarginally (Mankiw *et al.*, 2009). This becomes clear when the marginal and average tax rates are considered separately. When, for instance, the marginal tax rate increases for low earners, these individuals have a reason to lower their effort. However, for middle and high earners only the average rate would increase, not the marginal rate.[2] An illustration of this is presented in Appendix A. Second, Mirrlees (1971) finds that optimal marginal tax rates approach zero for high earners. In fact, due to the way the model is constructed, the highest earner must always have a marginal tax rate of zero in the optimal state. This logically follows from the previous results. Any positive marginal tax rate for the highest earning individual in society does not affect anyone inframarginally, since there are no individuals who earn more. A positive marginal tax rate for the highest earning individual does, however, produces the negative behavioural effects at the margin which disincentivize the highest earning individual to increase effort. This is costly, as the highest earning individual is extremely productive and can therefore have a strong impact on the society's total welfare. Since the negative marginal effects of positive marginal tax rates for the highest earning individual cannot be offset by positive inframarginal effects, a zero marginal tax rate is found to be optimal (Mirrlees, 1971). This result was later replicated by Seade (1977). Mirrlees (1971) and Seade (1982) also find that optimal marginal rates can never be negative or exceed 100%. Finally, Mirrlees (1971) finds that optimal marginal tax schemes become more redistributive as inequality in ability to earn income grows. There remains much discussion about the results of Mirrlees, but his work suggested that optimal marginal income tax rates are

[2] Additional tax revenue is generated from low, middle, and high earners, which can offset the efficiency loss from the marginal increase that only applies to low earners. The same trick does not work when increasing marginal rates for high earners, as the tax revenue gain relative to the efficiency loss would be substantially smaller, resulting in a net loss of welfare. A different intuition would be the following: consider the US government decides to put a high tax on the first 10,000 dollars earned each year. That would yield tax revenue from almost every earner in society (inframarginal effect), while only a handful of very low earners have an incentive to reduce effort since taxes on all of their income have increased (marginal effect). If the US government would apply the same trick to the second 10,000 dollars earned each year, it would be a little less effective since the inframarginal gain is now smaller whereas the marginal loss is now larger.

characterized by: the ratio of individuals affected marginally and inframarginally, declining marginal rates for high incomes, and the extent of inequality in the distribution of ability to earn income.

Since the mid-nineties, more studies have attempted to use the best insights gained from the traditional model by Mirrlees (1971) to build on the construction of new, more practical models which overcome the traditional technical difficulties. Significant progress was made in the late-nineties after a study by Feldstein (1995) about the effect of changes in marginal tax rates on earnings. Feldstein (1995) analyzed the behavioural responses to the Tax Reform Act of 1986 in the United States, and estimated what is referred to as taxable income elasticities. Taxable income elasticities indicate how many euros of taxable income are lost due to a tax reform which increases tax revenue by one euro. Feldstein (1995) estimated taxable income elasticity to be 2.14, and his study marked the start of an increased interest in the topic and its relation to optimal income taxation. Additionally, Feldstein (1995) notes that changes in taxable income as a response to changes in marginal tax rates are not necessarily due to changes in effort, as would be the response in Mirrlees' (1971) model. Instead, changes in the extent of investment in assets and spending on tax-deductible activities are behavioural responses which also affect taxable income. Essentially, taxable income elasticities capture a complete response to changes in marginal tax rates, which is what makes them useful for estimating welfare effects in the study of optimal income taxation. In order to use them, however, reliable estimates are needed. Following the methods of Feldstein (1995), Saez (2003) finds a taxable income elasticity of 0.4, significantly lower than Feldstein's estimate of 2.14. Gruber and Saez (2002) present an overview of the studies on taxable income elasticities from 1987 to 2000 and report that results range from 0 to 0.8. Differences are said to depend mostly on methodological details and on the definition of taxable income. Saez, Slemrod, and Giertz (2012) provide a more up to date review of studies on taxable income elasticities. They confirm that the use of elasticities can be fruitful in the study of optimal income taxation, and conclude that the best long-run estimates of taxable income elasticity are between 0.12 and 0.40. These estimates have been the benchmark for the latest studies on taxable income elasticity. The most recent studies, however, by Weber (2014) and Burns and Ziliak (2017), apply more robust controls and find taxable income elasticities to be 0.86 and 0.55 respectively. This suggests that the range between 0.12 and 0.40 estimated by Saez, Slemrod, and Giertz (2012) might be too conservative. More importantly, it suggests that consensus on the level of elasticity of taxable income is still absent. Despite the lack of consensus, the multitude of studies on taxable income elasticities produce a confidence interval ranging between 0.12 and 0.86 which can be applied in models of optimal income taxation.

Besides the use of taxable income elasticities, interest also arose for the use of social marginal welfare weights which reflect redistributive preferences of the government. Though the precise definition and use of social marginal welfare weights is not identical in every study, they all capture how much welfare the government wishes to redistribute to individuals at different levels of earnings. Social marginal welfare weights are what open models of optimal income taxation up to the inclusion of a pluralism of principles of distributive justice. Mirrlees' (1971) traditional approach is utilitarian, and only the degree of concavity of utility functions could be changed to modify the taste for equal redistribution to a limited extent. When social marginal welfare weights are included in a model, however, any possible set of redistributive tastes of the policymaker can be reflected in the optimal income taxations scheme. Atkinson (1990) explores the shortcomings of results obtained by Mirrlees (1971), and looks for possible improvements to the study of optimal income taxation which would not produce the unrealistic result of rates converging to zero for the highest earner. The case of the charitable conservative is considered in which the policymaker assigns a high social marginal welfare weight to the poor and a low weight to the nonpoor. Diamond (1998) continues on this road and develops a model which produces U-shaped optimal marginal income tax rates, contrary to the traditional result of declining rates. The reason that marginal tax rates by Diamond (1998) are not converging to zero is because of the Pareto parameter in the model. This is a parameter based on the distribution of skills/earnings, and its economic interpretation would be that it captures the relative impact of behavioural effects from a change in marginal tax rates at different levels of income.[3] Note that the Pareto parameter reflects directly the important result found by Mirrlees (1971), that a change in marginal tax rates has the smallest effect on efficiency when it affects few individuals at the margin and many inframarginally. Diamond (1998) computes the Pareto parameter as a ratio capturing the density of individuals at the margin relative to the density of individuals inframarginally. The Pareto parameter is, by definition, constant for Pareto distributions. For lognormal distributions, such as the distribution

[3] In most studies prior to 2001, including the studies by Mirrlees (1971) and Diamond (1998), the shape of the distribution of skills determines the shape of the distribution of earnings. Therefore, the Pareto parameter is computed using the distribution of skills rather than earnings in studies prior to 2001. In most recent studies, however, the Pareto parameter is based on the distribution of earnings due to advancements in the models of optimal income taxation.

of abilities in Mirrlees (1971), this ratio is always declining. Diamond (1998) shows that when using a distribution with a Pareto tail, optimal marginal tax rates could be U-shaped depending on other parameters in the model. Besides that, the study by Diamond (1998) also considers elasticity of labour supply and social marginal welfare weights for the determination of optimal marginal tax rates. This marks the start of a new wave of literature on optimal income taxation built around the Pareto parameter, elasticities, and social marginal welfare weights.

The new wave of literature revolves primarily around the studies by Diamond's doctoral student: Emmanuel Saez. The study by Saez (2001) is a breakthrough as it is the first to construct a model of optimal marginal tax rates based on the distribution of earnings directly, as opposed to using a distribution of abilities. Optimal marginal tax rates are found to depend on the level of earnings, the Pareto parameter, elasticities, and social marginal welfare weights.[4] Essentially, the model by Saez (2001) can be interpreted as the traditional utilitarian model by Mirrlees (1971) written using the variables from the model by Diamond (1998). As such, the model by Saez (2001) still involves the maximization of a welfare function, but due to the difference in variables used it can be applied to the distribution of earnings directly. Though still on the technical side, the model by Saez (2001) is significantly less technical than the traditional model by Mirrlees (1971) and yet yields more realistic results. First, Saez (2001) shows that through the use of elasticities and the Pareto parameter, optimal income tax rates for high incomes can be determined and are found to be as high as 80%. This is in sharp contrast with the unrealistic result of optimal marginal tax rates converging to 0% for high incomes. Second, Saez (2001) find that the optimal marginal tax rates for the distribution of earnings in the US in 1992 and 1993 are U-shaped, confirming results by Diamond (1998). It is however, not confirmed that this U-shaped pattern holds for all distributions of earnings, as the shape of the income distribution affects the Pareto parameter which in turn affects the pattern of optimal marginal rates. Though the model by Saez (2001) is modern in the sense that it relies on variables which are observable and has more room for normative tastes through the inclusion of social marginal welfare weights, it is still based on the traditional idea of maximizing a social welfare function. Total welfare is then calculated using a social utility function

[4] Elasticities are divided in compensated and uncompensated elasticities, due to the assumption of income effects in the model. These three concepts are connected through the Slutsky equation, more information can be found in section 3.1 in Saez (2001). Most studies consider the case where income effects are assumed to equal zero, and thus only one elasticity variable is required in a model. The latter approach is followed in the present study too, and therefore details on compensated and uncompensated elasticities are not considered relevant in the literature review.

of disposable income, where the concavity of this function captures society's concern for redistribution. Saez and Stantcheva (2016) are the first to present a non-welfarist model of optimal income taxation. They are able to drop the social welfare objective by applying social marginal welfare weights directly to observed earnings, rather than applying them to unobservable functions of utility. These weights are called generalized social marginal welfare weights, as opposed to (standard) social marginal welfare weights used in previous studies. Generalized social marginal welfare weights "represent the value that society puts on providing an additional dollar of consumption to any given individual", and thereby "directly reflect society's concerns for fairness" (Saez & Stantcheva, 2016, p. 24). Essentially, redistributive preferences are now reflected through generalized social marginal welfare weights rather than the degree of concavity of utility functions. The model by Saez and Stantcheva (2016) is very similar to the model used in Saez (2001), except that it is significantly less technical as there is no more maximization problem to be solved.

Abandoning the social welfare objective does not come without costs though (Fleurbaey & Maniquet, 2018). This can effectively be illustrated by reviewing the definition of 'optimal' in optimal income taxation literature. For a welfarist model such as that of Mirrlees (1971), where a social welfare function is maximized by solving a first order condition, one or multiple equilibria may be found. In the model by Saez and Stantcheva (2016) there is no social welfare function and therefore no first order condition to be solved. Instead, welfare gains and losses for each individual are weighted using welfare weights, which depend on both normative tastes of the government and the distribution of earnings. These weights, together with behavioural effects through the Pareto parameter and the taxable income elasticity, are used to compute optimal marginal tax schemes. The model is constructed in a way that it produces tax schemes such that no small tax reform could yield a social welfare gain, without having to solve a first order condition. Proof is mathematically complex, and can be found in Saez and Stantcheva (2016). Essentially, social welfare could be evaluated after using the model, as a sum of disposable incomes and behavioural effects weighted by the generalized social marginal welfare weights. The convenient property of the model by Saez and Stantcheva (2016) is that it always returns schemes which are optimal as long as social marginal welfare weights are nonnegative and decreasing over income, meaning that users of the model do not have to conduct the complex evaluation of social welfare. As is the case for models based on the maximization of a social welfare function, multiple equilibria could potentially satisfy this condition. The fundamental point is that although users of the model by Saez and Stantcheva

(2016) are liberated from working with a social welfare function, the model only returns one local optimal tax scheme. The important difference to welfarist approaches is that the solution to the first order condition problem can yield multiple optima, and the most optimal equilibrium can be identified by direct comparison of total welfare between these equilibria (Mirrlees, 1971). This is not the case for non-welfarist models using generalized social marginal welfare weights, as the model returns only one local optimum which can therefore not be ranked against others. Saez and Stantcheva (2016) acknowledge that this is a disadvantage in their approach compared to the welfarist approach, as produced schemes could suffer in terms of accuracy.[5] Essentially, welfarist models are able to find the optimal scheme such that after-tax incomes are least distorted by taxes and redistribution, and therefore possess an implicit defense of the laissez-faire allocation (Fleurbaey & Maniquet, 2018). The model by Saez and Stantcheva (2016) does not carry such a property, which is why resulting schemes could be less accurate than those based on traditional welfarist models. Saez and Stantcheva (2016) indicate that their model can best be applied to examine tax reforms given the extent of optimality the model can guarantee. Although this potential loss in accuracy should definitely be acknowledged, it does not outweigh the benefits of the approach by Saez and Stantcheva (2016) for the goals relevant to this specific paper. This is because accuracy is important, but not a priority in my study. Saez and Stantcheva (2016) make use of generalized social marginal welfare weights which give the model exceptional room for normative input by the policymaker. Piketty and Saez (2013) highlight the potential of the use of generalized social marginal welfare weights in models of optimal income taxation.[6] Important is that they point out that generalized social marginal welfare weights can be derived from principles of social (distributive) justice. Madden and Savage (2020) are among the first to explore generalized social marginal welfare weights based on different principles of distributive justice. They do, however, use many variables which are not easily observable for the policymaker and focus primarily on household tax reform.

[5] By accuracy, I refer to the ability of the model to provide tax schemes which are truly optimal for every individual in society given the inputs and assumptions in the model. Mirrlees' (1971) model is extremely accurate, since it rests on mathematical proof that results are optimal for every individual in society given the inputs and assumptions of the model. The model by Saez and Stantcheva (2016) returns optimal tax schemes which are less accurate in this sense.

[6] Piketty and Saez (2013) discuss the generalized social marginal welfare weights in the at that time working paper by Saez and Stantcheva. They refer to: 'Saez, E., & Stantcheva, S. (2013). Generalized Social Marginal Welfare Weights for Optimal Tax Theory. *NBER Working Paper*, (w18835)'. The concept is the same as in the final study published in 2016 though.

This study aims to improve on previous literature by exploring normative dimensions of optimal income taxation while continuing to work on the objective of using completely observable variables. Specifically, this study would be the first to construct optimal income taxation schemes which rely solely on the distribution of earnings, taxable income elasticity, and the normative tastes of the government. This is an advancement in literature for four reasons. First, as discussed by Piketty and Saez (2013) and Saez and Stantcheva (2016), using generalized social marginal welfare weights to represent different principles of distributive justice "brings back social preferences as a critical element for optimal tax theory analysis" (Saez & Stantcheva, 2016, p. 43). Critical, since Saez and Stantcheva (2016), Weinzierl (2014), and Madden and Savage (2020) report that people do not always share welfarist views on taxation issues. Moving beyond the traditional welfarist framework broadens the ethical scope of optimal income taxation theory and lifts the debate to a higher level, as it allows for discussion amongst a more diverse range of normative perspectives (Saez & Stantcheva, 2016). Second, as optimal income taxation schemes rely on the distribution of earnings in this study, schemes based on different principles of distributive justice can, ceteris paribus, be compared across different distributions of earnings. This can help answer the question raised by Saez (2001), on whether the U-shaped pattern of optimal tax rates is universal across different distributions of earnings. Third, my study contributes to the efforts made since Mirrlees' model in 1971 to close the gap between optimal income taxation in theory and practice. Especially since the mid-nineties, progress has been made to simplify models of optimal income taxation and use better observable variables. Mankiw *et al*. (2009, p. 147) analyze the differences between theory and practice in optimal income taxation and argue that throughout history, the two have been "far from parallel". Murray (2017) examines the gap between theory and practice and, like Maniquet and Neumann (2016), acknowledges the need for a more normative framework of optimal income taxation. Along with Fleurbaey and Maniquet (2018) and Kaplow (2007), Murray (2017) also recognizes the difficulty of abandoning the traditional welfarist approach given its strength in capturing the tradeoff between justice and economic efficiency. Nonetheless, Murray (2017) argues that it is of great importance that policymakers and analysts are able to understand the underlying mechanisms leading to results found in literature on optimal income taxation. This understanding is vital for the design of practical income taxation policies, which makes practical models with room for normative input highly relevant despite their inevitable sacrifice in traditional economic accuracy. The fourth and final area of knowledge to which my study can contribute is a more general one: the interplay between justice and economics. LeFevre (2016) provides as excellent analysis about the role of optimal taxation theory in the study of philosophy and economics. It is best summarized in his own words, which takes us back to the opening quote of this paper:

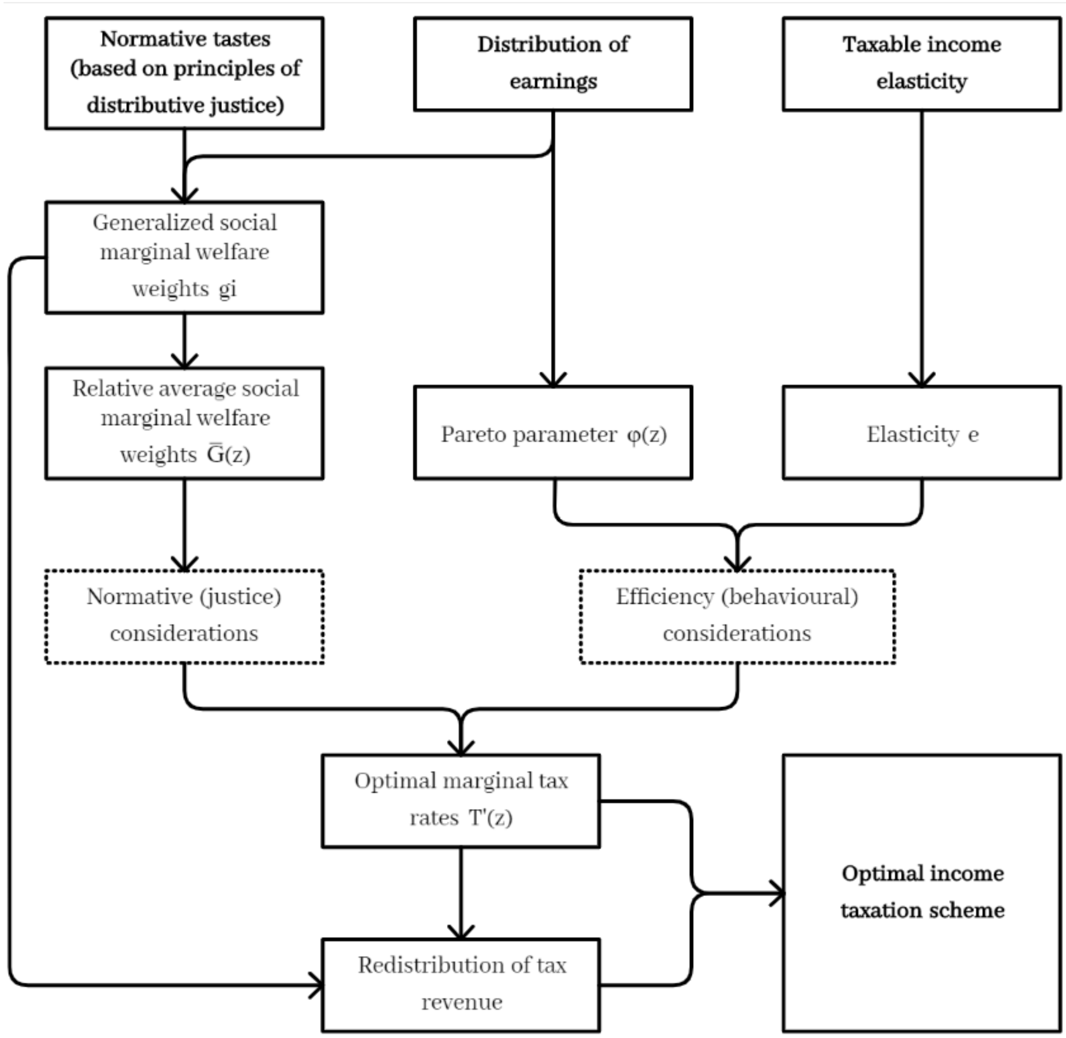
> Philosophy-free tax theory or practice does not exist; there is only tax theory and practice conducted with insufficient attention to underlying philosophical assumptions. Moral philosophy fixes the ends to which taxation properly aims. . . . Philosophy, economics, or any other single field of study cannot have a monopoly on useful contributions to tax theory. A living, meaningful tax theory requires uniting philosophy and science. (p. 768)

LeFevre (2016) continues to argue that economic demands of taxation are naturally intertwined with justice demands of taxation, and calls for the development of competing theories of optimal taxation based on different principles of justice. This is a different path to reach the aforementioned conclusion by Murray (2017), and further adds to the relevance of my study. In the broadest sense, my study could be interpreted as an exploration in the economic quantification of different philosophical perspectives and their potential implications for society.

Lastly, one might wonder whether this study is positive, normative, or instrumental? Albrecht (2017) states that all optimal policy models are positive, normative, and instrumental to some degree. My model is mostly instrumental, as it gives guidance to policymakers given their normative tastes. The model does not indicate what the normative tastes of the policymaker should be. Additionally, elements of my model are inherently positive as they aim to realistically capture the responses of individuals to policies. These parts of the model predict how society would react to actions taken by the policymaker. However, as Albrecht (2017) argues, results of my model could also imply that policymakers should follow certain tax policies as a result of the underlying mechanisms used in the model. This could be interpreted as a normative result. Though room for normative input is exceptionally large in my model, the goal of ‘optimization’ in the model still implies some sort of assumption about what society should strive for. Albrecht (2017, p. 13) suggests to regard an optimal policy model as “*an instrumental tool to generate hypotheses*”. My study mostly highlights the mechanisms underlying optimal income taxation, and ultimately advices policymakers on how to use this knowledge for efficient tax reforms. This advice could be interpreted as “tax reforms in the directions suggested by my model are preferable to no tax reforms”, which is a testable hypothesis in the political process.

3. Methodology

In order to construct optimal income taxation schemes relying solely on the distribution of earnings, taxable income elasticity, and the normative tastes of the government, a model of optimal marginal tax schemes is required. The model presented in this study primarily follows the model for optimal marginal tax rates by Saez and Stantcheva (2016), with some adjustments to make the model fit the goals of this study. Since the construction of optimal income taxation schemes can, even in its most simplified form, get quite complex for new readers, an overview of the construction of schemes in this study is presented below. The diagram below shows how the three inputs – normative tastes of the government, distribution of earnings, and taxable income elasticity – are used in this study to compute optimal marginal tax rates and its respective redistributive policy. Together, optimal marginal tax rates and redistribution of tax revenue form the optimal income taxation scheme.



