IN SEARCH OF NEWTON'S NATURAL PHILOSOPHICAL METHOD

AN EVALUATION OF THE METHODOLOGICAL STATEMENTS IN THE *PRINCIPIA* AND *OPTICKS*

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I, Melody van der Mierden, hereby declare and assure, that I have composed this Master's thesis independently, that I did not use any other sources or tools other than indicated, and that I marked those parts of the text derived from the literal content or meaning of other works – digital media included – by making them known as such by indicating their source(s).

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ABSTRACT

Isaac Newton is considered a giant of science for his achievements in mathematics, optics, and celestial mechanics. He distinguished himself from most of his contemporaries by means of his mathematised and empirical method of natural philosophy. Newton's methodological statements elicited controversial responses, both in his own time and in ours. Moreover, there are significant differences and incompatibilities in the contemporary scholarly literature on Newton's method. In this article, I discuss the methodological statements in Newton's *Opticks* and *Principia* – particularly the central terms 'analysis', 'synthesis', 'induction', and 'deduction', and Newton's objective of certainty – and compare four contemporary interpretations of this method. My evaluation will conclude with an endorsement of a historically accurate, but much overlooked, interpretation of Newton's method by Zev Bechler.

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1. INTRODUCTION

Sir Isaac Newton's works on natural philosophy are among the most influential contributions to the scientific compendium, and the method he used to discover and prove his natural philosophical theories is an important part of this. A first insight into this method might be taken from Voltaire, who writes that, in response to the question how he had discovered the theory of universal gravitation, Newton answered: "by thinking on it continually."[1] This strategy can be recognised in the account of Newton's roommate at Cambridge, John Wickins, and that of his niece, Catherine Conduitt, according to whom Newton would forget to eat and sleep when he was working on a problem, staying up all night and leaving his standing food for his cat.[2] As a result, Newton had postulated the gravitational attraction of the moon to the earth and of the planets to the sun in 1669, only one year after he had graduated from Cambridge as a Master of Arts.[3] The completed system would be published nearly twenty years later for the first time – in the *Mathematical Principles of Natural Philosophy* (from now on: *Principia*) (1687) – and almost sixty years later, a year before his death, for the last time.

Alongside with his own thought process, the continuation of the ideas of others played an important role in Newton's study of natural philosophy. In a letter to Robert Hooke, Newton stated: "If I have seen further it is by standing on ye sholders of Giants."[4] As he mentioned to Hooke, René Descartes was such a giant for Newton, both as an example he followed and as one he wished to distinguish himself from.[5] Like Descartes, Newton strove for a more certain method for the study of natural philosophy, and the principal strategy he and Descartes both applied was a mathematisation of their methods. Such an approach was a novelty in the 17th century. The Aristotelian tradition, with its separation between mathematics and natural philosophy, still dominated the academic world, and Cambridge was no exception in this regard.[6] Nevertheless, Newton considered Descartes' philosophy to fall short of the rigorous standard he himself had in mind: an empirical natural philosophy following a strict

* A note on the use of dates in 17th and 18th century texts: whereas on the continent and in Scotland the new year started on the first of January, England used Lady Day (25 March) as the start of the new year until the middle of the 18th century. A dual dating (e.g., 6 February 1671/72) was used between 1 January and 25 March to accommodate the different styles. In this article, I will follow England's dating system, but I will use the dual dating in footnotes when it is used in the original text. See also Isaac Newton, *The Correspondence of Isaac Newton, 1661-1675*, 8 vols., vol. 1, ed. H. W. Turnbull (Cambridge: Cambridge University Press for the Royal Society, 1959), xxvi.

[1] Westfall notes that this anecdote, the earliest mention of which he found in Voltaire's *Élements de la Philosophie de Newton*, is most likely not completely reliable. Richard S. Westfall, *Never at Rest: A Biography of Isaac Newton* (Cambridge: Cambridge University Press, 1980), 105n1.

[2] Ibid., 103-4.