Tax schemes are created from scratch in this study to examine the underlying mechanisms in optimal income taxation related to changes in normative input, taxable income elasticity, and the distribution of earnings. Please, however, recall that the model by Saez and Stantcheva (2016) produces tax schemes which are optimal such that no small tax reform could yield a welfare gain. This is true for my model too, and therefore policymakers are advised to only apply the model to examine the effectiveness of small tax reforms. Small tax reforms also limit potential long-term distortions to the earnings distribution.[7]

3.1. The model

The model used to construct optimal income taxation schemes which rely solely on the distribution of earnings, taxable income elasticity, and the normative tastes of the government primarily follows the model for optimal marginal tax rates by Saez and Stantcheva (2016). Their model uses variables which are relatively easy to observe such as the shape of the income distribution and income elasticities, and allows for normative input through the use of generalized social marginal welfare weights. Moreover, Saez and Stantcheva (2016) provide links to alternative (non-welfarist) principles of justice in their study, providing guidance on how to compute generalized social marginal welfare weights for non-welfarist principles of distributive justice. Overall, the study by Saez and Stantcheva (2016) incorporates the most important findings on optimal income taxation since the traditional model by Mirrlees (1971) and the modernization by Saez (2001) in a slightly less complex model, while being open for normative input. Therefore, the model by Saez and Stantcheva (2016) connects best to the objectives of this study.

Let us first be introduced to the model of optimal marginal tax rates constructed by Saez and Stantcheva (2016):

$$T'(z) = \frac{1 - \bar{G}(z)}{1 - \bar{G}(z) + \alpha(z) \cdot e(z)} \tag{1}$$

[7] Though the model produces tax schemes such that the impact of behavioural effects is minimized, a radical change to an existing tax scheme could potentially distort the future distribution of earnings to such an extent that the newly implemented tax scheme is no longer optimal. This potential problem is minimized when the model is only used to implement small tax reforms, as these do not result in large distortions to the future earnings distribution. Future research could examine an optimal taxation model in a setting with multiple time periods to capture such effects. Results could hint at a stable long run optimal tax policy, or an endless cycle of ever-changing optimal policies.

Where z is earnings (also known as before-tax or taxable income), $e(z)$ is the average elasticity of earnings z_i with respect to the retention rate $1 - T'$ for individuals earning $z_i = z$, $\alpha(z)$ is the local Pareto parameter defined as $zh(z)/\left(1 - H(z)\right)$, and $\bar{G}(z)$ is the relative average social marginal welfare weight for individuals who earn more than z, defined as:

$$\bar{G}(z) \equiv \frac{\int_{\{i:\, z_i \geq z\}} g_i \, di}{Prob(z_i \geq z) \cdot \int_i \; g_i \, di} \tag{2}$$

With g_i the generalized social marginal welfare weight on individual i.[8] Saez and Stantcheva (2016) apply social marginal welfare weights directly to observed earnings, rather than applying them to unobservable functions of utility. Essentially, the social welfare objective is abandoned by the use of relative average social marginal welfare weights $\bar{G}(z)$. The model is constructed in a way that it produces tax schemes such that no small tax reform could yield a social welfare gain, without having to solve a first order condition. Welfare gains and losses for each individual are weighted through $\bar{G}(z)$ by using generalized social marginal welfare weights g_i (equation (2)). Generalized social marginal welfare weights g_i "measure how much society values the marginal consumption of individual i", and thus reflect normative redistributive tastes of the government (Saez & Stantcheva, 2016, p. 26). These weights, together with behavioural effects through the Pareto parameter $\alpha(z)$ and taxable income elasticities $e(z)$, are used to compute optimal marginal tax rates $T'(z)$ such that no small reform could yield a welfare gain (equation (1)). Details on the derivation of this model are discussed in Saez and Stantcheva (2016). All the variables will now be covered, as well as their derivation and use of them in my model, which is presented in detail later in this section.

3.1.1. Efficiency (behavioural) considerations

Saez and Stantcheva (2016) provide very little information on the derivation of elasticity $e(z)$, but it is clear that the elasticity affects the change of earnings due to a change in taxes and transfers. The derivation of taxable income elasticities in Saez (2001) and Piketty and Saez (2013) is complex and distinguishes uncompensated and compensated elasticities, combined with income effects through the connection of the Slutsky equation. The model used by Saez and Stantcheva

[8] g_i is defined by: $g_i = g\left(c_i, z_i, x_i^s, x_i^b\right)$. For detailed information on these variables, see Definition 1 in Saez and Stantcheva (2016).

(2016), however, is constructed in such a way that it rules out income effects in order to simplify the optimal tax formula. Therefore, elasticity derivations as presented in Saez (2001) and Piketty and Saez (2013) are of no use in the model by Saez and Stantcheva (2016). In part of their illustrations, Saez and Stantcheva (2016) assume constant elasticities over income. Constant elasticities further simplify the model of optimal marginal tax rates. Despite a lack of consensus in literature, the multitude of studies on taxable income elasticities produce a confidence interval ranging between 0.12 and 0.86 which can be applied in models of optimal income taxation. As such, the variable $e(z)$ in this study will be a constant e, tested using lower limit $e = 0.12$, middle estimate $e = 0.40$, and upper limit $e = 0.86$. Constant elasticities are assumed for simplicity reasons, the same argument used by Saez and Stantcheva (2016).

The next variable in Saez and Stantcheva's (2016) model of optimal marginal tax rates is $\alpha(z)$, the local Pareto parameter defined as $zh(z)/\big(1 - H(z)\big)$. Here, $h(z)$ denotes the earnings density and $H(z)$ the cumulative earnings distribution function. The Pareto parameter is based on the distribution of earnings and captures the relative impact of behavioural effects from changes in marginal tax rates at different levels of earnings.[9] The cumulative earnings distribution function $H(z)$ is simply a function which returns the probability of an individual earning at most z_i, and is thus increasing with earnings. The density $h(z)$ is simply the density of the earnings distribution and could be thought of as a continuous probability function of the histogram of earnings. In this study, the density is estimated using the Epanechnikov kernel density function in Stata, which is the default method to estimate density functions.[10] Though the Pareto parameter in Saez and Stantcheva (2016) is defined as $\alpha(z) = zh(z)/\big(1 - H(z)\big)$, there exists another derivation of the Pareto parameter in literature, $a(z)$, which is used for high earners. In Saez (2001) and Diamond and Saez (2011), the Pareto parameter a is defined as $z_m/(z_m - z_i)$, with $z_m = \bar{z}\, when\, z > z_i$, where $\bar{z}$ is the mean of z. Thus, z_m is the average income above earnings level z_i. Figure 2 in Diamond and Saez (2011) illustrates the empirical Pareto parameters $\alpha(z)$ and $a(z)$ for gross incomes in the US, 2005. It can be observed that the Pareto parameters converge to a constant for

[9] Detailed information on the Pareto parameter and its respective components can be found in Saez (2001), Diamond and Saez (2011), and Piketty and Saez (2013).

[10] Saez (2001), Piketty and Saez (2013), and Saez and Stantcheva (2016) technically use the virtual density, often denoted as $h^*(z)$. This is the density which would hold at z_i if an optimal tax system linearized at z_i were to be applied. This however, is very complex to derive and only slightly affects the density. As part of the goal of this study is to make optimal income taxation theory more applicable, the regular density $h(z)$ is used.

high earners, and that parameter $a(z)$ is substantially less volatile for high earners. The Pareto parameter $a(z)$ is inaccurate for low earners though, as it has a lower bound of 1 instead of 0. My testing yields similar results, which are discussed in the Data section and in Appendix E. The most noticeable difference compared to Saez (2001) and Diamond and Saez (2011) is that the Pareto parameter $\alpha(z)$ is even more volatile for this study at high levels of earnings, which would yield extremely volatile tax rates for high earners. As such, in my model of optimal marginal tax rates I define the Pareto parameter $\alpha(z) = zh(z)/\big(1 - H(z)\big)$ for low and medium earnings, and the Pareto parameter $\rho(z) = z_m/(z_m - z_i)$ with $z_m = \bar{z}\, when\, z > z_i$ for high earnings.[11] Specifically, the $\rho(z)$ replaces $\alpha(z)$ when they first intersect, as they always intersect at the maximum of $\rho(z)$, when earnings are high (around the 75th percentile). This is a consistent point where the difference in volatility becomes noticeable, and can thus be corrected for. The point of intersection is defined as I. The final Pareto parameter is then defined as:

$$\varphi(z) \equiv \begin{Bmatrix} \alpha(z)\; when\; z_i \leq z_I \\ \rho(z)\; when\; z_i > z_I \end{Bmatrix} \tag{3}$$

Where z_I denotes the earnings level at the point of intersection I.

3.1.2. Normative (justice) considerations

The final component in the model of optimal marginal tax rates by Saez and Stantcheva (2016) is $\bar{G}(z)$, the relative average social marginal welfare weight for individuals who earn more than z. As can be observed in equation (2), $\bar{G}(z)$ is a function of the generalized social marginal welfare weights g_i. Generalized social marginal welfare weights g_i measure how much the marginal consumption of individual i is valued by society. Essentially, $\bar{G}(z)$ can be interpreted as the sum of g_i for every $z_i \geq z$, divided by the probability that earnings $z_i \geq z$ multiplied by total sum of g_i. For the lowest earner, this ratio equals 1 as the equation will simplify to the total sum of g_i, over 1 multiplied by the total sum of g_i. Consequently, the ratio will decrease over earnings until it approaches its minimum, which equals the level of $g_{i=highest\ earner}$ corresponding to the very highest earner in society. The relative average social marginal welfare weight for individuals who

[11] The Pareto parameter for high incomes is denoted using $\rho(z)$, and not $\alpha(z)$ as in Saez (2001) and Diamond and Saez (2011), to make the distinction from Pareto parameters for low and medium incomes $\alpha(z)$ easier. Moreover, the use of $\rho(z)$ instead of $a(z)$ makes the distinction from the parameter a, used for generalized social marginal welfare weights g_i, introduced later in this section, easier.

earn more than z, $\bar{G}(z)$, is thus decreasing over earnings, but the shape of this decrease is not straightforward. This depends entirely on generalized social marginal welfare weights g_i. Please recall that generalized social marginal welfare weights g_i "measure how much society values the marginal consumption of individual i", and thus reflect normative redistributive tastes of the government (Saez & Stantcheva, 2016, p. 26). Saez and Stantcheva (2016) define these as follows: $g_i = g\left(c_i, z_i, x_i^s, x_i^b\right)$, where c_i is consumption, x_i^s is a set of characteristics affecting only the social welfare weights, whereas x_i^b is a set of characteristics also affecting utility. This in turn connects to the individual utility function used in Saez and Stantcheva (2016). Such a definition of generalized social marginal welfare weights g_i allows the model to open up to some unique features, such as extensions about freeloaders and tagging.[12] However, it also makes the model reliant on and sensitive to the construction of an unobservable individual utility function. This goes very much against the purpose of my study, as the use of such unobservable utility functions are part of the bridge between theory and practice in optimal taxation theory. Conveniently though, Saez and Stantcheva (2016) constructed their model in such a way that standard social welfare weights used in previous studies can simply be substituted and used as generalized social welfare weights in their model. Therefore, this study follows the approach by Gruber and Saez (2002) on the construction of the generalized social marginal welfare weights g_i.[13] Gruber and Saez (2002) simply make assumptions about the redistributive preferences of the government under different principles of distributive justice, which are reflected in the values of g_i. They distinguish a Rawlsian objective using zero weights to maximize tax revenue for redistribution to the poor, a utilitarian progressive objective using steeply declining weights over earnings, a utilitarian conservative objective using only declining weights for the poor, and finally a no redistribution (libertarian) objective which uses constant weights. Fundamentally, the difference in the construction of generalized social marginal welfare weights g_i between Saez and Stantcheva (2016) and Gruber and Saez (2002) is that the former uses a combination of technical functions

[12] Freeloaders are nonworking individuals who are able but not willing to work. It can be argued that freeloaders do not deserve financial support though redistribution. Saez and Stantcheva (2016) provide a method in section II B to identify freeloaders and redistribute only to those considered deserving. Tagging concerns the separation of the population into different groups based on inelastic and observable attributes which correlate with earnings, such as sex. Section II C in Saez and Stantcheva (2016) shows how these tags can be used to achieve horizontal equity goals.

[13] Though they are referred to as '(standard) social marginal welfare weights' in previous studies such as Gruber and Saez (2002) and Saez (2001), they are from now referred to as 'generalized social marginal welfare weights' in this study since they can be substituted. This is done to avoid confusion about the terminology used across studies.

which allow for redistributive tastes and desert-based characteristics, whereas the latter simply use broad assumptions about redistributive tastes based on principles of distributive justice. The generalized social marginal welfare weights g_i in Saez and Stantcheva (2016) are most definitely more precise on an individual level, distinguishing for instance people with low incomes due to disabilities or laziness.[14] On an aggregate level, however, the use of assumptions as in Gruber and Saez (2002) make the model significantly simpler to apply with only a minimal sacrifice in precision. Given that the purpose of my study is to simplify optimal income taxation schemes which rely on the normative tastes of the government, it is clear that an approach to g_i similar to Gruber and Saez (2002) suits the nature of this study best. The aggregate effects are primarily relevant in my study, as well as the normative input based on different principles of distributive justice.

For the construction of the generalized social marginal welfare weights g_i, the weights used by Gruber and Saez (2002) can be used as a guideline, but no more than that. This is because Gruber and Saez (2002) divide their sample in four brackets of earnings, after which aggregated generalized social marginal welfare weights g_i are applied to each bracket. Though this is a totally viable method, it cannot be applied to other distributions as the bracket cutoff points are based on absolute levels on earnings.[15] Therefore, it is more convenient to construct g_i based on relative properties of the earnings distribution, such as the mean of earnings. Moreover, g_i as a continuous function of z rather than discrete in brackets vastly simplifies the process of redistribution of tax revenue (discussed in more detail in Section 3.1.4). Therefore, the limitation of following Gruber and Saez (2002) is that they only provide good guidelines regarding the relative difference in weights between different principles of distributive justice, but not a good starting point for a function of g_i. This is where the studies by Saez (2001) and Madden and Savage (2020) are helpful. In Saez (2001), utilitarian social marginal weights are approximately equal to $\frac{1}{c}$, where c is

[14] By precision, I refer to the ability of the model to provide tax schemes which are consistently optimal for a given individual in society when varying the inputs in the model. Models which incorporate many individual characteristics are able to return more consistent and closer to optimal tax schemes for specific (sets of) individuals since the information on behavioural effects of these individuals can be considered when for instance the distributions of earnings changes. The model combined with the specific generalized social marginal welfare weights presented by Saez and Stantcheva (2016) is very precise as it incorporates such individual characteristics. The approach by Gruber and Saez (2002) does not furnish such a level of precision.

[15] For instance, a cutoff point of bracket 1 at $z = 10{,}000\$$ for the distribution of earnings in the US in 1992 would be meaningless for any other distribution of earnings.

disposable (after-tax) income. The ratio $\frac{1}{c}$ is decreasing roughly proportional to earnings but at a lower rate. This is consistent with utilitarian weights g_i for households computed by Madden and Savage (2020), who use a ratio of income relative to the poorest household, combined with an inequality aversion parameter. In this study, disposable income is not used to compute generalized social marginal welfare weights g_i as disposable income depends on the marginal tax rates which again depend on weights g_i. This makes disposable income very complex to observe unless a tax system is already in place, which is not the case in this study. Moreover, this study is about earnings of individuals and not households. Following the guidelines set by Saez (2001) and Madden and Savage (2020) for the context of this study, the exogenous parameter $\frac{1}{z+\bar{z}}$ is used instead, since this ratio is also decreasing roughly proportional to earnings but at a lower rate. As such, weights computed using the ratio $\frac{1}{z+\bar{z}}$ share the utilitarian nature of weights computed in Saez (2001) and Madden and Savage (2020) to a reasonable extent. Most important to note is that weights computed using the ratio $\frac{1}{z+\bar{z}}$ depend on the distribution of earnings z, and are decreasing roughly proportional to earnings but at a lower rate, consistent with both Saez (2001) and Madden and Savage (2020). The fraction $\frac{1}{z+\bar{z}}$ therefore forms the basis of the computation of generalized social marginal welfare weights g_i based on different principles of distributive justice in this study. As mentioned earlier, the generalized social marginal welfare weights g_i measure how much the marginal consumption of individual i is valued by society. This means that if $g_1 = 1.33$ and $g_2 = g_1/2$, then society is indifferent between giving 1.33 euros to individual 1 and $\frac{1.33}{2} = 0.67$ euros to individual 2. Assume that total tax revenue is 3 euros and the government decides to redistribute 100% of this amount over society, which has a population of $n = 2$. The government would then give $\frac{3\ Euros}{n} \cdot g_1 = 2$ euros to individual 1, and $\frac{3\ Euros}{n} \cdot g_2 = 1$ euros to individual 2. Notice that this only holds when the mean of g_i is normalized to one. This does not affect the relative weights of g_i and therefore has no effect on the computation of $\bar{G}(z)$, but does make the interpretation and mathematics behind redistribution very straightforward. As such, the mean of g_i is always normalized to one in this study. Moreover, normalization of the mean makes the comparison of g_i using different principles of distributive justice considerably easier to comprehend.

In this study, generalized social marginal welfare weights are computed using the following function:

$$g_i = g(z) = \gamma(z) + q \cdot \gamma(z)_{max} \;\; with \;\; \gamma(z) = \frac{1}{(z + \bar{z}) + az - b\bar{z}} \tag{4}$$

Using this method, the generalized social marginal welfare weights g_i are a function of earnings z and depends on the level of parameters a and b, which are captured by $\gamma(z)$, and parameter q. First, consider the basic situation where $a = 0$, $b = 0$, and $q = 0$. In this situation, $g(z) = \gamma(z) = \frac{1}{(z+\bar{z})}$, which means that g_i is decreasing over earnings z, and this decrease is proportional to the increase in $(z + \bar{z})$, the sum of earnings z and the mean of earnings $\bar{z}$. Now, assume that parameter $a = 1$, such that $g(z) = \gamma(z) = \frac{1}{(z+\bar{z})+z}$. This increases the weight for individuals with very low earnings, at the expense of medium and medium-high earners, as the effect of z in the denominator is relatively largest for the lowest earners and almost nonexistent for very high earners. Alternatively, let parameter $b = 0.5$, such that $g(z) = \gamma(z) = \frac{1}{(z+\bar{z})-0.5\bar{z}}$. A positive b means that part of the mean of z, a constant, is subtracted from the denominator. This increases the weight for low earners, at the expense of high earners. The most important effect of b is that it flattens the curve of weights for high earners, meaning that the weights for high earners become almost constant. Finally, we can consider the case where parameter $q = 0.5$, such that $g(z) = \gamma(z) + 0.5\gamma(z)_{max} = \frac{1}{(z+\bar{z})} + 0.5\left(\frac{1}{(z+\bar{z})}\right)_{max}$. Now, 0.5 times the maximum value of $\gamma(z)$, which is a constant depending on $\gamma(z)$, is added to $\gamma(z)$. When plotting g_i over z, a positive value of q simply rotates the curve counterclockwise around the level of z where $g_i = 1$. Parameters are bounded such that $a \geq 0$, $0 \leq b < 1$, and $q \geq 0$. Illustrations of the relationships between weights g_i and parameters a, b, and q are presented in Appendix B. The reason for using such a function of generalized social marginal welfare weights g_i is because it allows for computation of continuous decreasing weights g_i over earnings z, which is a requirement for optimality in the model by Saez and Stantcheva (2016). Moreover, weights g_i computed this way always decrease proportional to increases in earnings z to at least some degree, meaning that the earnings distribution affects the weights g_i. This is consistent with social marginal weights computed by Saez (2001) and Madden and Savage (2020). My function of g_i is able to satisfy all the aforementioned properties, yet still provides the flexibility of yielding sets of g_i which correspond to different principles of distributive justice. Essentially, my function of g_i satisfies all the criteria it needs to satisfy for the purpose of my study and leaves enough room for normative input. There is, though, by no means a perfect way of deriving generalized social marginal welfare weights, as they ultimately depend on the normative tastes of the policymaker. As such, it would be senseless to call my approach to g_i perfect. Imperfection, however, does not mean incorrect in this context, but simply denotes that there is no single correct answer. As such, weights g_i should be considered estimations in this study which satisfy certain properties while reflecting principles of distributive justice as closely as possible, similar to the weights g_i in Gruber and Saez (2002).

The knowledge from literature on distributive justice in economics is used to estimate parameters a, b, and q such that the corresponding generalized social marginal welfare weights g_i represent normative tastes based on different principles of distributive justice. Graphical illustrations of generalized social marginal welfare weights g_i using parameters which reflect different principles of distributive are presented in the beginning of Section 5.1. As discussed in section 2.1, the trilogy of egalitarianism, utilitarianism, and libertarianism is used as a starting point. Two extremes which are often used in studies on optimal income taxation are the extreme Rawlsian and extreme libertarian. The extreme Rawlsian is an egalitarian policymaker that cares about the welfare of only the poorest individual in society. Essentially, the extreme Rawlsian choses a tax policy such that the difference principle is most effectively implemented, which is done through maximizing tax revenue and redistributing to the poor. When using weights g_i, the extreme Rawlsian assigns extremely high weights to the poorest member in society and extremely low weights to the richer members in society. In this study the extreme Rawlsian is represented using parameters $a = 0.5, b = 0.9, q = 0$. Opposingly, the extreme libertarian policymaker prioritizes the idea of self-ownership to such an extent that leaves no room taxation and redistribution. Therefore weights g_i assigned by the extreme libertarian are equal to one for all individuals, which produces the laissez-faire situation. Parameters $a = 0, b = 0, q = 10$ resemble the extreme libertarian in this study.[16] Additionally, a moderate Rawlsian and moderate libertarian are depicted using parameters $a = 0, b = 0.6, q = 0$ and $a = 0, b = 0.2, q = 1.4$ respectively. Utilitarian policymakers are somewhere in between, since they advocate redistribution to the poor through diminishing marginal utility of consumption on the one hand, and lower taxes to incentivize effort – which implies less redistribution – on the other hand (Mirrlees, 1971). Saez

[16] Since the use of constant weights g_i causes errors in the computation of $\bar{G}(z)$ and violates the requirement for optimality of decreasing weights g_i over earnings z in this model, the weights g_i representing the extreme libertarian in this study are ever so slightly decreasing over earnings z. The difference in results compared to using constant weights is negligible.

(2001) uses the ratio $\frac{1}{c}$, and Madden and Savage (2020) use a ratio of income relative to the poorest household to represent utilitarian weights g_i. It should be clear that utilitarian weights are in between Rawlsian and libertarian weights g_i and are highly dependent on an individual's relative level of earnings. As such, parameters a, b, and q should be close to zero to best represent utilitarian policymakers. This study follows Gruber and Saez (2002) by distinguishing a liberal and conservative utilitarian policymaker, which in this study are represented by parameters $a = 0.5, b = 0, q = 0.1$ and $a = 0, b = 0, q = 0.4$ respectively. The liberal utilitarian is leaning slightly more towards the egalitarian side, whereas the conservative utilitarian slightly advocates the libertarian side. Finally, a novel type of policymaker is considered in this study: the libertarian-Rawlsian. Such a policymaker is not commonly found in literature, but perfectly illustrates the flexibility of the computation of weights g_i in my model, thereby demonstrating its potential range of applications. The libertarian-Rawlsian is mostly libertarian, except for a slight concern about the poorest in society, based on the Rawlsian difference principle. Another way to interpret this is by using the idea of a 'social minimum', which broadly implies that every individual in society should be able to meet basic needs such as food and minimum health-care (White, 2015). White (2015) and Gamble (2013) state that some libertarians argue that the provision of some form of social minimum would be a just violation of self-ownership. Empirically, the political developments since the second half of the twentieth century in the United States could be relevant for defending this attitude. Gamble (2013) describes how economic libertarian doctrines gained popularity in the 1970s, but also highlights the many arguments made against these doctrines in recent decades, mostly from 'liberal' perspectives. The libertarian tradition is still present in the United States today, but so is an increasing support for basic needs such as healthcare. The libertarian-Rawlsian allows for some minor redistribution towards the very poorest members of society and minimizes violations of self-ownership beyond that. Weights g_i are therefore a little higher than 1 for the poorest members of society, and almost equal to one for all other members. The libertarian-Rawlsian is depicted using parameters $a = 0, b = 0.9, q = 2$ in this study, sharing the high values for q and b which characterize libertarians and Rawlsians respectively. These seven configurations for weights g_i using different principles of distributive justice are used to examine the normative dimensions of optimal income taxation in this study, as they capture the boundaries of, and popular configuration within, the trilogy of egalitarianism, utilitarianism, and libertarianism. Many more configurations for weights g_i could be explored, but that would harm the overview in this study.

3.1.3. Optimal marginal tax rates

Finally, after defining all variables, we reach the model of optimal marginal tax rates, which is a modified version of the model used in Saez and Stantcheva (2016):

$$T'(z) = \frac{1 - \bar{G}(z)}{1 - \bar{G}(z) + \varphi(z) \cdot e} \quad (5)$$

Where e is the constant elasticity of earnings z, $\varphi(z)$ is the local Pareto parameter defined as $\varphi(z) \equiv \begin{Bmatrix} \alpha(z) = zh(z)/\big(1 - H(z)\big), \text{when } z_i \leq z_I \\ \rho(z) = z_m/(z_m - z_i), \text{when } z_i > z_I \end{Bmatrix}$, and $\bar{G}(z)$ the relative average social marginal welfare weight for individuals who earn more than z. $\bar{G}(z)$ is defined as $\bar{G}(z) \equiv \frac{\int_{\{i:\, z_i \geq z\}} g_i \, di}{Prob(z_i \geq z) \cdot \int_i \; g_i \, di}$, where g_i are generalized social marginal welfare weights $g_i = g(z) = \gamma(z) + q\gamma(z)_{max}$, with $\gamma(z) = \frac{1}{(z+\bar{z}) + az - b\bar{z}}$. This model follows the model by Saez and Stantcheva (2016), except that elasticity e is now set constant over earnings z and the Pareto parameter $\varphi(z)$ now follows parameter $\rho(z)$ when earnings are sufficiently high. Additionally, the generalized social marginal welfare weights g_i are now derived based solely on the distribution of earnings and normative input, not on individual sets of characteristics and utility functions. Optimal marginal tax rates $T'(z)$ are determined through one constant and two main variables: e, and $\bar{G}(z)$ and $\varphi(z)$. An increase in elasticity e results in a larger behavioural effect (discouragement to devote effort towards labour) of taxation. As such, optimal marginal tax rates decrease to offset these increased behavioural effects. Note that the behavioural effect of e is directly connected to $\varphi(z)$ in the model. Pareto parameter $\varphi(z)$ reflects the density of taxpayers at earnings level z_i relative to number of taxpayers above this earnings level. As such, $\varphi(z)$ reflects the relative impact on the economy of behavioural effects captured by e. The impact of high marginal tax rates on the economy is smallest at low earnings levels. This is because high marginal tax rates at these levels result in increased average tax rates for a large share of the population, while the damage through behavioural effects at the margin is limited. The product $\varphi(z) \cdot e$ in the model of optimal marginal tax rates captures precisely the relative negative effects of marginal tax rates at each earnings level z_i on the economy, and therefore negatively affects $T'(z)$. The product $\varphi(z) \cdot e$ mostly shapes where $T'(z)$ is increasing and decreasing over earnings z, but the overall height of $T'(z)$ is predominantly determined by the desired level of tax revenue, via $\bar{G}(z)$, the relative average social marginal welfare weights for individuals who earn more than z. When weights $\bar{G}(z)$ are high, the effect of the product $\varphi(z) \cdot e$ on $T'(z)$ is relatively high too. This means that optimal marginal tax rates are high overall when generalized social marginal welfare weights g_i are progressive.[17] Remember that weights $\bar{G}(z) = 1$ for the lowest earner in society and decrease over earnings z until $\bar{G}(z)$ approach the level of $g_{i=highest\ earner}$ corresponding to very highest earner in society.[18] Hence, when the level of $g_{i=highest\ earner}$ is high, overall optimal marginal tax rates are lower due to the relatively large negative effect of the product $\varphi(z) \cdot e$ on $T'(z)$. This is in line with expectations, as weights g_i relatively close to one imply little desire for redistribution and therefore little desire for a high tax revenue, which is captured through the relatively large negative effect of the product $\varphi(z) \cdot e$ on $T'(z)$.

The fruitful property of this model is that it depends completely on observable variables and normative tastes. The distribution of earnings affects every variable in the model and normative tastes based on different principles of distributive justice enter the social welfare weights via parameters a, b, and q. Only constant e, elasticity, is not necessarily straightforward to observe. An increase in elasticity e, however, always leads to lower optimal marginal tax rates. Since this is true for every level of earnings, any change in e merely shifts the optimal marginal tax rates up or down. To put it differently, two curves of optimal marginal tax rates over earnings using identical parameters, only e different, will never intersect. This means that the effect of elasticity e on the optimal marginal tax rates relative to changes in earnings z is limited, and therefore the consequences of potential inaccuracies of e are limited too.

[17] Values of weights $\bar{G}(z)$ depend on the values of generalized social marginal welfare weights g_i. Essentially, when weights g_i for low earners are increased (g_i are always decreasing over earnings z), the desire for redistribution towards these low earners increases and therefore also the desire for a higher level of tax revenue. As such, when weights g_i for low earners become very large, the optimal marginal tax rates are set to maximize the level of tax revenue (while still limiting the damage to the economy through behavioural effects).

[18] The use of $\bar{G}(z) = 1$ in equation (5) would lead to an optimal marginal tax rate of zero for the single lowest earner in society when using a bounded earnings distribution. As this is highly unrealistic and not in line with the relationships in the model, a correction is applied such that the optimal marginal tax rate of the single lowest earner is equal to that of the second lowest earner in society. This yields much more realistic results.

3.1.4. Redistribution of tax revenue

Using equation (5), optimal marginal tax rates can be computed for each level of earnings z given any earnings distribution and any desired set of generalized social marginal welfare weights g_i. This is the most important part of the optimal income taxation scheme, but the final step of redistribution is still missing. Following the definition of g_i, which is how much the marginal consumption of individual i is valued by society, the amount of redistribution towards individual i is based on its respective g_i. Moreover, the amount of redistribution depends on the share of tax revenue dedicated to redistributive purposes. The total income tax revenue is calculated as $R = \sum_{i=1}^{n} r_i$, where $r_i = \sum_{i=1}^{i}\left((z_i - z_{i-1}) \cdot T_i'(z)\right)$ is the income tax revenue of individual i and n the total amount of individuals in society. Note that this calculation of R is simply the sum of taxes paid by all individuals based on the optimal marginal tax rates $T'(z)$ calculated using equation (5), assuming that the dataset is sorted by earnings z. Income tax revenue of individual i, r_i, is computed as the sum of marginal taxes paid at each respective marginal level of income. Thus, individual 1 (lowest earner) pays $z_1 \cdot T_1'(z)$, whereas individual 2 (2nd lowest earner) pays $z_1 \cdot T_1'(z) + (z_2 - z_1) \cdot T_2'(z)$, and so forth. If 100% of total income tax revenue is dedicated to redistribution, then the disposable income c_i of individual i is calculated as:

$$c_i = (z_i - r_i) + g_i \cdot \bar{R} \tag{6}$$

which is the individual's after-tax earnings plus the social weight g_i times the mean of total tax revenue. This works since the mean of social weights g_i is normalized to one, so the equation of total tax revenue is:

$$R = \sum_{i=1}^{n} (g_i \cdot \bar{R}) \tag{7}$$

, which always holds. Therefore, by substitution equation (6) can be written in aggregate form as follows:

$$C = (Z - R) + R = Z \tag{8}$$

where C represents the total amount of disposable income in society and Z is the total amount of earnings in society. Equation (8) indicates that there is no deadweight loss from income taxation collection in this model. Since most studies of optimal income taxation do not consider deadweight losses from income taxation collection, I see no value in adding it to this study (Diamond & Saez, 2011; Mirrlees, 1971; Piketty & Saez, 2013; Saez, 2001; Saez & Stantcheva, 2016). If, however, there exists a desire to add such deadweight losses, an extension to this model to include deadweight losses from income taxation collection can be found in Appendix C.

The previous paragraph considered the case where 100% of total income tax revenue R is used for redistributive purposes according to the generalized social marginal welfare weights g_i. This is also a common assumption in literature, as Saez and Stantcheva (2016) assume no government funded public goods in their study for the sake of simplicity. Piketty and Saez (2013), however, provide data on the public spending of several OECD countries from 2000-2010. It can be observed that on average 27% of total public spending in OECD countries is spent on education and healthcare alone. These two categories do not represent any redistributive tastes of the government, but instead every individual in society is able to benefit from them equally. Therefore, benefits for individuals gained in these two categories could be considered a demogrant, or public good (Piketty & Saez, 2013). In my model, a share of total income tax revenue R dedicated to government funded public goods can be included by making slight adjustments to equations (6) and (8). Let Γ be the share of R dedicated to the funding of public goods, with $0 \leq \Gamma \leq 1$, and assume that every individual i would benefit equally from the public good. This would transform equation (6) to $c_i = (z_i - r_i) + (1 - \Gamma)(g_i \cdot \overline{\mathrm{R}}) + \Gamma \cdot \overline{\mathrm{R}}$, and again by the substitution of equation (7), change equation (8) to $C = (Z - R) + (1 - \Gamma)R + \Gamma \cdot R = Z$. Thereby, the dedication of a share of total income tax revenue to government funded public goods would change individual disposable income c_i, but not affect aggregate disposable income C. Note that the benefit which comes from government funded public goods for individuals is still considered disposable income in this context, for the sake of simplicity of comparisons. If desired, however, this could of course be split up into disposable income net of public goods benefits, and public goods benefits. Moreover, the share of total income tax revenue dedicated to government funded public goods Γ can always be set to $\Gamma = 0$ for robustness tests to represent the situation often found in literature. As a considerable amount of public funds are spent without redistributive intent, however, $\Gamma = 0.27$ is used in this study, following the data on public spending in OECD countries by Piketty and Saez (2013).

The final point of discussion regarding redistribution and the computation of disposable income c_i is the exceptional case where redistribution towards the lowest incomes relative to medium incomes is so high that disposable income c becomes decreasing over earnings z. This is a highly unrealistic scenario, as it would be profitable for individuals to decrease their earnings levels in order to receive substantially more redistributive benefits and thereby reach a higher level of disposable income. Such a scenario, however, can occur when using very progressive (Rawlsian) weights g_i for certain combinations of earnings distributions and a low taxable income elasticity. In this case, a correction to the redistribution is applied such that disposable income c is always constant or increasing over earnings z. More specifically, the minimum of c when c is plotted over z, defined as (z_j, c_j), is localized. Then, the area below the curve of c where $z_i < z_j$ and $c_i > c_j$ is calculated, which equals $\varepsilon = \int_0^{\{i:\, z_i < z_j\}} c_i \, d_i - z_j \cdot c_j$, where ε is defined as the amount of tax revenue falsely redistributed. This area of the curve is subtracted from the original disposable incomes c_i, after which the amount of ε is redistributed again such that disposable income c is constant over earnings z for every $z_i \leq z_j$, and then increasing over earnings z when $z_i > z_j$. A graphical illustration of this correction applied to a Rawlsian extreme set of weights g_i can be found in Appendix D. This concludes the part of the methodology regarding the construction of optimal income taxation schemes.

3.2. Analyses

Next, the effects of the schemes using different principles of distributive justice on the distribution of income can be examined. Firstly, this can be done graphically by plotting the relationship between disposable income c and earnings z, in literature also referred to as the relationship between after tax income and before tax income. The graph shows disposable income levels as a consequence of tax schemes on the vertical axis plotted against earnings levels on the horizontal axis. A 45-degree line represents the *laissez faire* situation, in which there is no taxation and $c = z$. The graph essentially shows the development of disposable income c_i for each individual as its respective earnings z_i increase. This results in an excellent visual representation of the redistributive effects of the corresponding tax scheme. As multiple tax schemes can be applied to the same distribution and graphed in the same plane, they can conveniently be compared visually.

Different income taxation schemes have different effect on the redistribution of disposable income, both through differences in tax rates and differences in redistribution of tax revenue. Measures of (disposable) income inequality can be used to examine and compare these effects. As argued by Cowell (2018a), inequality measurement is relevant for economists, policymakers, and philosophers. Such measures can therefore be of considerable value in the understanding of the implications of income taxation schemes across different fields of study. Therefore, Lorenz curves and Gini coefficients are used to provide both graphical and quantitative measures of inequality. Decomposition of inequality is not relevant for this study since there are no subgroups to distinguish, which further explains the use of relatively simple inequality measures in this study. Besides accessibility reasons, there is another reason supporting the choice of these inequality measures. Gini coefficients are directly related to both Lorenz curves and generalized beta of the second kind (GB2) coefficients. Essentially, coefficients of GB2 distributions can be chosen based on the respective Gini coefficients such that optimal marginal tax schemes for equal and unequal distributions of earnings can be compared quantitatively (Gini coefficients before and after taxation can be compared). Lorenz curves then provide accurate visual representations of these Gini coefficients as the two are mathematically connected. As such, GB2 distributions, Gini coefficients, and Lorenz curves share a unique connection which is highly convenient for the consistency of inequality measurement across different stages in this study.

4. Data

Traditionally, studies used simulations of lognormal skill distributions to determine optimal marginal tax rates (Tuomala, 1990). The distribution can significantly affect these rates. In this study, recent methods are followed to use distributions of earnings, as opposed to distributions of skills. Distributions of earnings are far easier to observe, and therefore most studies based on distributions of earnings use empirical data (Saez, 2001; Diamond & Saez, 2011). The aim of these studies is to provide as much accuracy as possible in the determination of optimal taxation schemes, such that they could be implemented in the real world without any loss of welfare. This explains the preference for using empirical distributions of earnings over simulated distributions of earnings, as no data is more accurate than empirical data. For this study, however, accuracy is not the main objective. Instead, normative dimensions of optimal income taxation schemes using different principles of distributive justice are explored. Moreover, the question raised by Saez (2001), on whether the U-shaped pattern of optimal tax rates is universal across different distributions of earnings, is analyzed. For this purpose, it is highly valuable to have a flexible set of distributions of earnings, some more equal than others. The use of simulated distributions of earnings is therefore of high value, as it provides the option to change parameters affecting the shape of the distribution without empirical limits.[19] The use of simulated distributions, however, comes at the expense of a slight loss in accuracy. Essentially, the decision between empirical and simulated distributions of earnings can be seen as a tradeoff between accuracy and flexibility. Where previous studies have focused mostly on accuracy, this study explores how the role of normative dimensions in optimal income taxation schemes is connected to changes in the distribution of earnings and is therefore suited for the more flexible approach.

Simulated earnings distributions are generated using 50 sets of 10,000 random draws from a generalized beta of the second kind (GB2) distribution with desired parameters, where for all 50 distributions the very right tail is replaced using 50 sets of 10,000 random draws from a Pareto distribution. The average of these 50 distributions is taken to correct for possible outliers. Finally, the means of the resulting average distributions are normalized to 50,000 to benefit the interpretation of comparisons using different distributions of earnings. The GB2 distribution was first introduced by McDonald (1984), and is a distribution based on four parameters. It gained popularity for its good fit to data, as well as the inclusion of other popular distributions such as lognormal, gamma, Singh-Maddala, and Dagum as special cases by setting specific parameters equal to 0, 1 or infinity. Moreover, the parameters of a GB2 distribution can directly be used to compute various indices of inequality and poverty, such as the Gini coefficient (Bandourian *et al.*, 2002; Chotikapanich *et al*., 2018; McDonald & Ransom, 2008). Various studies, such as those by Parker (1999) and Jenkins (2009), confirm that the GB2 distribution is the best distribution to describe earnings. McDonald, Sorensen, and Turley (2013) further compare different earnings distributions and examine their ability to model levels of kurtosis and skewness, which are important for modelling earnings distributions. Again, they confirm that the GB2 distribution provides the best fit. The superior fit of the GB2 distribution combined with the parameters' inherent connection to indices of inequality make the GB2 distribution perfect for this study. Configurations of parameters fitting empirical income distributions for a wide range of countries and years up to 1997 can be found in Bandourian *et al.* (2002). Using the results by Bandourian *et al.* (2002), distributions based on the GB2 parameters fitted to the empirical earnings distributions of Germany in 1981, the United States in 1994, and Mexico in 1994 are simulated. The GB2 parameters are $a = 4.094$ $b = 55333$ $p = 0.605$ $q = 1.1111$ for Germany 1981, $a = 3.685$ $b =$

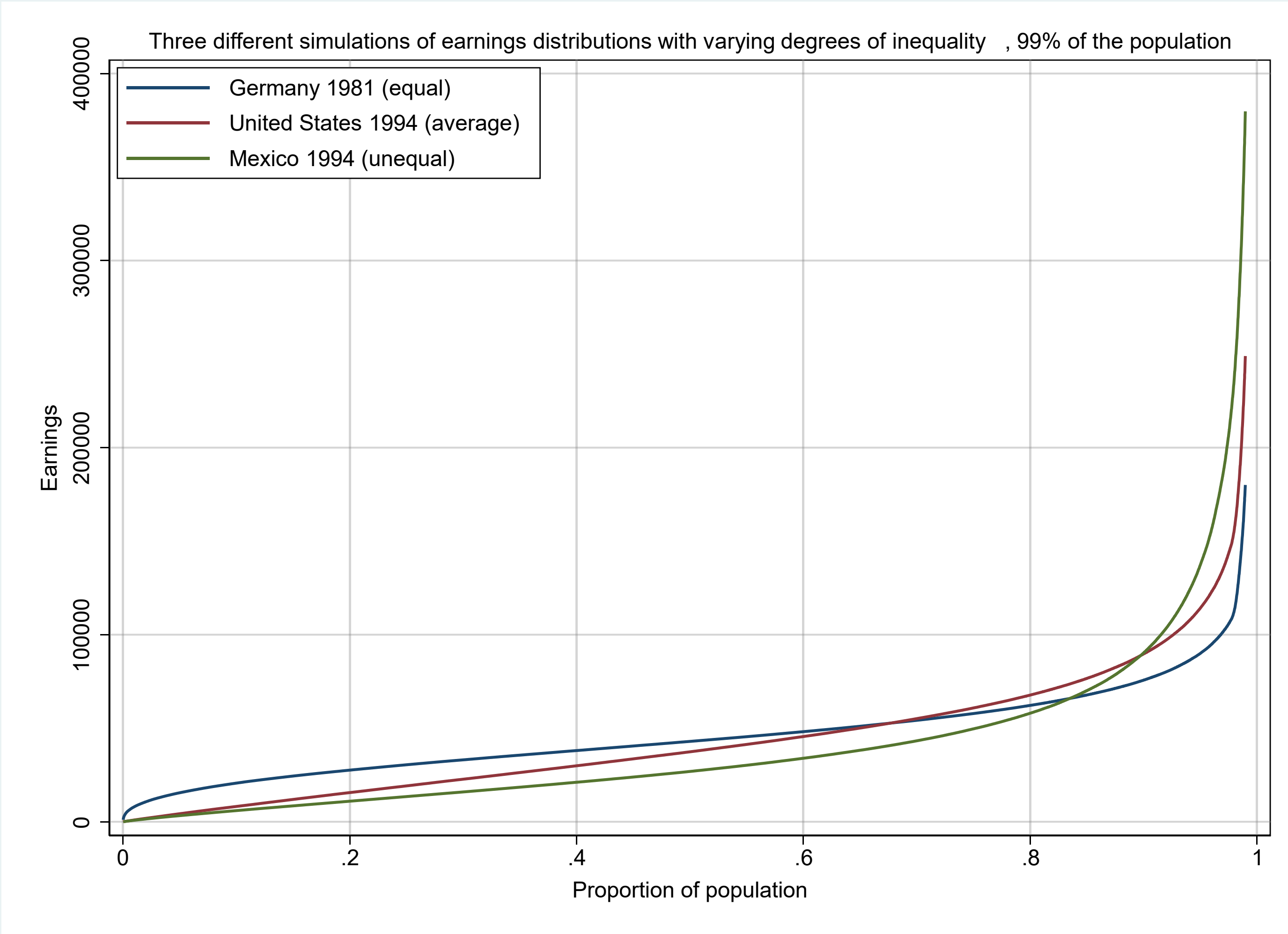
[19] Empirical earnings distributions are only observable using tax return data, which not many countries publish. Policymakers, however, should have no trouble accessing these data.