[3] Portsmouth Collection in the University Library, Cambridge, MS Add. 3958.5, fol. 87, in Newton, *Correspondence*, 1:297-301; Westfall, *Never at Rest*, 180.

[4] Newton to Hooke, 5 February 1675/76, in *Correspondence*, 1:416.

[5] Westfall, *Never at Rest*, 96-97, 381.

[6] Ibid., 81-83.

mathematical method in which hypotheses had no place. The metaphysical foundation of Descartes' cosmology and physics, with its invisible particles and its general *res extensa* (extended matter), were hypotheses in Newton's eyes and should therefore not be used in natural philosophy.

Newton's scientific method is described in his two main natural philosophical works: the *Opticks* (1704), on the nature of light and colour, and the *Principia* (1687), on celestial mechanics. The methodological statements in these two works will be the focus of this article. Let us consider therefore, as a preliminary outline, the following two central passages from the *Principia* and *Opticks*, respectively.

> [T]hose whose natural philosophy is based on experiment [...]. [T]hey proceed by a twofold method, analytic and synthetic. From certain selected phenomena they deduce by analysis the forces of nature and the simpler laws of those forces, from which they then give the constitution of the rest of the phenomena by synthesis.[7] (Editor's Preface to the *Principia*, 1713)

> As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition [synthesis[8]]. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction [...]. And the Synthesis consists in assuming the Causes discover'd, and establish'd as Principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanations.[9] (*Opticks*, 1717/18)

[7] Roger Cotes' Editor's Preface to the second edition (based on Newton's descriptions of his method), in Isaac Newton, *Philosophiae Naturalis Principia Mathematica*, 2nd ed., ed. Roger Cotes (Cambridge: Cornelius Crownfield, 1713), [9] (translation by Cohen and Whitman, 386). In this article, we will consider the differences between Newton's methodological statements in the several editions of his *Principia* and *Opticks*. The order of these editions will therefore be relevant, and I have accordingly indicated the details with every quoted passage. To provide a most accurate view of the development of these statements I have supplied every quote with the reference to the edition in which the passage first appears in this form (i.e., if a passage was changed in a later edition, I will refer to this later edition). I have also included an overview of all quoted passages in the appendix. In addition, in every reference to the first or second edition of the *Principia*, I have also included the page number to Cohen and Whitman's translation of the third edition. Isaac Newton, *The Principia: Mathematical Principles of Natural Philosophy*, trans. I. Bernard Cohen and Anne Whitman assisted by Julia Budenz, preceded by "A Guide to Newton's Principia" by Bernard I. Cohen (Berkeley: University of California Press, 1999).

[8] The terms 'analysis' and 'synthesis' are also known under their Latin equivalents 'resolution' and 'composition'. I will, as a rule, refer to them by their Greek terms and supplement these when the Latin equivalents are used in the literature.

[9] Query 31, in Isaac Newton, *Opticks: Or, a Treatise of the Reflections, Refractions, Inflections and Colours of Light*, 2nd English ed. (London: W. and J. Innys, printers to the Royal Society, 1718), 380. Newton published both English and Latin editions of his *Opticks* and numbered them separately. While the first Latin is a translation of the first English edition, Newton separately made revisions in both the

The interpretation of these methodological statements has proven to be challenging. In the contemporary scholarly literature, significantly different explanations are presented that diverge with regard to three aspects: (a) the meaning of the key terms Newton used to describe his method: ‘analysis’, ‘synthesis’, ‘induction’, and ‘deduction’, (b) the interrelations between these components (i.e., whether they are part of the same method or separate methods), and (c) the relative importance of the components within the method (i.e., whether they are equally essential). Conspicuously, there is no literature that compares these different views of Newton’s methodological statements and terminology, even though such a comparison would be useful in obtaining insight in some of the more elusive aspects of the method and the difficulties of its interpretation. This is the aim of the present article: to consider Newton’s descriptions of his natural philosophical method in the *Principia* and *Opticks*, and present, and in doing so, compare, a number of interpretations of the method by contemporary authors. I have therefore selected four critical interpretations, provided by James Garrison, Jaakko Hintikka & Unto Remes, Alan Shapiro and Zev Bechler. They each provide a comprehensive discussion of Newton’s methodological statements, while considering them from distinct perspectives, which enables us to constructively evaluate their interpretations.