61524 $p = 0.293$ $q = 0.8759$ for the United States 1994, and $a = 2.68$ $b = 16251$ $p = 0.43$ $q = 0.6368$ for Mexico 1994. The empirical earnings distributions of Germany in 1981 and Mexico in 1994 serve as extreme boundaries of very equal and very unequal distributions, with Gini coefficients of 0.274 and 0.577 respectively.[20] The empirical earnings distribution of the United States in 1994 is used as a more average distribution with a Gini coefficient of 0.425. Moreover, distributions representing the United States dominate literature on optimal income taxation, which means results based on distributions representing the United States are well suited for comparison to previous results in literature. Please note that the use of simulated distributions of earnings based on slightly old data does not pose a problem in this study, since they are merely used for illustrative purposes of equal, average, and unequal distributions. The three simulated distributions used in this study are illustrated in figure 1 below.[21]

[20] Gini coefficients for the distributions simulated in this study are always slightly higher, due to the correction of the very right tail of the distribution. This is explained in detail in the next paragraph.

[21] Figure 1 shows the 99% lowest earners of the population, as the vertical scale would be extremely large with the top 1% included.

Figure 1



Though the GB2 distributions based on parameters fitting empirical earnings distributions are very accurate, they come short in one area: the very right tail, covering the highest earners in society. This part of earnings is best described by the Pareto distribution (Bandourian *et al.*, 2002). The GB2 distribution still does a respectable job at fitting these incomes, but the tail will always be just slightly too thin. For most purposes this does not pose any problems, but for optimal income taxation models it does. The right tail of the distribution being too thin is the reason for the declining marginal tax rates for high earners in traditional models (Saez, 2001). The use of a lognormal skill distribution, which has a thin right tail compared to a fitting Pareto distribution, can explain Mirrlees' (1971) result of declining marginal rates for high earners (Saez, 2001; Diamond & Saez, 2011). Following the model by Saez and Stantcheva (2016), the Pareto parameter negatively affects the marginal tax rates. For a Pareto distribution, the Pareto parameter is constant, but for a thinner right tail the Pareto parameter will diverge to infinity at very high levels of earnings. As was found by Saez (2001), Diamond and Saez (2011), and Piketty and Saez (2013), empirical earnings distributions converge to a Pareto parameter of around 1.5 for roughly the highest 1% of earners.[22] Using such a Pareto tail is crucial for the model to work, as this is one of the main reasons it yields more realistic marginal tax rates for high earners than previous models did. As such, the GB2 distributions in this study are modified to represent a Pareto tail as observed in empirical data. More specifically, the top 2% of GB2 observations are replaced by the top 2% of observations from a random draw from a Pareto distribution (scaled to fit the respective GB2 distribution) with parameter 1.4. Essentially, the very right tail of the distribution representing the top 2% of earners is replaced by a fitting Pareto tail with parameter 1.4. The top 2% of observations using a Pareto parameter of 1.4 are chosen because, from testing, this is the smallest change that can be made to the original GB2 distribution that yields consistent results similar to empirical data. An example of Pareto parameters with and without a modified right tail is presented in Appendix E. When using a modified GB2 distribution, the Pareto parameter $\alpha(z) = zh(z)/\big(1 - H(z)\big)$ works best for low and medium earners. The high-income Pareto parameter $\rho(z) = z_m/(z_m - z_i)$ works best after the first intersection I of the two curves though, as it does not suffer from increased volatility and steadily converges to a constant Pareto parameter for high earners.

[22] Limited evidence exists about the Pareto parameter in countries other than the United States. It is safe to assume that the Pareto parameter is constant for other countries as well, given the empirical evidence on the fit of the Pareto distribution for the top earners. Consensus on the size of the Pareto parameter, however, is not extensive beyond the United States earnings distribution. The Pareto parameter of 1.5 is used in this study as it replicates previous optimal income taxation studies. Additional research on the exact size of the Pareto parameter for countries other than the United States would be welcome.

5. Results

Results based on different principles of distributive justice are presented using distributions with different levels of inequality, and different levels of taxable income elasticity. First, the case of an average distribution of earnings based on the United States in 1994 combined with an average elasticity assumption of 0.4 is presented in Section 5.1. This setting is most comparable to results presented in literature. Second, Section 5.2 presents results using an average distribution of earnings based on the United States in 1994, while using two extreme levels of taxable income elasticities of 0.12 and 0.86. This section illustrates the effect of the level of taxable income elasticities on the role that normative dimensions play in optimal marginal tax schemes. Finally, Section 5.3 presents results using extremely equal and unequal distributions of earnings representing Germany in 1981 and Mexico in 1994, while keeping the elasticity constant at the average level of 0.4. This section illustrates the effect of the level of inequality of earnings on the role that normative dimensions play in optimal marginal tax schemes. Additionally, in Appendix F3 results are presented where both the level of inequality of earnings and the level of taxable income elasticities vary. In this appendix section, the potential presence of interaction effects between the earnings distribution, elasticities, and normative tastes are investigated. Some graphs presented exclude the top 1% highest earners in society to keep the vertical scale at a readable level. Please recall that the means of earnings distributions are normalized to 50,000, and that numbers presented in this section are only relevant when interpreted as a relative distance to this mean.[23]

5.1. Average distribution and average elasticity

Results in Section 5.1 are based on an average distribution of earnings representing the United States in 1994 combined with an average taxable income elasticity assumption of 0.4. As this is the first section in the study where results on optimal marginal tax rates, disposable incomes, and effects on inequality are presented, the general mechanisms behind these results are only explained in this section to avoid repetition.

[23] In other words: an income level of 25,000 should be interpreted as an income level of $0.5 \cdot mean\ of\ earnings$, since only then it can be compared to empirical earnings distributions.

Figure 2

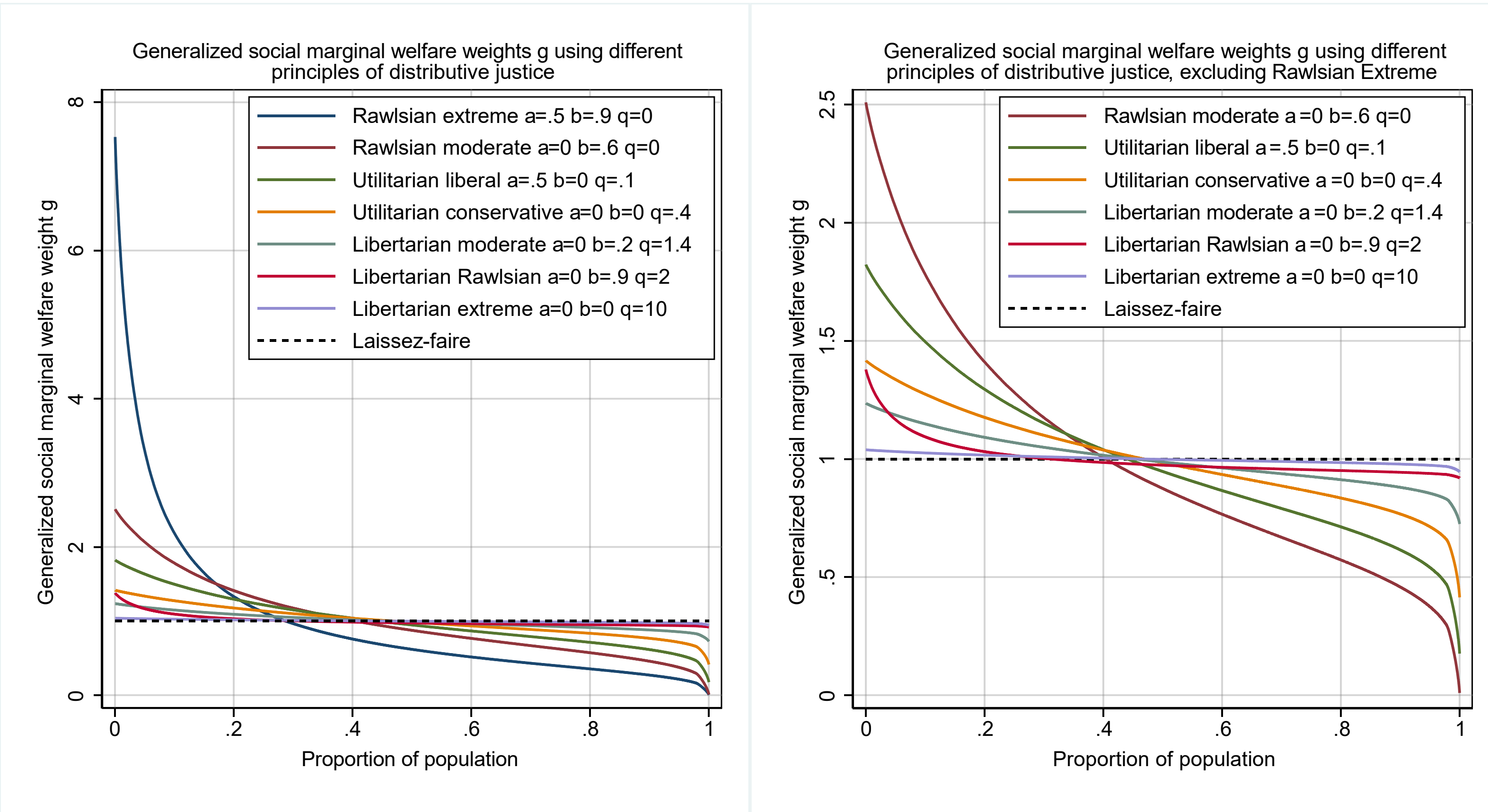
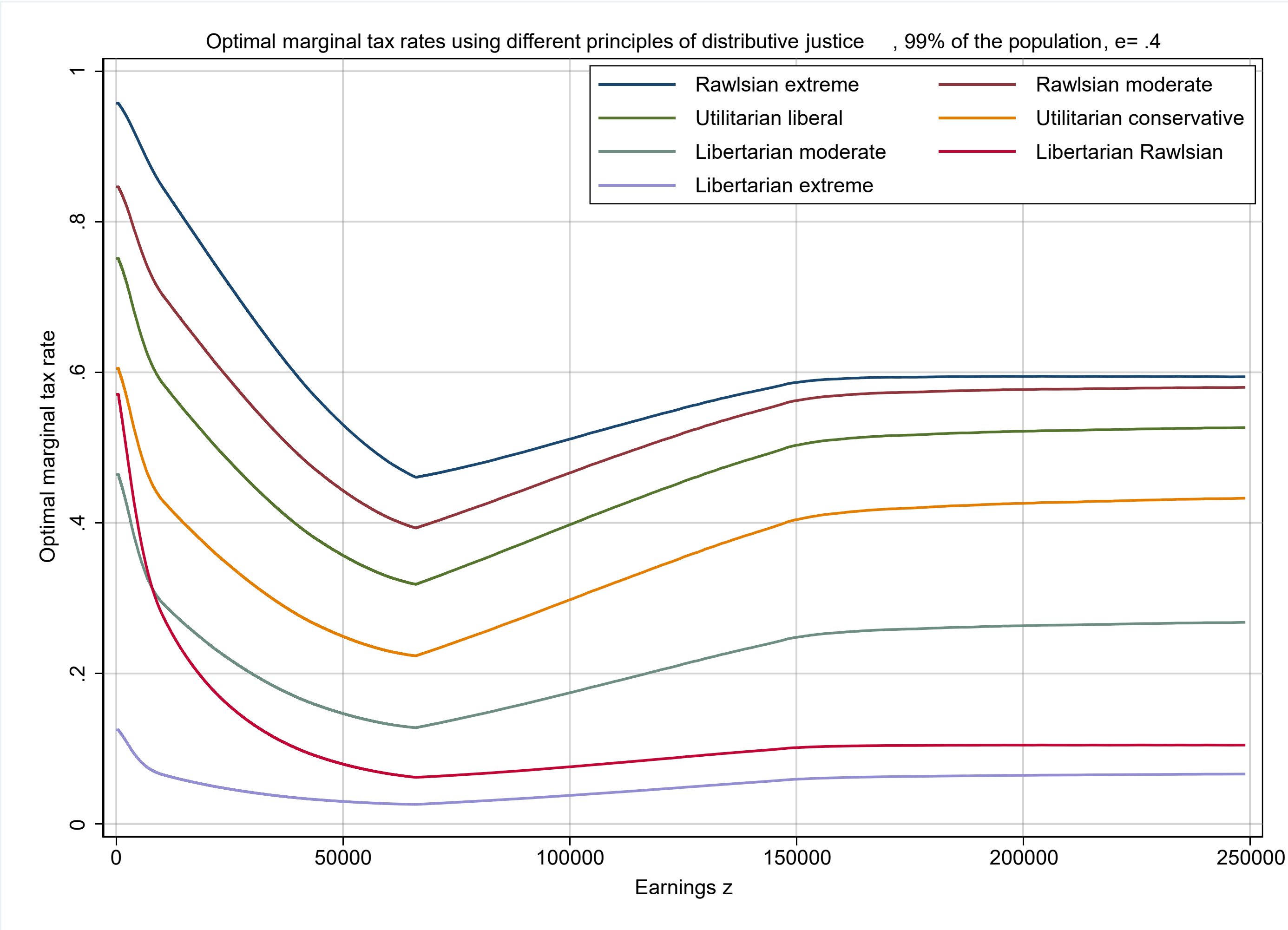


Figure 2 shows generalized social marginal welfare weights using different principles of distributive justice, and is mostly illustrative.[24] The graph on the left includes all seven principles of distributive justice, whereas the graph on the right excludes the Rawlsian extreme principle such that the vertical scale of the graph benefits interpretation for the other six principles. The larger the deviation from $g_i = 1$, the larger the desire the redistribute to $(g_i > 1)$ or from $(g_i < 1)$ this earnings level. As the model demands, generalized social marginal welfare weights are non-negative and decreasing over earnings for all principles of distributive justice. Rawlsian weights deviate the most from the neutral weight of $g_i = 1$, whereas libertarian weights are close to $g_i = 1$. This implies that Rawlsians maximize tax revenue, whereas libertarians are close to the laissez-faire situation. Utilitarian weights are in between Rawlsian and Libertarian weights, and decrease roughly proportional to earnings but at a slightly lower rate, consistent with utilitarian weights in literature.

[24] where $g_i = g(z) = \gamma(z) + q \cdot \gamma(z)_{max}$ with $\gamma(z) = \frac{1}{(z+\bar{z})+az-b\bar{z}}$.

Figure 3

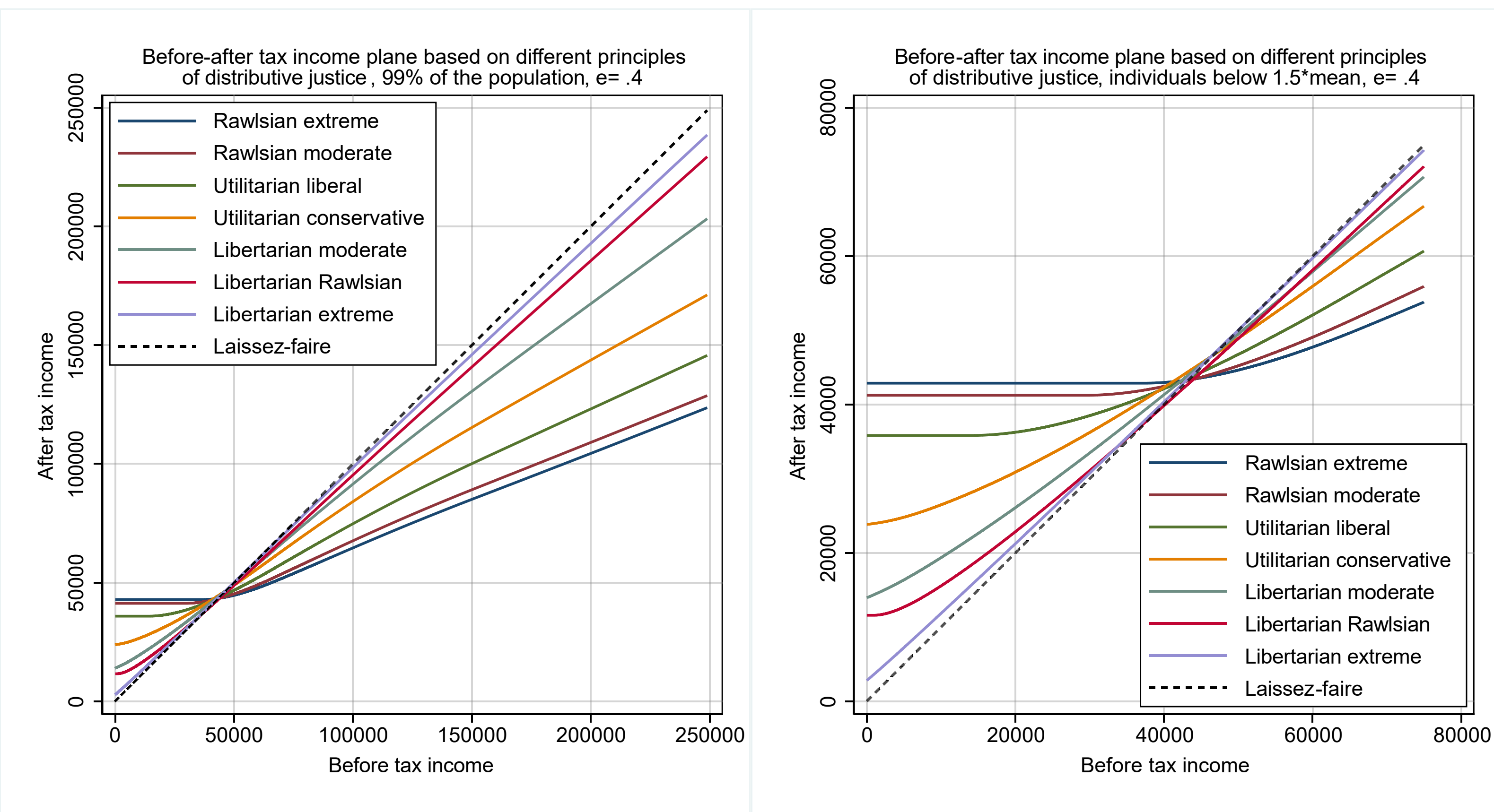


The optimal marginal tax rates according to different principles of distributive justice are presented in Figure 3. Generally, it can be observed that optimal marginal tax rates are highest for the lowest earners and decrease until just after the mean of earnings (50,000), where they become increasing for higher earners until they finally stabilize for individuals earning more than three times the mean of earnings. The patterns of optimal marginal tax rates are U-shaped for all principles of distributive justice, but with varying degrees. The U-shape can be explained by the inverse U-shaped pattern of the United States 1994 Pareto-parameter. The Pareto parameter determines where rates should be relatively high and relatively low based on the relative impact of behavioural effects from changes in marginal tax rates at different levels of earnings. The Pareto parameter based on the United States 1994 earnings distribution is increasing for low incomes until it becomes decreasing for higher earners and then constant for the highest earners. The inverse of the shape of the pattern of the Pareto parameter is, as expected, reflected in the shape of the pattern of optimal marginal tax rates. The overall height of the optimal marginal tax rates can be explained by the generalized social marginal welfare weights. The more weights deviate from $g_i = 1$, the larger the desired tax revenue, and therefore the larger the respective optimal marginal tax rates. This explains why Rawlsian weights produce the highest overall marginal tax rates, whereas the opposite is true for libertarian weights. Finally, the relative shape of the pattern of generalized social marginal welfare weights also determines the shape of the pattern of optimal marginal tax rates. When weights for middle and high earners are relatively flat, it means that the burden of supporting the poor should be distributed relatively equally between these middle and high earners. This is precisely the case for (extreme) Rawlsian principles of distributive justice, where all the non-poor should give up as much income as they can to support the poor. This explains why the Rawlsian extreme marginal tax rates are a little flatter for medium and high earners than the utilitarian rates, since Rawlsian extreme weights are relatively flat for medium and high earners. Utilitarian weights, however, steadily decrease roughly proportional to earnings and therefore produce the most extreme U-shaped pattern. An examination of the libertarian-Rawlsian principle illustrates this mechanism even better. Libertarian-Rawlsian weights are very flat and close to $g_i = 1$ for all but the very lowest earners. Weights are such that society can raise a little tax revenue to support the very poor, but the burden of doing so should be carried in libertarian fashion. As such, the burden should be carried equally for each individual in society. This results in high and decreasing optimal marginal tax rates for the lowest earners, and very low and flat rates for the rest of society. The situation looks somewhat like a poll tax for every working individual, as high taxes are paid over the first little money earned, after which individuals are freed from their burden and can earn almost without paying any more taxes. Libertarian extreme weights are bordering laissez-faire flat weights, and therefore produce extremely low optimal marginal tax rates with a minimal U-shaped pattern.

Generally speaking, the mechanisms underlying the results in Figure 3 are consistent with what would be expected based on literature about distributive justice and optimal income taxation. Simulations presented in figure 5 in Saez (2001) are based on the empirical earnings distribution of the United States in 1992 and are tested using constant elasticity values of 0.25 and 0.5. Results using utilitarian and Rawlsian generalized social marginal welfare weights are presented. The simulation is thus based on data which are extremely similar to the specific case presented above.

This means results for libertarian and Rawlsian principles can effectively be compared to Saez (2001). As can be observed, the patterns of optimal marginal rates are also U-shaped in Saez (2001). Moreover, the rates and patterns for both utilitarian and Rawlsian principles of distributive justice are almost identical to those in figure 5 in Saez (2001). This indicates that my model produces optimal marginal tax rates which are consistent with results found in literature. Favourably, room for normative input is exceptionally large in my model. Though all patterns of optimal marginal tax rates are technically U-shaped – meaning they are decreasing for low earners and increasing for high earners – libertarian extreme and libertarian-Rawlsian patterns could also be described as flat and L-shaped respectively. Libertarian extreme rates represent the case close to laissez-faire, which should be a completely flattened (U-shaped) pattern at a rate of zero. Therefore, the libertarian extreme result is not surprising at all. L-shaped patterns, however, have not been found in literature before, probably because the range of principles of distributive justice investigated so far is limited and mostly revolved around utilitarian principles.

Figure 4



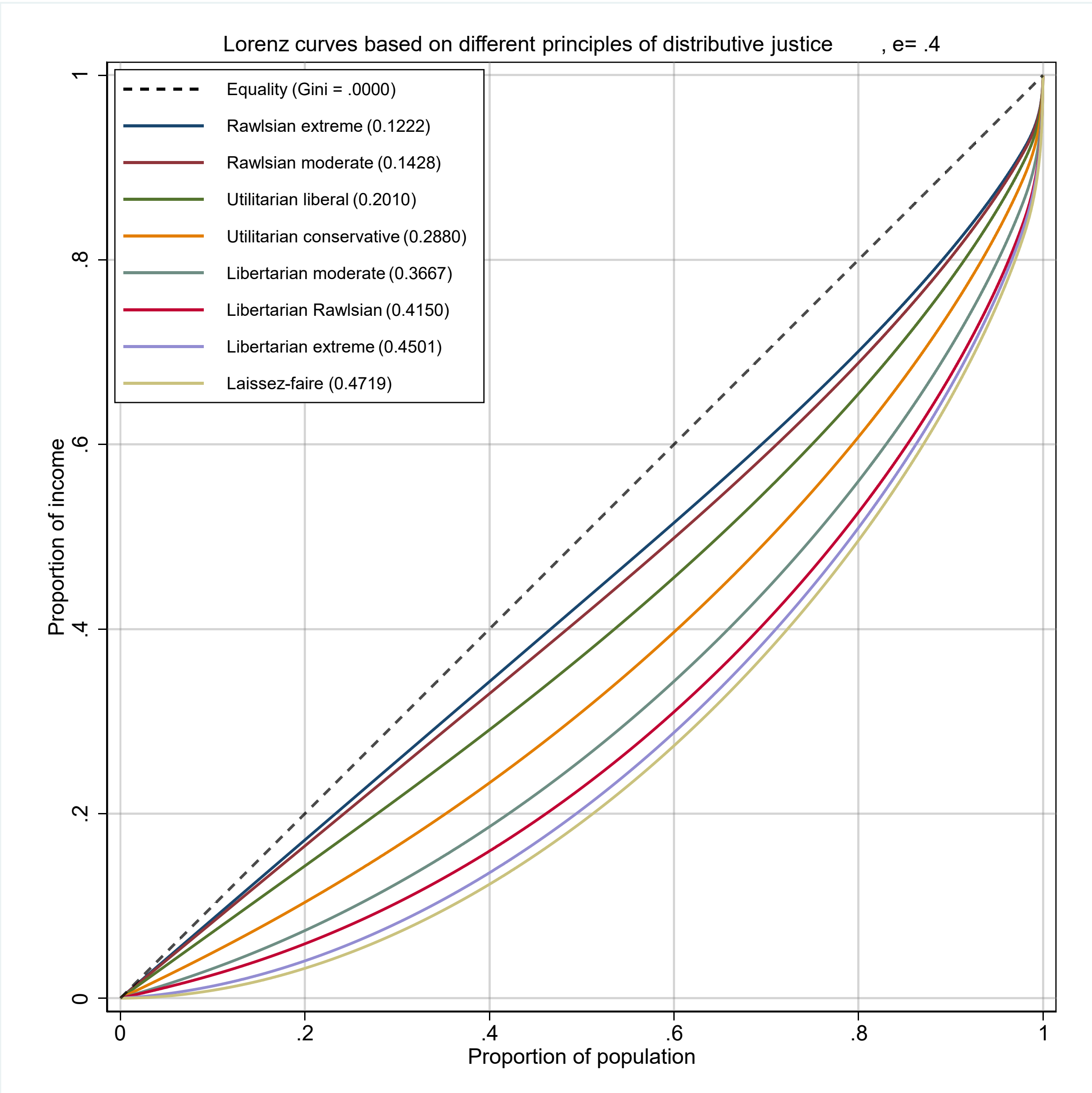
The relationship between before- and after-tax income – or earnings and disposable income – according to different principles of distributive justice is projected in Figure 4. The graph on the left shows before- and after-tax incomes for the 99% lowest earners, whereas the graph on the right only displays individuals earning at most 1.5 times the mean of earnings $(1.5 \cdot 50{,}000)$ to make redistribution of low and middle earners better visible. After-tax income follows directly from optimal marginal tax rates and redistributive policy applied to the United States 1994 earnings distribution with an average taxable income elasticity level of 0.4. Therefore, the optimal marginal tax rates presented in Figure 3 combined with the redistributive policy based on generalized social marginal welfare weights presented in Figure 2 can be used to explain the results in Figure 4. Since redistributive policy is always progressive, as generalized social marginal welfare weights are always decreasing over earnings, higher tax revenue results in more progressive redistribution. More progressive redistribution consequently leads to a larger deviation of disposable incomes to the laissez-faire situation. It can be observed that the principles of distributive justice which raise the highest tax revenue – the ones with the highest overall optimal marginal tax rates – result in disposable incomes with the largest variance to the laissez-faire situation. Generally speaking, Figure 4 indicates that principles which yield high tax revenue make the poor better off at the expense of individuals earning roughly more than the mean. Furthermore, disposable income is always increasing over earnings except when weights for the lowest earners are relatively very high, in which case disposable income is constant over earnings.

Again, results are mostly as expected. The Rawlsian extreme tax scheme maximizes tax revenue for redistribution to the poorest individual in society, which should result in a high demogrant for as many individuals as possible. This is exactly what my model produces. Recall that the mean of earnings is 50,000, which means that theoretically the maximum demogrant equals 50,000, which would represent complete equity. This would, however, not be optimal for society due to the high costs of disincentivizing high earners to work. The Rawlsian extreme tax scheme shows that the boundary of a demogrant which respects economic efficiency is around 42,000. As a result, disposable income for high earners suffers, but is still increasing over earnings. Note how the disposable income based on the Rawlsian moderate principle closely follows disposable income based on the Rawlsian extreme principle. This indicates that under these circumstances, the Rawlsian moderate policymaker is already relatively close to maximizing tax revenue. The extreme libertarian tax scheme produces the opposite, and disposable income is close to earnings with an extremely low demogrant. Disposable incomes based on the libertarian-Rawlsian principle illustrate very nicely the role of generalized social marginal welfare weights in the redistributionary process. As can be observed, libertarian-Rawlsian disposable incomes are generally very close to the laissez-faire situation, except for a positive deviation for the lowest earners. This reflects the pattern of generalized social marginal welfare weights, which are also generally close to the laissez-faire $(g_i = 1)$ weights, except for the higher weights for the lowest earners. The sets of disposable incomes based on the remaining three principles of distributive justice are in between the Rawlsian and libertarian extremes, as logically follows from Figures 2 and 3.

Figure 5 below shows the Lorenz curves and Gini coefficients after the implementation of optimal tax schemes using different principles of distributive justice. The role of different principles of distributive justice in inequality reduction can be examined. As expected, Rawlsian tax schemes are the most effective in reducing the level of income inequality and libertarian tax schemes are the least effective. Also notice how inequality can be reduced more significantly for low and middle earners than for high earners, illustrated by the distance between the Rawlsian extreme and laissez-faire Lorenz curves. The Rawlsian extreme Lorenz curve is relatively close to the laissez-faire Lorenz curve for the top 1% of incomes, suggesting that inequality reduction is least effective around this tail of the income distribution.

The role of the shape of the pattern of generalized social marginal welfare weights can be used to explain the difference in inequality reduction around both tails of the distribution. An exceptional case which can be used as an illustration is produced by the libertarian-Rawlsian principle. Notice how the libertarian-Rawlsian Lorenz curve closely follows the libertarian moderate Lorenz curve for low earners, but hugs the libertarian extreme curve for high earners. In other words, libertarian-Rawlsian tax schemes are only effective in reducing income inequality at low levels of income, not at high levels of income. This is because the respective generalized social marginal welfare weights are only relatively high for very low earners, but relatively flat for the rest of society. This results in two effects. First, the average tax rate is slightly decreasing for high earners, given the L-shaped pattern of marginal tax rates. As a result, the loss from taxes paid is not progressive and therefore does not help to reduce income inequality for high earners. Second, the progressive redistributive policy is only targeting the lowest earners in society. Consequently, inequality is only reduced around the left tail of the income distribution. The unique shape of the pattern of the libertarian-Rawlsian weights is thus reflected in the unique shape of the respective Lorenz curve.

Figure 5



Implications

In short, the patterns of optimal marginal tax rates are found to be U-shaped for most principles of distributive justice, but with varying degrees. Besides the Pareto parameter, the relative shape of the pattern of generalized social marginal welfare weights also determines the shape of the pattern of optimal marginal tax rates. When weights for middle and high earners are relatively flat, optimal marginal tax rates are also relatively flat for these earners. This result is relevant for extreme Rawlsian and libertarian principles. Rawlsian and utilitarian results are in line with literature. Libertarian-Rawlsian rates are more L-shaped and closer to a poll tax for every working individual, due to the unique pattern of generalized social marginal welfare weights. This has not been found before. The relationship between disposable income and earnings is found to reflect the shape of the pattern of generalized social marginal welfare weights for all levels of earnings. Total tax revenue is the largest determinant of the demogrant for low earners, and for high earners disposable income is mostly determined by the height of the respective marginal tax rates. The shape of the pattern of generalized social marginal welfare weights is also found to be related to the relative effectiveness of inequality reduction at different parts of the income distribution. The libertarian-Rawlsian tax scheme reduces inequality primarily for the lowest earners, due its unique pattern of generalized social marginal welfare weights.

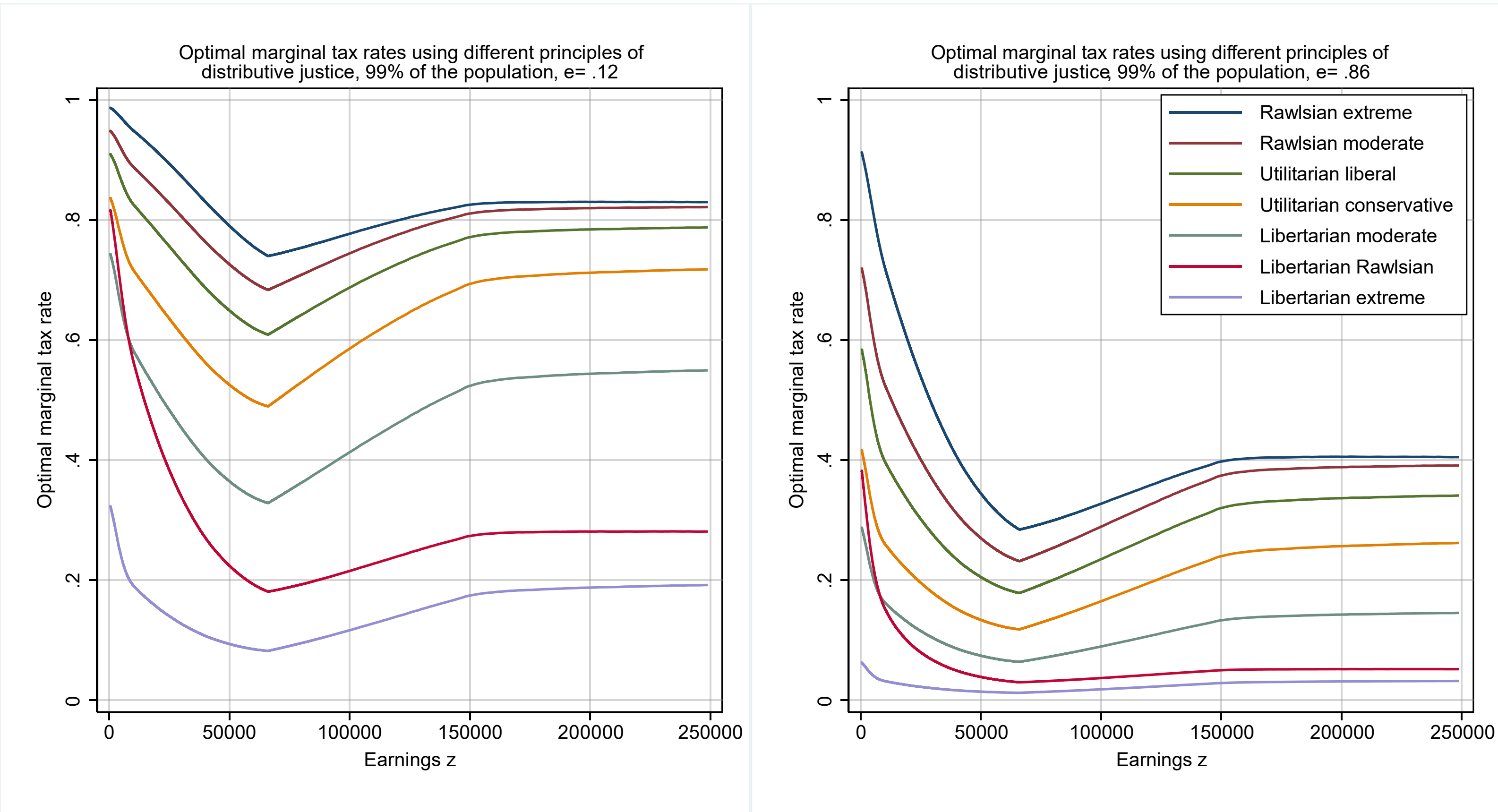
Results presented in Section 5.1 highlight the importance of the shape of the pattern of generalized social marginal welfare weights for optimal marginal tax rates, disposable income, and inequality reduction. Policymakers should therefore carefully estimate society's perceptions of justice and redistributive tastes over different levels of earnings.[25] The studies by Weinzierl (2014) and Kuziemko, Norton, Saez, and Stantcheva (2015) provide guidance on how to survey such perceptions and tastes of society, but additional research would be needed to develop more advanced methods to measure this.

5.2. Average distribution and extreme elasticities

Section 5.2 presents results based on an average distribution of earnings representing the United States in 1994 using two extreme taxable income elasticity assumptions of 0.12 and 0.86. Some of the general mechanisms behind results are explained in Section 5.1 and have been omitted in this section to avoid repetition.

[25] This advice is only relevant under the assumption of a well-functioning democracy. In a society where the policymaker's actions are not in line with the interests of society, the policymaker would be best off using utilitarian weights (in the interest of economic growth) or weights reflecting the policymaker's personal perceptions of redistributive justice.

Figure 6



The relationship between optimal marginal tax rates according to different principles of distributive justice and the level of taxable income elasticity is illustrated in Figure 6. The graph on the left shows optimal marginal tax rates according to a low taxable income elasticity assumption of 0.12, whereas the graph on the right displays rates for the high taxable income elasticity assumption of 0.86. Overall, it can be observed that optimal marginal tax rates decrease at every level of earnings when taxable income elasticity increases. Hence, the relationship between optimal marginal tax rates and taxable income elasticity is negative. This result is exactly as expected and in line with literature. When the behavioural response to an increase in taxation is larger, the consequences of high tax rates are more harmful to the economy and hence optimal rates are lower. The results in Figure 6 are more interesting when analyzing the impact of a change in elasticity on the shape of the pattern of optimal marginal tax rates. Notice that an increased taxable income elasticity has a different effect on the shape of the pattern of optimal marginal tax rates for different principles of distributive justice. The effect for Rawlsian principles is similar to that in figure 5 in Saez (2001), as rates decline mostly for medium and high earners when elasticity increases and this moves the pattern more towards an L-shape. For libertarian principles, however, rates for both low and very high earners are more strongly affected compared to rates for middle to high earners ($z \approx 65{,}000$). Hence, the shapes of the patterns of optimal marginal tax rates are flattened when elasticity increases, resulting in an L-shaped pattern for the libertarian-Rawlsian principle and a flat pattern for the libertarian extreme principle. This has not been observed in literature before, due to the limited range of principles of distributive justice examined in previous studies. For utilitarian principles, the shape of the pattern of optimal marginal tax rates seems stable when elasticity varies.

The results for utilitarian and Rawlsian principles are very similar to those presented in figure 5 in Saez (2001). The effect of an increase in elasticity on the optimal marginal tax rates for the lowest earners is relatively small when generalized social marginal welfare weights are high and decline steeply for these earners.[26] This is the case for the Rawlsian principles. Economically, this makes sense as these very low earners are compensated in the form of redistribution anyways. As such, high marginal tax rates at these earnings levels can be used to maximize tax revenue for said redistribution. Furthermore, the difference in the effect of an increase in elasticity on the optimal marginal tax rates for middle to high earners ($z \approx 65{,}000$) when using libertarian principles can be explained by the Pareto parameter. Recall that the Pareto parameter measures the relative impact of behavioural effects from changes in marginal tax rates at different levels of earnings, and is highest for middle to high earners. This explains why optimal marginal tax rates are always lowest at this level of earnings. There is, however, a lower bound on rates in this model such that rates are never negative. Essentially, the lower bound is, in this scenario, the reason that patterns are flattened. The effect of an increased elasticity diminishes when rates approach zero. The utilitarian principles are the only principles not affected by both of the effects discussed above, and therefore the shape of the pattern of optimal marginal tax rates declines steadily when elasticity increases.

[26] The steep decline in weights g_i results in a steep decline in $\bar{G}(z)$ for low earners too. Because the Pareto parameter is so low at this level of earnings, this small difference in $\bar{G}(z)$ is relatively large in the denominator of the formula for optimal marginal tax rates. Since the elasticity parameter is also in the denominator, the effect of an increase in this parameter is now relatively small. This effect is only present at very low levels of earnings, though, as the Pareto parameter quickly grows large enough to cancel this effect out.

Implications

Concisely, an increase in taxable income elasticity results in a decrease of optimal marginal tax rates, especially when the shape of the pattern of generalized social marginal welfare weights is rather flat for medium and high earners. As such, patterns of utilitarian optimal marginal tax rates remain U-shaped, whereas Rawlsian patterns become more L-shaped when elasticity increases. A figure and detailed analysis about the role of taxable income elasticity in the relationship between disposable income and earnings are presented in Appendix F1. As expected, disposable income deviates less from earnings when taxable income elasticity is high, due to lower overall optimal marginal tax rates and lower tax revenue for redistribution. Additionally, when elasticity is high, the Rawlsian extreme and Rawlsian moderate schemes produce almost identical levels of disposable income. The level of taxable income elasticity can thus affect the relative difference between disposable incomes produced by tax schemes using different principles of distributive justice. Appendix F1 also includes a figure and detailed analysis regarding the effect of a change in taxable income elasticity on inequality reduction. It is confirmed that an increased taxable income elasticity makes inequality reduction considerably less effective. Moreover, Rawlsian schemes can almost completely eliminate income inequality when elasticity is low.

Results presented in Section 5.2 further emphasize the role of generalized social marginal welfare weights in the determination of the shape of the pattern of optimal marginal tax rates. Moreover, Section 5.2 and Appendix F1 highlight just how important it is to correctly estimate the level of taxable income elasticity, as results based on a misestimation could be extremely misleading. Though consensus on the level of taxable income elasticity is still lacking in literature, progress has been made on the development of different methods to estimate taxable income elasticity. Policymakers could consult the study by Saez, Slemrod, and Giertz (2012), where the latest overview of these methods can be found. Conducting research on the changes in earnings as a response to income tax reforms using empirical data on their own country/region would be the best approach for policymakers.

5.3. Extreme distributions and average elasticity

Section 5.3 presents results assuming an average taxable income elasticity of 0.4, while considering two extreme distribution of earnings. The relatively equal distribution is based on the earnings distribution of Germany in 1981, whereas the relatively unequal distribution represents earnings in Mexico in 1994. Gini coefficients are 0.32 and 0.59 respectively, whereas the United States earnings distribution used for Sections 5.1 and 5.2 has a Gini coefficient of 0.47. As the distributions of earnings change, the generalized social marginal welfare weights change too. In Appendix F2, Figure 14 and its detailed analysis show how generalized social marginal welfare weights using different principles of distributive justice change when the distribution of earnings is relatively equal or unequal. For the relatively equal distribution, patterns of generalized social marginal welfare weights are shaped much steeper at both tails of the distribution but flatter around the middle compared to the unequal distribution. This directly reflects the difference between the shapes of the relatively equal and unequal earnings distributions, which can be found in Figure 1 in Section 4. Some of the general mechanisms behind results are explained in Section 5.1 and have been omitted in this section to avoid repetition.