Shapiro and Bechler have specialised on Newton’s methodology, while Garrison and Hintikka & Remes mainly pursue different subjects in the history of philosophy. The latter two take the geometrical origin of Newton’s method of analysis and synthesis into close consideration, and especially Garrison regards Newton’s statements as accurate descriptions of a uniform and stable method.[10] Shapiro, on the other hand, contests the trustworthiness of Newton’s methodological pronouncements and contrasts them with other passages from his correspondence and unpublished works to demonstrate that the method was not stable and uniform, as Garrison claims.[11] He argues that it underwent a development – particularly with regard to Newton’s objective of certainty – which is masked by the generality of his methodological statements.[12] In response to Shapiro’s interpretation, we will also briefly consider part of Kirsten Walsh’ reading of the method. She refutes Shapiro’s claim that Newton’s objective of certainty underwent a development and her arguments are relevant for our last interpretation, that of Zev Bechler. Bechler considers the methodological statements

Latin and English editions, and in this regard they may be viewed as independent editions – as opposed to the Latin editions being mere translations of the revised English editions.

[10] Jaakko Hintikka and Unto Remes, *The Method of Analysis: Its Geometrical Origin and its General Significance*, Boston studies in the philosophy of science, (Dordrecht: Reidel, 1974), 105; James W. Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," *Journal of the History of Ideas* 48, no. 4 (1987): 627, https://doi.org/10.2307/2709690.

[11] Alan E. Shapiro, "Newton’s ‘Experimental Philosophy’," *Early Science and Medicine* 9, no. 3 (2004): 186.

[12] Ibid., 190.

in relation to a number of objections to the method. He too draws the conclusion that Newton's method and objective of certainty underwent a development, though in a very different manner than Shapiro argues.[13] Bechler's interpretation, although much overlooked in recent Newtonian literature, offers a historically reliable representation of Newton's views and portrayal of his method. It incorporates the more subtle differences of the methodological statements in the different editions of Newton's works and thereby presents a comprehensive reading of the methodological pronouncements.

To compare these different interpretations, we will first, in chapter two, consider a general outline of Newton's methodological statements and the early development of the method. This will include a consideration of the four methodological components and their relation to other relevant aspects, among which Newton's objective of certainty. In chapter three, the aforementioned interpretations of Newton's method, by Garrison, Hintikka & Remes, Shapiro and Bechler, are considered. And in the conclusion, we will evaluate the differences and similarities between these interpretations and reflect on their implications.

Note that the present objective is to obtain insight into Newton's methodological statements, not to ascertain how the method functioned in practice. The latter will occasionally be discussed, but only when this is relevant for understanding the methodological statements.

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[13] Zev Bechler, *Newton's Physics and the Conceptual Structure of the Scientific Revolution* (Dordrecht: Kluwer Academic Publishers, 1991), 361.

2. NEWTON'S METHOD

The foundations of Newton's natural philosophical method, his interest in mathematics and his views on methodological certainty were established during his studies and early career. I will outline this early development and discuss the method and its methodological components to provide a context for the interpretations of Newton's method in the next chapter.

Three years after Newton was admitted into Trinity College in 1661, he was introduced to the study of mathematics.[14] Isaac Barrow, the first Lucasian professor of mathematics, who had started his inaugural series of lectures in March 1664, presumably had a role in the development of Newton's interest in this field of study.[15] Newton applied for a scholarship in mathematics in April, and when he was granted this, his mathematical studies started in earnest.[16] Only a year later he had begun experimenting with his own mathematical and geometrical ideas, challenging and expanding the views he had just introduced himself to.[17] The knowledge he acquired and developed in this period would form the basis for his pursuit of certainty in natural philosophy. During the following years, in his research on optics and celestial mechanics, Newton routinely emphasised the importance of his mathematical approach for a secure natural philosophical method. He thereby distinguished himself from almost all of his contemporaries. In a series of lectures on optics written between 1670 and 1672, Newton formulated his mathematical approach and his objective of certainty as follows:[18]

> I hope to show – as it were, by my example – how valuable mathematics is in natural philosophy. I therefore urge geometers to investigate nature more rigorously, and those devoted to natural science to learn geometry first. […] But truly with the help of philosophical geometers and geometrical philosophers, instead of the conjectures and

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[14] Mathematics was not yet part of the curriculum when Newton started his studies, and also during his previous education, the Grammar School in Grantham, he was not taught any mathematics. Westfall, *Never at Rest*, 56-58, 81-83.

[15] Ibid., 99.

[16] Ibid., 105. At the time of his application for the scholarship, the extent of Newton's mathematical expertise consisted of his having read Descartes' *Geometry*, however, without reading any of the expected preparatory works (such as Euclid's *Elements*). This can also explain why Barrow was not particularly impressed by his interview with Newton, even though its result was successful. In addition, it should be noted that Newton's application for the scholarship was not solely motivated by his interest in mathematics. Newton had, for financial reasons, already been preparing to apply for a scholarship at this time. Westfall, *Never at Rest*, 99-102.

[17] Westfall, *Never at Rest*, 106.

[18] In 1669, Newton had been named the second Lucasian professor of mathematics – following Isaac Barrow – for which purpose he developed these optical lectures. Westfall, *Never at Rest*, 180, 206.

probabilities that are being blazoned about everywhere, we shall finally achieve a natural science supported by the greatest evidence.[19] (*Optical Lectures*, 1670-72)

Newton presented his mathematical approach to natural philosophy here as an alternative capable of providing a degree of certainty which exceeds that of the "conjectures and probabilities" offered by the methods of others.[20] Among these others were Robert Hooke and Robert Boyle, who considered Newton's standard of certainty inapplicable, since, they argued, no more than a probabilistic evaluation could be achieved in natural philosophy.[21] For example, Boyle explains that it is, "in experiments where we are dealing with gross matter, […] almost impossible to obtain (unless sometimes by accident) a mathematical exactness."[22]

Another important philosopher Newton likely had in mind when composing the passage above is René Descartes.[23] Newton had actively engaged with Descartes' mechanical philosophy and mathematical ideas during his studies, but he soon started to reject his views more and more. This shift toward an anti-Cartesian position was instrumental in the development of Newton's own distinctive mathematical, methodological and natural philosophical views.[24] In 1669-1670, during a project to revise and annotate Gerard Kinckhuysen's *Algebra* – a leading introductory work on (among others) Descartes' algebra – Newton began to reflect on Descartes' method and its relation to ancient analysis.[25] This project set Newton to study the ancient, geometrical method of analysis and synthesis, which would become essential for his own natural philosophical method.[26]

The geometrical method of analysis and synthesis, which had influenced Descartes' methodology, later also influenced Newton's.[27] It was already used by ancient Greek geometers, among whom Euclid, Apollonius of Perga and Aristaeus the Elder, and was documented by the Greek mathematician Pappus of Alexandria in circa 300 AD. As the name suggests, the method of analysis and synthesis is a twofold method: the analytic branch is used for the discovery of theories and the solutions of problems, and the synthetic branch for