Figure 7

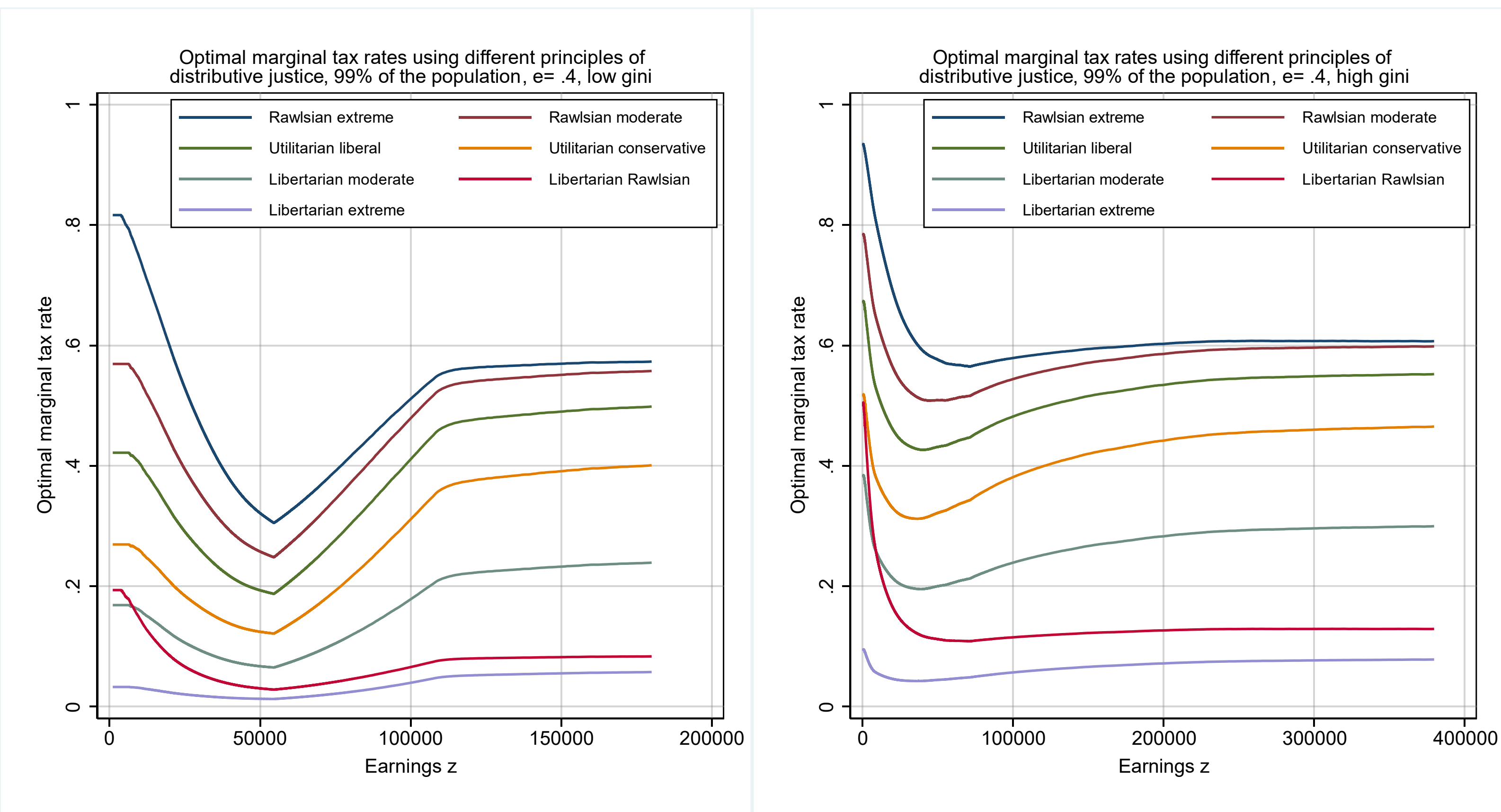


Figure 7 presents the optimal marginal tax rates using different principles of distributive justice for an equal and unequal distribution of earnings. The graph on the left represents the equal earnings distribution, whereas graph on the right represent the unequal earnings distribution. Two general results can be observed in Figure 7. First, optimal marginal tax rates are higher when the distribution of earnings is relatively unequal. Second, the shape of the pattern of optimal marginal tax rates is very U-shaped when the distribution of earnings is relatively equal, but more L-shaped when the distribution of earnings is relatively unequal.

Both results can be explained by differences in the Pareto parameter between the two distributions of earnings. The unequal distribution is much wider, meaning that earnings are relatively further away from the median. As such, the density of the unequal earnings distribution is relatively low at the medium compared to the equal earnings distribution. Moreover, density values of the unequal earnings distribution are spread over a wider range of earnings, as indicated by the difference in the horizontal scale in Figure 7. As such, the density curve of the unequal earnings distribution is flatter and lower overall than the density curve of the equal distribution. This difference in earnings density is reflected in the Pareto parameter, which is lower for the unequal distribution, especially around the median level of earnings. A lower Pareto parameter results in higher optimal marginal tax rates, which explains the higher rates for the unequal earnings distribution. Because the difference in Pareto parameters is largest around middle earners, optimal marginal tax rates increase mostly around these earnings levels when the distribution of earnings is more unequal. This explains the shift in the direction of L-shaped optimal marginal tax rates as the earnings distribution becomes more unequal. For the relatively equal distribution of earnings, the density of earnings for low-medium earners is relatively high which results in optimal marginal tax rates for low earners being lower than rates for high earners, when using utilitarian and libertarian moderate principles. The result regarding the shift towards L-shaped patterns of optimal marginal tax rates is very important, as it answers the question raised by Saez (2001) about the relationship between the shape of the pattern of optimal marginal tax rates and the distribution of earnings. My results suggest that U-shaped patterns of optimal marginal tax rates are not universal, and most likely specific to distributions with equality levels similar to or lower than the United States.

Implications

In short, results suggest that as inequality of the distribution of earnings increases, optimal marginal tax rates increase. Rates increase primarily for middle earners, which means that the shape of the pattern of optimal marginal tax rates becomes less U-shaped and more L-shaped as inequality of the earnings distribution of earnings increases. This is because the relative impact of behavioural effects, measures by the Pareto parameter, changes as the distribution of earnings changes. An unequal distribution is generally characterized by an overall lower Pareto parameter due to lower densities of earnings, especially for middle earners. As such, my results indicate that U-shaped patterns of optimal marginal tax rates are not universal, but could in fact be L-shaped when earnings inequality is very high. A figure and detailed analysis about the role of the distribution of earnings in the relationship between disposable income and earnings are presented in Appendix F2. As expected, redistribution towards the poor increases as earnings inequality increases due to the higher tax revenue raised by the increased optimal marginal tax rates. Despite the higher tax revenue, the size of the demogrant under the tax revenue maximizing case suffers due to the increased costs of providing a high demogrant (as a larger share of society is a net receiver of redistribution). Additionally, Appendix F2 includes a figure and detailed analysis regarding the role of the distribution of earnings on inequality reduction. Inequality reduction is more effective when earnings inequality is high. The final level of income inequality, however, will always be lower when earnings inequality is low due to the relatively equal laissez-faire position for equal distributions.

The results in Section 5.3 and Appendix F2 indicate that the distribution of earnings can have a strong impact on the height and pattern of optimal marginal tax rates, which in turn has consequences for redistribution and inequality reduction. Policymakers representing countries/regions with a high level of earnings inequality, such as developing countries in Africa or Latin America, might be better off using an L-shaped pattern of marginal tax rates, contradicting the conventional U-shape found in most literature. By using data on tax returns, the distribution of earnings should readily be observable for the policymaker.

6. Conclusion

My model of optimal income taxation schemes using different principles of distributive justice, taxable income elasticity, and the distribution of earnings yields multiple insights. First, it can be concluded that optimal marginal tax schemes are sensitive to changes in generalized social marginal welfare weights and taxable income elasticity. Policymakers are therefore suggested to be thorough in their estimations of redistributive preferences and taxable income elasticities.

Additionally, the sensitivity indicates that results obtained in previous studies are of limited relevance due to the uniformity in assumptions about normative tastes. Next, my model produces highly similar results to those presented in Saez (2001) when using comparable inputs and circumstances, despite the significant simplifications and increased flexibility of normative inputs in my model. This suggests that, at least for utilitarian and Rawlsian policymakers in the United States, accurate optimal marginal tax schemes can be constructed without using the welfarist approach. This simplification is a step forward in closing the gap between optimal tax theory and practice, as it makes the understanding of underlying mechanisms more accessible for a wider audience. Third, the question raised by Saez (2001) on the universality of U-shaped patterns of optimal marginal tax rates beyond the United States earnings distribution is finally addressed. My study suggests that as earnings inequality increases, optimal marginal tax rates for middle earners could increase substantially resulting in a shift towards L-shaped patterns. This result could be relevant for developing countries in Africa and Latin America as they are often characterized by high levels of earnings inequality. Fourth, I demonstrate the exceptional flexibility in describing redistributive preferences in my model by considering a libertarian-Rawlsian principle. The libertarian-Rawlsian policymaker represents the idea that the provision of a social minimum is the only just violation of self-ownership, which would be impossible to capture in welfarist models. Optimal tax schemes using the libertarian-Rawlsian principle yield unique results, suggesting that a poll tax for every working individual could be preferred in societies where taxable income elasticity is high.

Besides contributing to the understanding of underlying mechanisms in optimal income taxation, my model can also be used for small tax reforms in two ways. First, policymakers could use the model to identify preferred tax reforms based on efficiency grounds. There already is an existing tax scheme in place, and the distribution of earnings and taxable income elasticity should be known. As such, the model can be solved for generalized social marginal welfare weights (simply by rearranging equation (5)). Negative weights or weights increasing over earnings would indicate inefficiency. Weights can be adapted such that they are nonnegative and decreasing over earnings. Using these adapted weights to construct a new tax scheme would guide the policymaker about what type of reforms would produce a tax schemes that are preferred to the existing tax scheme. Second, policymakers could use my model to identity preferred tax reforms from a normative view. Again, generalized social marginal welfare weights describing the existing tax scheme should be solved for first. As an example, the policymaker could, for normative reasons, prefer increased weights for middle earners. Weights can be adapted to reflect this normative preference, and a new tax scheme can be constructed using the model. The new tax scheme would again inform the policymaker about the direction of reform needed to produce a tax scheme which is preferred to the existing tax scheme.

The exploration of normative dimensions of optimal income taxation using observable variable can be extended in various ways. First, the relationship between redistributive preferences and generalized social marginal welfare weights could be studied in more detail. Studies by Weinzierl (2014) and Kuziemko *et al.* (2015) are promising starting points, but more progress on the quantification of perceptions of distributive justice would be valuable. This could support the development of weights describing novel types of policymakers, such as the libertarian-Rawlsian, which could be tested in optimal income taxation models. Second, the role of the earnings distribution on optimal income tax schemes could be examined in more detail. The three distributions used in this study are simulated using generalized beta of the second kind distributions with a modified right tail representing a Pareto distribution. It would be interesting to construct optimal income tax schemes based on different and more up to date parameters. In addition, empirical earnings distributions could be used to evaluate the external validity of results presented in this study. Third, it could be fruitful to explore the underlying mechanisms in optimal income taxation in more detail by developing methods to compare results quantitatively rather than visually. This could also benefit the process of identifying possible interaction effects between parameters in the model.

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Appendix

A

Figure 8

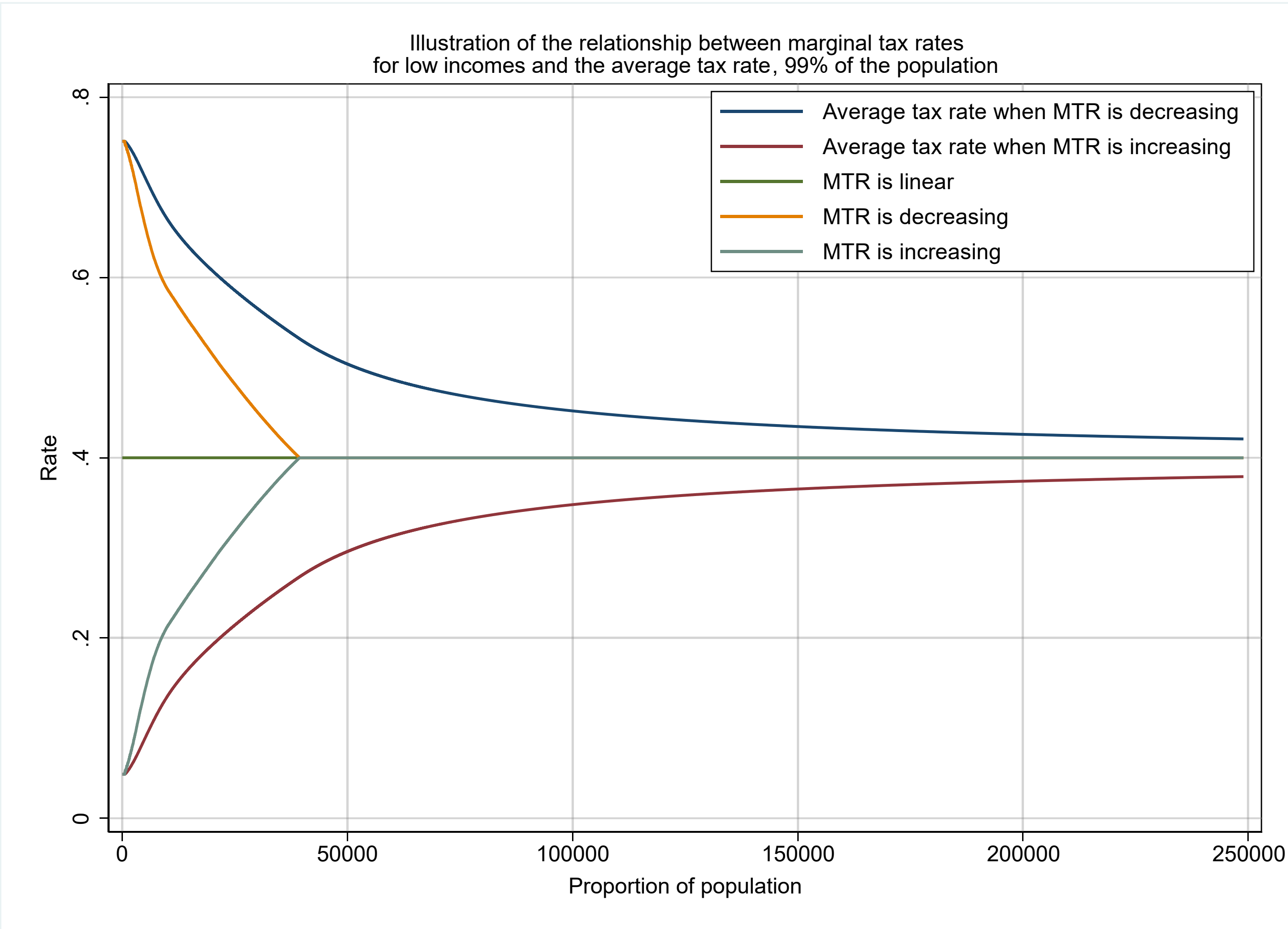
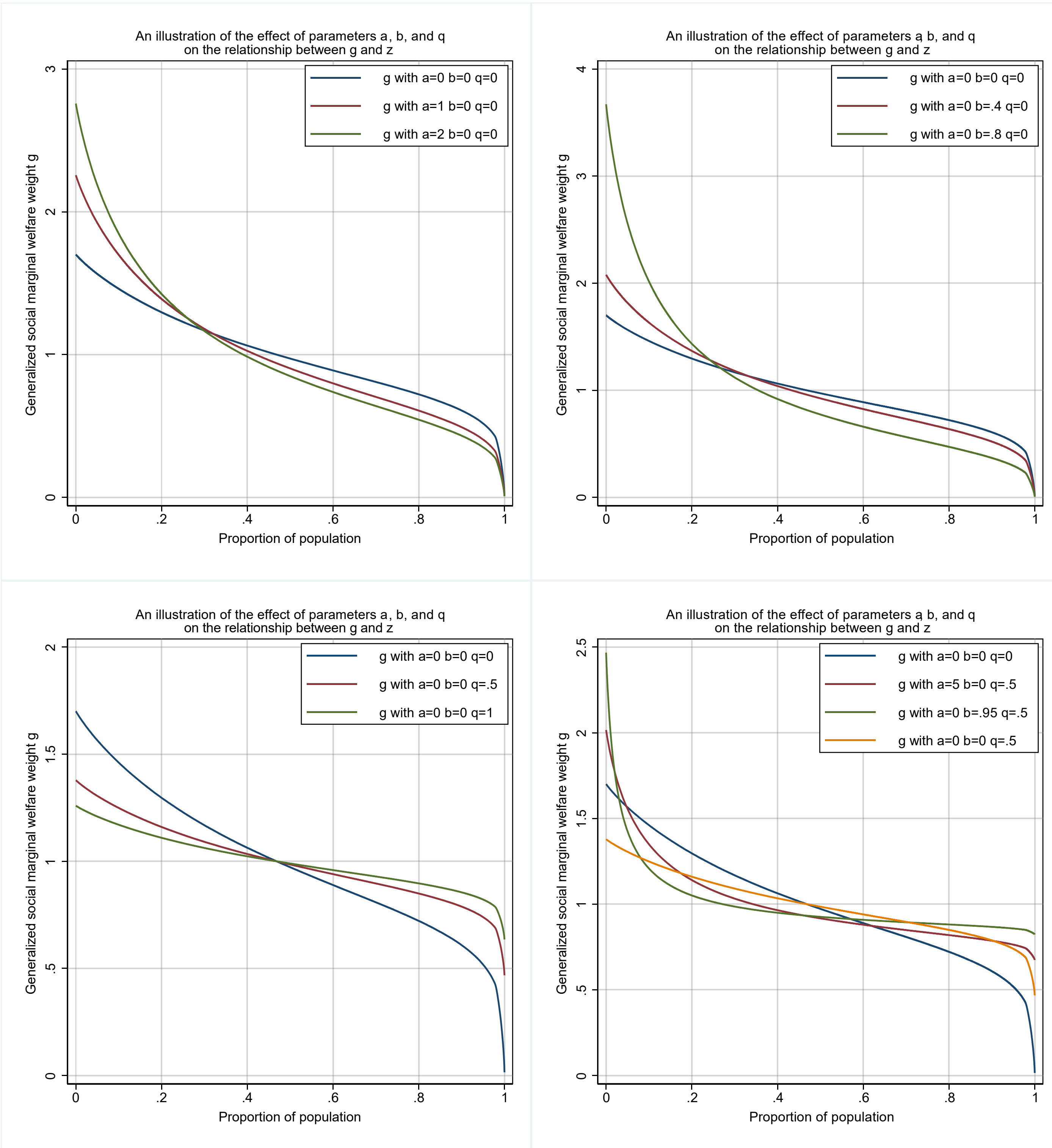


Figure 8 shows how average tax rates are affected when the marginal tax rate for low earners changes. In this illustration, three marginal tax schemes are used: a linear scheme, a decreasing scheme, and an increasing scheme. Note that the linear scheme is equal to a rate of 0.4, and that the decreasing and increasing schemes also converge to this rate of 0.4. As such, only the marginal tax rates for low earners are different between these schemes. The average tax rate is obviously equal to 0.4 for the linear scheme, regardless of the level of earnings. However, the average tax rate is higher (lower) for the increasing (decreasing) marginal tax scheme, and higher (lower) than 0.4. This is because higher earners are still affected by the difference in marginal tax rates for low earnings. The marginal tax rates, however, are identical for high earners in all three schemes. As such, the net gain for earning one extra euro is always 0.60 euros for high earners in all three schemes. This means that the behavioural effects for high earners are not affected by a change in marginal tax rates for low earners. Only those at the margin (low earners) are affected. The average tax rate, however, changes for all earners due to a change in marginal tax rates for low earners. Mirrlees (1971) uses this relationship between marginal tax rates at the margin and average tax rates beyond the margin to maximize the amount of taxes paid while minimizing the consequences of behavioural effects.

B

Figure 9



An illustration of the effect of parameters a, b, and q
on the relationship between g and z
g with a=0 b=0 q=0
g with a=1 b=0 q=0
g with a=2 b=0 q=0
Generalized social marginal welfare weight g
Proportion of population
An illustration of the effect of parameters a, b, and q
on the relationship between g and z
g with a=0 b=0 q=0
g with a=0 b=.4 q=0
g with a=0 b=.8 q=0
Generalized social marginal welfare weight g
Proportion of population
An illustration of the effect of parameters a, b, and q
on the relationship between g and z
g with a=0 b=0 q=0
g with a=0 b=0 q=.5
g with a=0 b=0 q=1
Generalized social marginal welfare weight g
Proportion of population
An illustration of the effect of parameters a, b, and q
on the relationship between g and z
g with a=0 b=0 q=0
g with a=5 b=0 q=.5
g with a=0 b=.95 q=.5
g with a=0 b=0 q=.5
Generalized social marginal welfare weight g
Proportion of population

Figure 9 above shows the effect of changes in parameters a, b, and q on the relationship between generalized social marginal welfare weights g_i and earnings z, where $g_i = g(z) = \gamma(z) + q \cdot \gamma(z)_{max}$ with $\gamma(z) = \frac{1}{(z+\bar{z})+az-b\bar{z}}$. Earnings z are scaled using the proportion of population, as a scale using absolute values of z would stretch the curves massively over the very right tail, making interpretation difficult. As can be observed in the upper left graph, an increase of parameter a, ceteris paribus, results in higher g_i for low earners at the expense of medium to high earners. Weights for the very highest earner, however, remain unchanged. Note that when a increases, the share of individuals who receive net benefits from redistribution, meaning where $g_i > 1$, decreases slightly. The upper right graph shows the effect of an increase in parameter b, ceteris paribus. On first sight, the effect seems similar to the effect of an increase in parameter a, but closer inspection shows otherwise. Note how the shapes of the curves are different, mainly towards the right tail. Though an increase in both a and b leads to higher g_i for low earners, the effect is much higher for b at the expense of the highest earners in society. As b approaches $b = 1$, g_i will approach zero for high earners as the concavity of the curve decreases substantially. When only increasing a, however, the very right tail will always remain very concave, and g_i will mostly decrease for medium to high earners without approaching zero. Next, we consider increasing parameter q in the lower left graph in Figure 9. As can be observed, an increase in q essentially rotates the curve of g_i with parameters a and b already set in a counterclockwise direction, in this case g_i with $a = 0$ and $b = 0$. The rotation happens around the level of earnings z where $g_i = 1$. As can be observed, an increase in parameter q has a very large effect on g_i for the highest and lowest levels of earnings, where g_i decreases for low earners at the expense of an increase for high earners. The lower right graph in Figure 9 shows how the effect of an increase in q changes when parameters a and b are not equal to zero. As can be observed, the properties of a high a and a high b as discussed earlier are still present. For very low earners g_i is higher and for high earners the curve of g_i is less concave for a high b compared to a high a. Using different combinations of parameters a, b, and q, generalized social marginal welfare weights g_i corresponding to different principles of distributive justice can be computed. These configurations are presented in results Section 5.1.

C

If there exists the desire to add deadweight losses from income taxation collection to this model, for instance due to administrative costs, then please follow the extension presented here. The addition of deadweight losses from collection could easily be achieved by rewriting equation (6) as $c_i = (z_i - r_i) + (1 - DWL)(g_i \cdot \overline{\mathrm{R}})$, where DWL denotes the percentage of total tax revenue lost due to deadweight losses. This would, again by the substitution of equation (7), change equation (8) to $C = (Z - R) + (1 - DWL) \cdot R = Z - DWL \cdot R$, which would result in the aggregate amount of disposable income C being lower than the aggregate amount of earnings Z. Specifically, the aggregate amount of disposable income C would be lower than the aggregate amount of earnings Z by the product of percentage of total tax revenue lost due to deadweight losses and total tax revenue, $DWL \cdot R$.