[19] Isaac Newton, *The Optical Papers of Isaac Newton.* Vol. 1, *The Optical Lectures 1670-1672*, ed. and trans. Alan E. Shapiro. (Cambridge: Cambrdige University Press, 1984), 86-89.
[20] Ibid.
[21] Niccolò Guicciardini, *Isaac Newton on Mathematical Certainty and Method* (Cambridge, Massachusetts: MIT Press, 2009), 23.
[22] Robert Boyle, *The works of the Honourable Robert Boyle*, A new edition ed., 6 vols., vol. 5 (London: W. Johnston, S. Crowder, T. Payne, G. Kearsley, J. Robson, B. White, T. Becket et al., 1772), 480.
[23] Guicciardini, *Mathematical Certainty and Method*, 21-23; Niccolò Guicciardini, *Isaac Newton and Natural Philosophy* (London, UK: Reaktion Books Ltd, 2018), 83.
[24] Westfall, *Never at Rest*, 381; Guicciardini, *Mathematical Certainty and Method*, 15-16.
[25] Christoph J. Scriba, "Mercator's Kinckhuysen-Translation in the Bodleian Library at Oxford," *The British Journal for the History of Science* 2, no. 1 (1964): 45; Westfall, *Never at Rest*, 222-23.
[26] Guicciardini, *Mathematical Certainty and Method*, 14.
[27] Howard Duncan, "Descartes and the Method of Analysis and Synthesis," in *An Intimate Relation: Studies in the History and Philosophy of Science*, ed. I. R. Brown and I. Mittelstrass (Dordrecht: Springer, 1989), 67; Guicciardini, *Mathematical Certainty and Method*, 35.

their demonstration and proof. Pappus gave the following description of the method in his *Collection*:

> Analysis is the way from what is sought – as if it were admitted – through its concomitants in order to something admitted in synthesis. For in analysis we suppose that which is sought to be already done, and we inquire from what it results, and again what the antecedent of the latter is, until we on our backward way light upon something already known and being first in order. And we call such a method analysis, as being a solution backwards. In synthesis, on the other hand, we suppose that which was reached last in analysis to be already done, and arranging in their natural order as consequents the former antecedents and linking them one with another, we in the end arrive at the construction of the thing sought.[28]

Let us paraphrase Pappus' description of this method: in the analysis, a theorem is postulated from which the precursory steps are deduced until already established truths or axioms are reached.[29] There are two possible results of the analysis: either the geometer will not be able to arrive at any axioms or established truths, or he will be able to do so. In the first case, the theorem is proven false or impossible and can be discarded. In the second case it is provisionally accepted and the geometer can proceed to the synthesis: arguing – now in reverse order – from established truths and axioms to the postulated theorem. If the geometer succeeds in deriving the theorem from the already known truths and axioms, it is proven.

The purpose of the method is to discover and prove geometrical theorems. The analysis is instrumental in identifying the steps from the conclusion to the premise. And the synthesis is consequently performed to establish the truth of the premises uncovered in the analysis.

An adaptation of the method of analysis and synthesis was required to enable its use in natural philosophy: instead of geometrical data, Newton deduced the causes of natural phenomena (e.g., the force of gravity) from experimental and observational data (e.g., motions

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[28] Pappus, *Collection*, 7.634.11-22, in, and translated by, Hintikka and Remes, *The Method of Analysis*, 8-9. (With a minor change: I have moved "is" in the second sentence, which I presume is erroneously placed: "and again what ~~is~~ the antecedent of the latter [is].") Also, Pappus' clarification of the term 'analysis' "as being a solution backwards" refers to the Greek word for analysis: ἀνάπαλιν λύσις, which can be transliterated as 'backward or reverse solution'.

[29] I use the term 'deduced' here, but Pappus does not explicate the type of reasoning that is used in the method. Mäenpää clarifies that it is topic of discussion among contemporary interpreters whether 'deduction' or 'reduction' is used. Nevertheless, Mäenpää explains, "almost all examples of analysis in the Greek mathematical corpus are in fact deductions." Petri Mäenpää, "From Backward Reduction to Configurational Analysis," in *Analysis and Synthesis in Mathematics*, ed. Michael Otte and Marco Panza (Dordrecht: Kluwer Academic Publishers, 1997), 201.

of the planets) in the analysis and consequently explained and proved these deductions by deriving the phenomena from their causes in the synthesis.