D

Figure 10

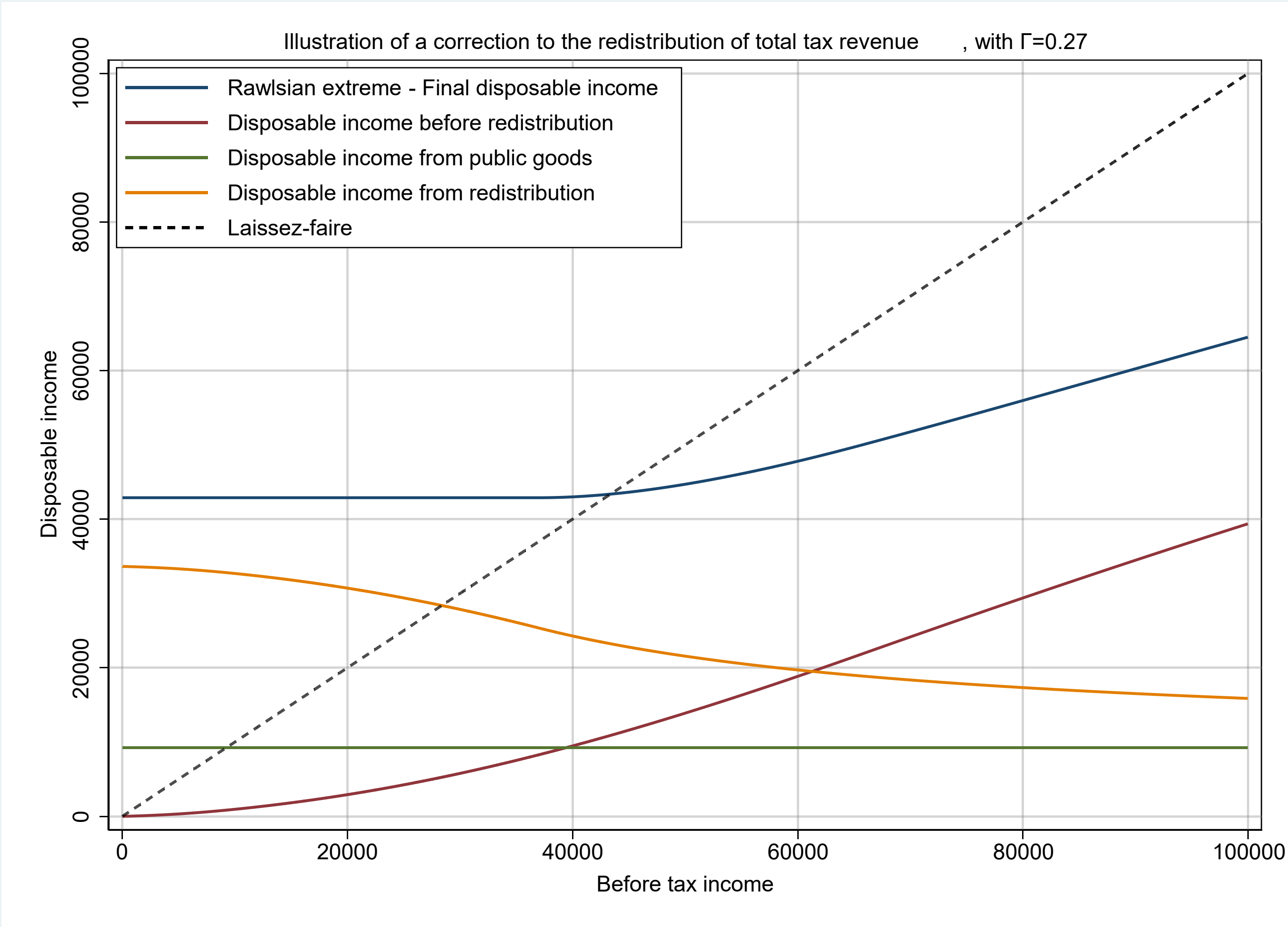


Figure 10 graphically illustrates the results of a correction to the redistribution of total tax revenue net of public goods spending. Specifically, it shows how total tax revenue net of public goods spending is redistributed such that disposable income c is always constant or increasing over earnings z. This results in a stable guaranteed income level until earnings $z_i > z_j$, where disposable income c becomes increasing over earnings z.

E

Figure 11

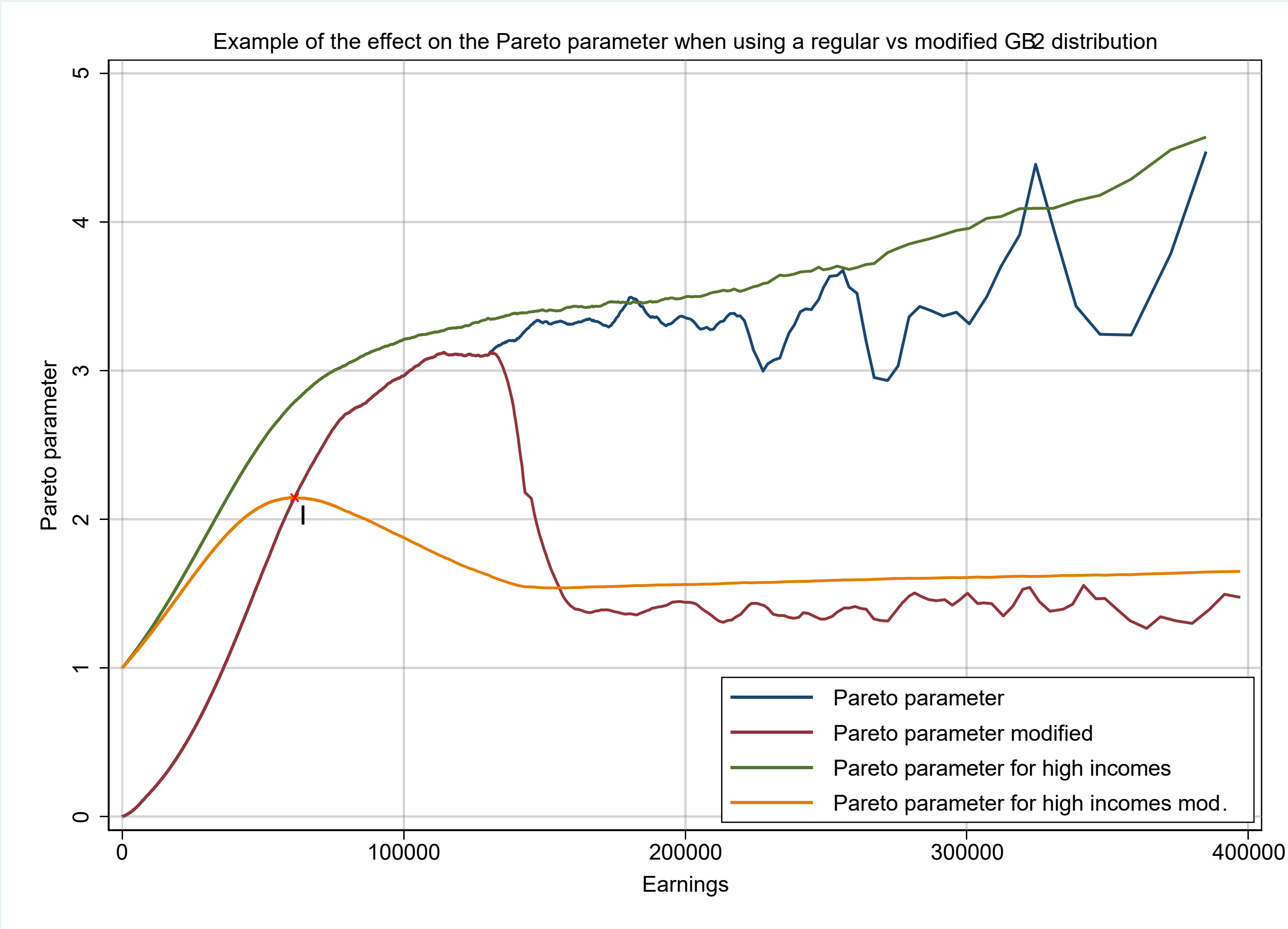
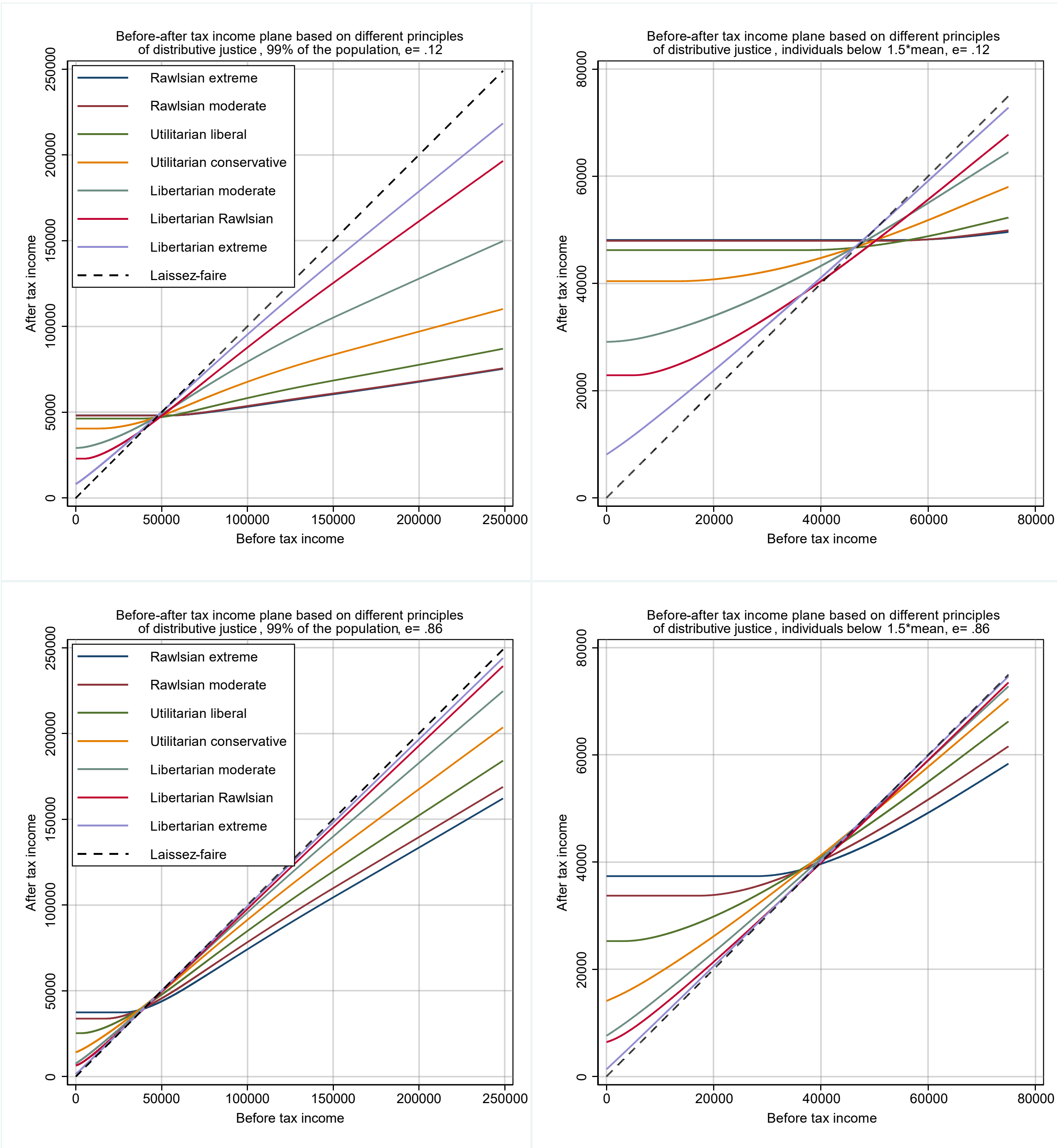


Figure 11 follows figure 2 in Diamond and Saez (2011). The example in Figure 11 is based on 50 sets of 10,000 random draws from a GB2 distribution fit to the United States 1994 earnings distribution. In the modified distribution, the top 2% of earnings are replaced by the top 2% earnings from 50 sets of 10,000 observation random draws from a Pareto distribution. The Pareto distribution uses a shape parameter of 1.4, and was fitted to the generated GB2 distribution such that they smoothly intersect at the 98th percentile. This essentially means that the modified distribution is identical to the original distribution, only with a thicker very right tail which follows a Pareto distribution. Pareto parameters for low and medium incomes are computed using $\alpha(z) = zh(z)/\left(1 - H(z)\right)$, whereas Pareto parameters for high incomes follow $\rho(z) = z_m/(z_m - z_i)$. As can be observed, the modified GB2 distribution produces high-income Pareto parameter curves shaped very much like the empirical curves in Figure 11 in Diamond and Saez (2011). The difference in results is especially noticeable after the first intersection I, between the regular and high-income Pareto parameters. Figure 11 shows that when using a modified GB2 distribution, the regular Pareto parameter works well for low and medium incomes. The high-income Pareto parameter works best after the intersection of the two curves though, as it does not suffer from increased volatility and steadily converges to a constant Pareto parameter for high incomes.

F1

Figure 12

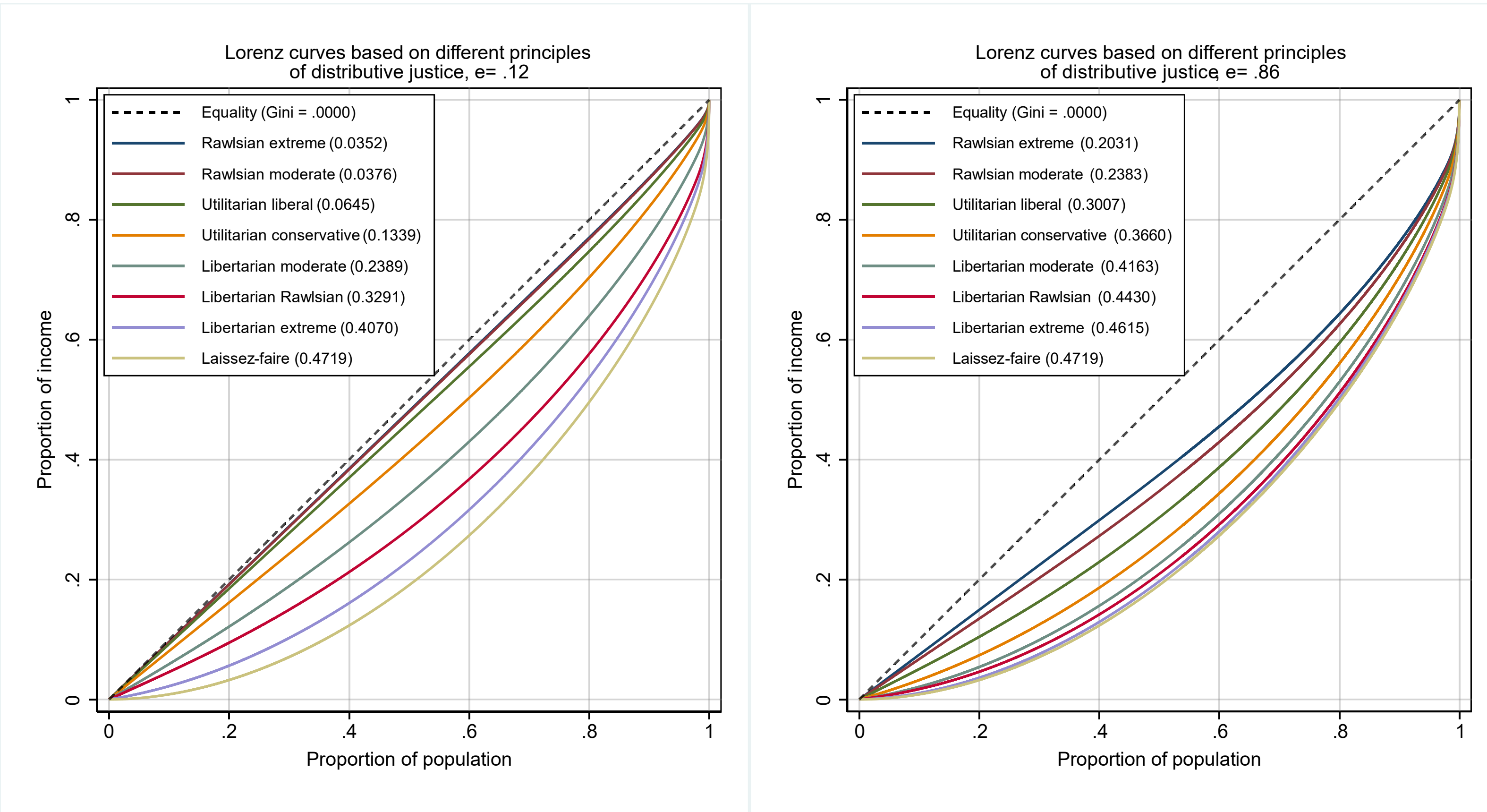


Before-after tax income plane based on different principles of distributive justice , 99% of the population, e= .12
Before-after tax income plane based on different principles of distributive justice , individuals below 1.5*mean, e= .12
Before-after tax income plane based on different principles of distributive justice , 99% of the population, e= .86
Before-after tax income plane based on different principles of distributive justice , individuals below 1.5*mean, e= .86
Rawlsian extreme
Rawlsian moderate
Utilitarian liberal
Utilitarian conservative
Libertarian moderate
Libertarian Rawlsian
Libertarian extreme
Laissez-faire
After tax income
Before tax income
0
50000
100000
150000
200000
250000
20000
40000
60000
80000

Figure 12 demonstrates how the relationship between before- and after-tax income – or earnings and disposable income – is affected by the level of taxable income elasticity, using different principles of distributive justice. The graphs on the top row show the situation where elasticity is assumed to be low, at 0.12, whereas a high elasticity of 0.86 is assumed in the bottom row. The graphs on the left show before- and after-tax incomes for the 99% lowest earners, whereas the graphs on the right only display individuals earning at most 1.5 times the mean of earnings $(1.5 \cdot 50{,}000)$ to make disposable incomes of low and middle earners better visible. Figure 6 showed how much optimal marginal tax rates could be affected by a change in taxable income elasticity. In Figure 12, it can be observed that this has considerable consequences for the relationship between before- and after-tax income. As expected, disposable income deviates less from earnings when taxable income elasticity is high, which can be explained by two reasons. First, lower overall optimal marginal tax rates mean that high earners – the net losers from taxation – lose a smaller share of their earnings to taxation, leaving them with higher disposable income. Second, and related to the first reason, total tax revenue is lower. A lower total tax revenue implies that less money is available for progressive redistribution to the poor, which results in smaller net gains for these individuals. As such, the demogrant is much lower for an increased elasticity, whereas disposable income for high earners becomes much larger. This is the same for all principles of distributive justice. These results follow logically from Figure 6 and are mostly in line with expectations and literature.

The only remarkable result produced by Figure 12 relates to the tax revenue maximizing case. The Rawlsian extreme principle shows the boundaries of an optimal tax revenue maximizing scheme, but when elasticities are low the Rawlsian moderate principle produces an almost identical distribution of disposable income. The Rawlsian extreme and Rawlsian moderate outcomes are not exactly the same, but sufficiently close to suggest that a Rawlsian moderate stance is already touching the boundaries of achieving equity whilst respecting economic efficiency when elasticity is low. The level of taxable income elasticity can thus affect the relative difference between disposable incomes produced by tax schemes using different principles of distributive justice. This highlights just how important it is to correctly estimate the level of taxable income elasticity.

Figure 13



The effect of a change in taxable income elasticity on inequality reduction using different principles of distributive justice is illustrated in Figure 13. The graph on the left shows Lorenz curves and respective Gini coefficients when elasticity is assumed to be low, at 0.12, whereas the graph on the right depicts Lorenz curves and respective Gini coefficients according to a high elasticity assumption of 0.86. As follows from the results presented in Figures 6 and 12, an increased taxable income elasticity makes inequality reduction considerably less effective. This effect is smallest for the libertarian tax schemes, simply because those are close to the laissez-faire allocation. The difference in effectiveness of inequality reduction between both tails of the distribution, as was discussed in Section 5.1, still holds regardless of the level of elasticity. Generally, these results are as expected. Figure 12 showed that the Rawlsian extreme and Rawlsian moderate schemes produce almost identical distributions of disposable income when elasticity is low, which explains why results in Figure 8 are also almost identical. When elasticity is low, the Rawlsian schemes are able to almost completely eliminate inequality, especially for low and medium incomes. The difference between potential inequality reduction is large, as the tax revenue maximizing scheme is only able to achieve a Gini coefficient of 0.2031 when elasticity is high, compared to 0.0352 when elasticity is low. The consequence is that the libertarian moderate tax scheme achieves the same Gini coefficient when elasticity is low as the Rawlsian moderate achieves when elasticity is high. This further demonstrates the importance of a correct estimation of the level of taxable income elasticity, as it significantly affects the relative effectiveness in inequality reduction between tax schemes using different principles of distributive justice.

F2

Figure 14

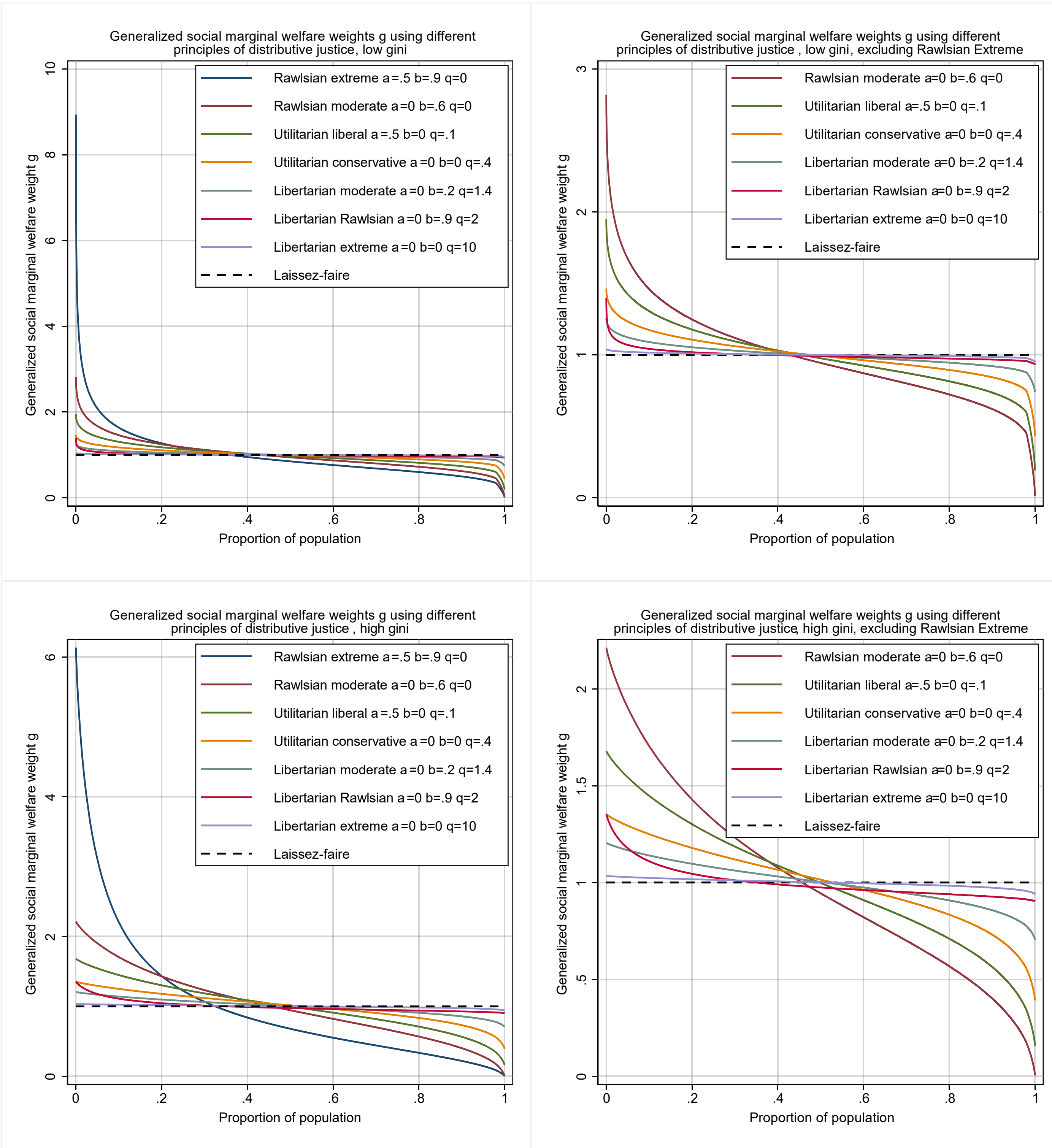

Generalized social marginal welfare weights g using different principles of distributive justice, low gini
Generalized social marginal welfare weight g
Proportion of population
Rawlsian extreme a=.5 b=.9 q=0
Rawlsian moderate a=0 b=.6 q=0
Utilitarian liberal a=.5 b=0 q=.1
Utilitarian conservative a=0 b=0 q=.4
Libertarian moderate a=0 b=.2 q=1.4
Libertarian Rawlsian a=0 b=.9 q=2
Libertarian extreme a=0 b=0 q=10
Laissez-faire
0
2
4
6
8
10
0
.2
.4
.6
.8
1
Generalized social marginal welfare weights g using different principles of distributive justice , low gini, excluding Rawlsian Extreme
Generalized social marginal welfare weight g
Proportion of population
Rawlsian moderate a=0 b=.6 q=0
Utilitarian liberal a=.5 b=0 q=.1
Utilitarian conservative a=0 b=0 q=.4
Libertarian moderate a=0 b=.2 q=1.4
Libertarian Rawlsian a=0 b=.9 q=2
Libertarian extreme a=0 b=0 q=10
Laissez-faire
0
1
2
3
Generalized social marginal welfare weights g using different principles of distributive justice , high gini
Generalized social marginal welfare weight g
Proportion of population
Rawlsian extreme a=.5 b=.9 q=0
Rawlsian moderate a=0 b=.6 q=0
Utilitarian liberal a=.5 b=0 q=.1
Utilitarian conservative a=0 b=0 q=.4
Libertarian moderate a=0 b=.2 q=1.4
Libertarian Rawlsian a=0 b=.9 q=2
Libertarian extreme a=0 b=0 q=10
Laissez-faire
0
2
4
6
Generalized social marginal welfare weights g using different principles of distributive justice, high gini, excluding Rawlsian Extreme
Generalized social marginal welfare weight g
Proportion of population
Rawlsian moderate a=0 b=.6 q=0
Utilitarian liberal a=.5 b=0 q=.1
Utilitarian conservative a=0 b=0 q=.4
Libertarian moderate a=0 b=.2 q=1.4
Libertarian Rawlsian a=0 b=.9 q=2
Libertarian extreme a=0 b=0 q=10
Laissez-faire
0
.5
1
1.5
2

Figure 14 shows how the generalized social marginal welfare weights using different principles of distributive justice change when the distribution of earnings is equal or unequal. The graphs in the top row represent the equal earnings distribution, whereas graphs on the bottom represent the unequal earnings distribution. Graphs on the right exclude the Rawlsian extreme principle to benefit vertical scaling for the other six principles. It can be observed that for the equal distribution, patterns of generalized social marginal welfare weights are shaped much steeper at both tails of the distribution but flatter around the middle compared to the unequal distribution. This directly reflects the difference between the shapes of the equal and unequal earnings distributions, which can be found in Figure 1 in Section 4. For the relatively equal distribution, inequalities only exist in the tails of the distribution and therefore these are the only areas where redistributive tastes are relatively different. Individuals around the middle of the distribution already earn relatively similar salaries, so there is no reason for the policymaker to have very different redistributive tastes between these individuals. For the unequal earnings distribution, earnings inequality extends more towards the middle of the distribution and hence generalized social marginal welfare weights are still steep here.

Figure 15

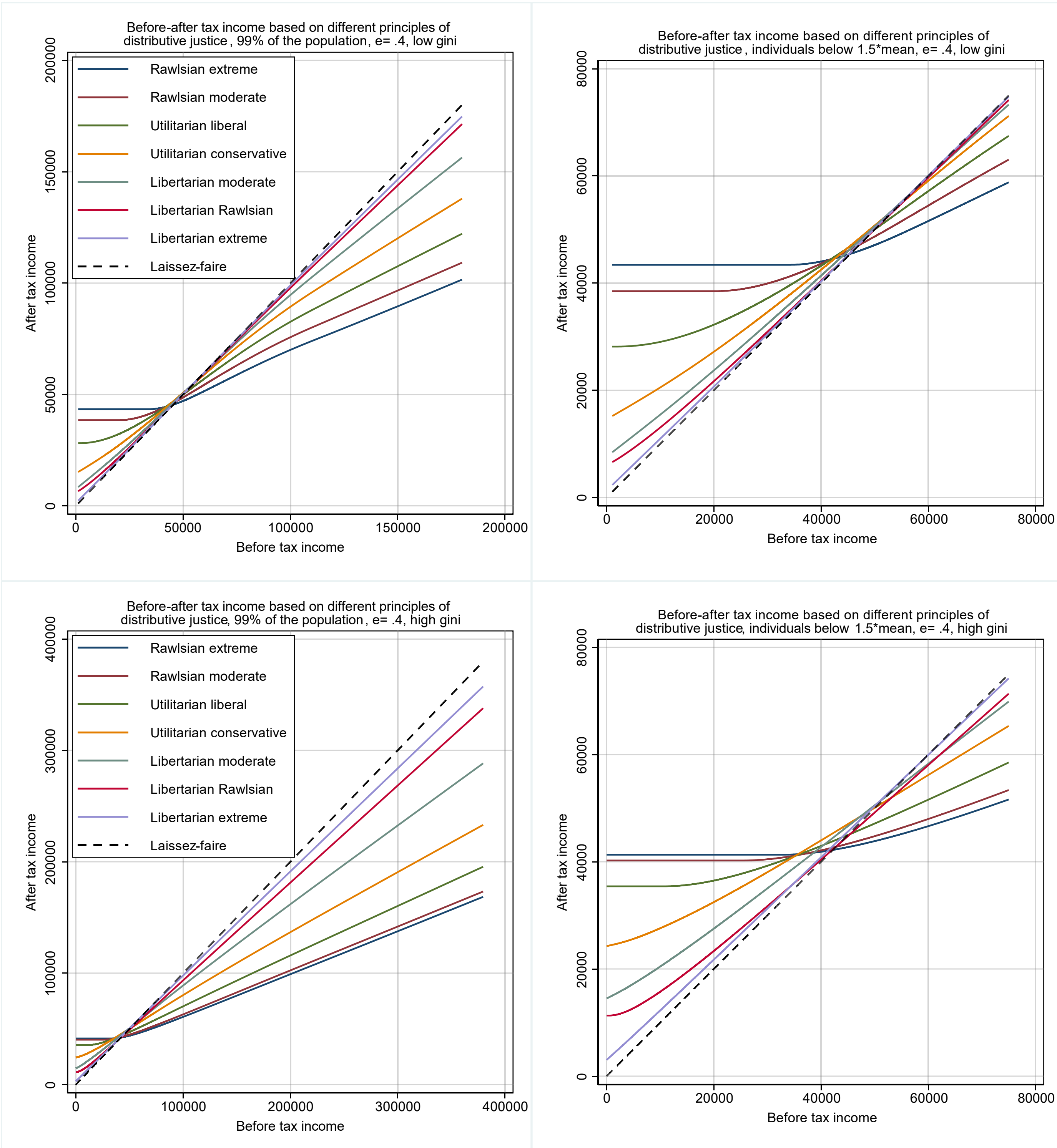


Figure 15 demonstrates how the relationship between before- and after-tax income – or earnings and disposable income – is affected by the degree of equality in the earnings distribution. The graphs on the top show the results related to the equal earnings distribution, whereas graphs on bottom represent the situation for the unequal earnings distribution. The graphs on the left show before- and after-tax incomes for the 99% lowest earners, whereas the graphs on the right only display individuals earning at most 1.5 times the mean of earnings $(1.5 \cdot 50{,}000)$ to make disposable incomes for low and middle earners better visible.

The optimal marginal tax rates are higher overall for the unequal earnings distribution, as was shown in Figure 7, which results in higher tax revenue and therefore more room for redistribution towards the poor. This is reflected in Figure 15, as the demogrant is higher when libertarian and utilitarian optimal tax schemes are applied to the unequal distribution. The demogrant of the Rawlsian extreme tax scheme, however, is lower for the unequal distribution. This is because for the unequal earnings distribution, a larger share of society has an earnings level far below the mean. Hence, redistribution such that an individual reaches the level of the demogrant is relatively more expensive when the distribution of earnings is unequal. These increased costs are the reason that the optimal demogrant is lower for the Rawlsian extreme case when inequality is high, despite the higher total tax revenue. Thus, higher tax revenue benefits redistribution towards the poor when the earnings distribution is unequal, but the size of the demogrant under the tax revenue maximizing policy is constrained due to the increased costs of providing the demogrant to a large share of society.

Figure 16

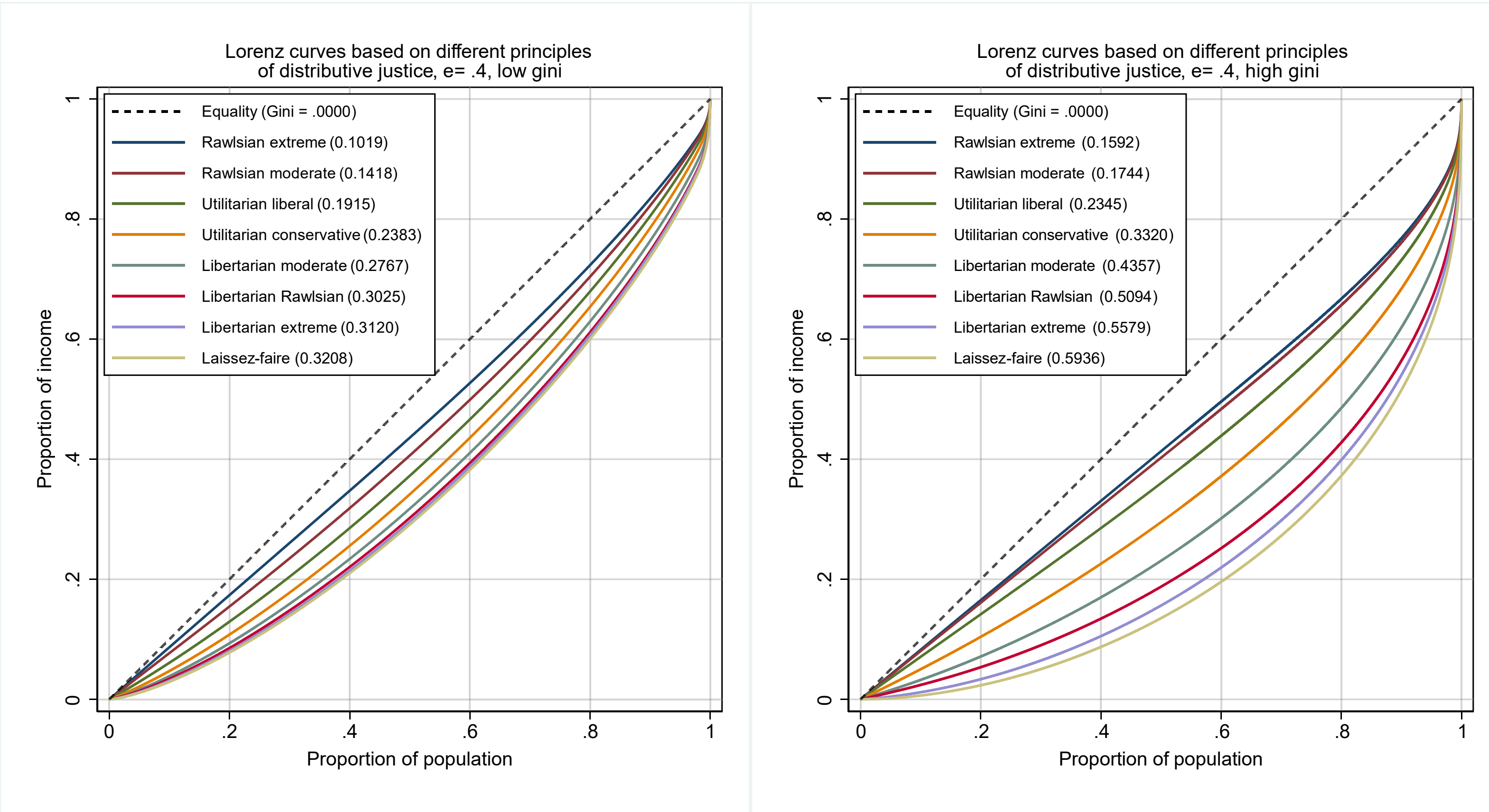
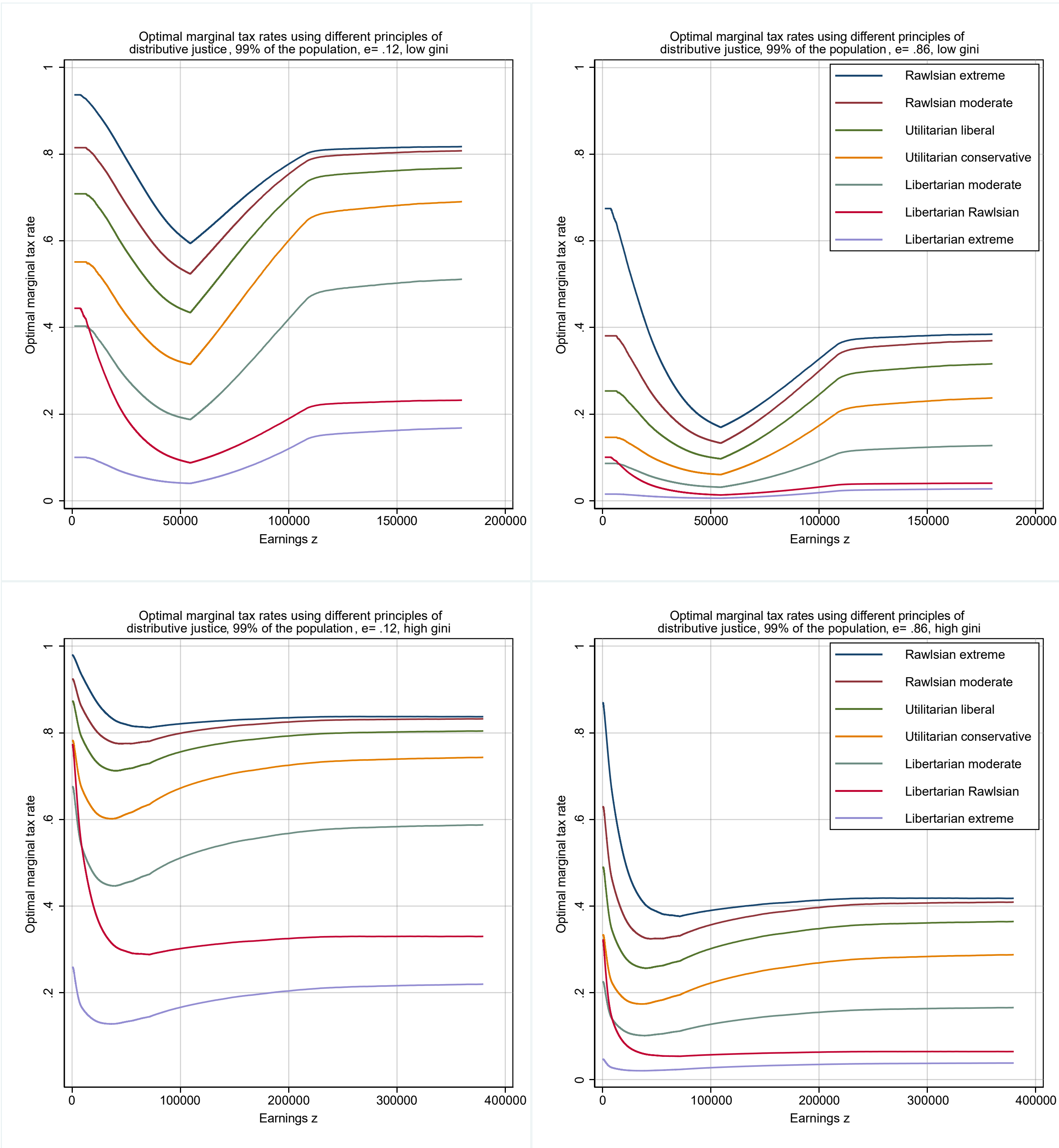


Figure 16 illustrates the difference in inequality reduction using different principles of distributive justice based on an equal and unequal earnings distribution. The graph on the left shows Lorenz curves and respective Gini coefficients for optimal tax schemes applied to a relatively equal earnings distribution, whereas the graph on the right depicts Lorenz curves and respective Gini coefficients according to a relatively unequal earnings distribution. As follows from Figures 7 and 15, inequality reduction is much larger for all principles of distributive justice when the distribution of earnings is relatively unequal, as tax revenue is higher and redistributive policy is progressive. The resulting distributions of disposable income, however, are more equal when the earnings distribution is relatively equal due to the fact that the laissez-faire situation is more equal. These results are very much in line with expectations.

F3

Figure 17



Optimal marginal tax rates using different principles of distributive justice, 99% of the population, e= .12, low gini
Optimal marginal tax rate
Earnings z
0
.2
.4
.6
.8
1
0
50000
100000
150000
200000
Optimal marginal tax rates using different principles of distributive justice, 99% of the population, e= .86, low gini
Rawlsian extreme
Rawlsian moderate
Utilitarian liberal
Utilitarian conservative
Libertarian moderate
Libertarian Rawlsian
Libertarian extreme
Optimal marginal tax rates using different principles of distributive justice, 99% of the population, e= .12, high gini
100000
200000
300000
400000
Optimal marginal tax rates using different principles of distributive justice, 99% of the population, e= .86, high gini

Figure 17 illustrates the potential interaction effects on optimal marginal tax rates when both the level of taxable income elasticity and the level of equality of the earnings distribution vary, using different principles of distributive justice. The graphs in the top row represent the relatively equal earrings distribution of Germany in 1981, whereas the bottom row shows results based on the relatively unequal distribution based on Mexico in 1994. The graphs on the left show optimal marginal tax rates according to a low taxable income elasticity assumption of 0.12, whereas the graphs on the right display rates for the high taxable income elasticity assumption of 0.86. The main results from Sections 5.2 and 5.3 can be reconsidered in Figure 17. In Section 5.2, it is found that an increase in taxable income elasticity results in lower optimal marginal tax rates, especially for middle and high earners. As can be observed in Figure 17, this result holds regardless of the equality level of the distribution of earnings. Section 5.3 demonstrates that as the level of inequality in the earnings distribution grows, optimal marginal tax rates increase. This effect is particularly strong for earners close to the middle of the earnings distribution, resulting in a shift towards L-shaped patterns of optimal marginal tax rates. Figure 17 indicates that this result holds for both low and high levels of taxable income elasticity. Results from Sections 5.2 and 5.3 thus seem consistent when parameters in the model vary, and therefore no general interaction effects can be found in Figure 17.

The extreme cases presented in Figure 17, however, do yield some interesting outcomes. In the bottom left graph, it can be observed that when taxable income elasticity is extremely low and the distribution of earnings is extremely unequal, Rawlsian optimal marginal tax rates become extremely high (consistently over 80%) and the pattern is relatively flat. In the bottom right graph, however, elasticity is extremely high and the pattern of rates becomes very L-shaped. This indicates that the effect of an increase in taxable income elasticity has a larger effect on the shape of the pattern of optimal marginal tax rates for tax revenue maximizing principles when the distribution of earnings is relatively unequal. The reason for this to happen relates to what is described in Section 5.2, that the effect of an increase in elasticity on the optimal marginal tax rates is relatively small when generalized social marginal welfare weights are high and decline steeply for low earners. Since the pattern of especially Rawlsian weights becomes less steep relatively quickly for unequal distributions of earnings, the effect for low to medium earners of an increase in elasticity is large. This is a result which has not been found before, since previous literature only focused on the United States earnings distribution and revolved mostly around utilitarian principles of distributive justice. More research and testing are required to properly examine this finding. To gain better insights, future studies could test interaction effects using a wider variety of earnings distributions. Furthermore, quantitative rather than visual examination of results might help to accurately identify interaction effects.