Now that we have a better understanding of the first two terms, 'analysis' and 'synthesis', we can take another look at the methodological statements of the *Principia* and *Opticks*, which were quoted in the introduction:

> [T]hose whose natural philosophy is based on experiment [...]. [T]hey proceed by a twofold method, analytic and synthetic. From certain selected phenomena they deduce by analysis the forces of nature and the simpler laws of those forces, from which they then give the constitution of the rest of the phenomena by synthesis.[30] (*Principia*, 1713)

> As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition [synthesis]. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction [...]. And the Synthesis consists in assuming the Causes discover'd, and establish'd as Principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanations.[31] (*Opticks*, 1717/18)

Analysis is described here as a method that reasons from experimental and observational data to derive causes and establish them as principles – such as the forces and laws of nature. Synthesis is consequently used to prove these principles and to explain other phenomena by means of them. The terms 'deduction' and 'induction' are presented in relation to the method of analysis and synthesis as the type of reasoning employed in the method. We will consider a number of descriptions of both terms in Newton's works.

In a letter to Henry Oldenburg, Newton clarified his understanding and use of deductive reasoning and how it differed from that of others.[32]

> [T]he proper Method for inquiring after the properties of things is to deduce them from Experiments. And I told you that the Theory wch I propounded was evinced to me, not by inferring tis thus because not otherwise, that is not by deducing it onely from a

[30] Editor's Preface, in Newton, *Principia*, 2nd ed. (1713), [9] (trans. C&W, 386).

[31] Query 31, in Newton, *Opticks*, 2nd English ed. (1718), 380-81.

[32] Henry Oldenburg was the secretary of the Royal Society and editor of the *Philosophical Transactions*, in which Newton's optical paper (which is the subject of this letter) was published. He also initially served as an intermediary for the correspondence between Newton and, among others, Robert Hooke, Christiaan Huygens and Ignace-Gaston Pardies, and was therefore intimately involved in various discussions regarding Newton's works and method. In the letter quoted here, Newton responds to a question of Oldenburg relating to such a discussion.

confutation of contrary suppositions, but by deriving it from Experiments concluding positively & directly.[33] (Newton to Oldenburg, 1672)

Newton specified his use of deduction here as a direct and affirmative derivation, as opposed to disproving alternatives. In the *Principia* and *Opticks*, respectively, Newton's understanding of the term is further clarified in the following passages: "we *derive* [*derivantur*] from celestial phenomena the gravitational forces by which bodies tend toward the sun and toward the individual planets. Then the motions from the planets, the comets, the moon, and the sea are *deduced* [*deducuntur*] from these forces" (*Principia*, 1687).[34] And: "the main Business of Natural Philosophy is to *argue from Phaenomena* without feigning hypotheses, and to *deduce* causes from effects" (*Opticks*, 1717/18).[35] To summarise, Newton's deduction is a derivation from phenomena, experiments or observations in a direct and affirmative manner: propositions are derived directly from the phenomena, without mediation by a process of disproving contrary suppositions.[36]

The remaining term, 'induction', occurs only in a few passages in the *Principia* and *Opticks* and is there consistently described in terms of a generalisation of propositions deduced from phenomena. In the *Principia* (1713), Newton states: "in this experimental philosophy, propositions are deduced [*deducuntur*] from the phenomena and are *made general by induction* [*per Inductionem*]."[37] Similarly, in the *Opticks* (1717/18) he writes: "*drawing general Conclusions* from them ['Experiments and Observations'] *by Induction*."[38] Hence, by the term 'induction' Newton refers to the generalisation of empirically deduced propositions. It remains to be determined how he viewed this generalisation. As will be demonstrated in the next chapter, 'induction' appears to be one of the more elusive terms in Newton's method, even though it is described with such consistency.

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[33] Newton to Oldenburg, 6 July 1672, in *Correspondence*, 1:209.

[34] Author's preface to the first edition, in Isaac Newton, *Philosophiae Naturalis Principia Mathematica* (Cambridge: Cornelius Crownfield, 1687), [5] (trans. C&W, 382) (italics and additions mine).

[35] Query 28, in Newton, *Opticks*, 2nd English ed. (1718), 344 (italics mine).

[36] For ease of reading, I will regard the terms 'phenomena', 'experiments' and 'observations' in this article as part of the same group of 'empirical input'. This is somewhat of a simplification, though not one that interferes with the arguments we will consider. Newton uses these terms in the same context and regularly combines two of them (mostly 'experiments' with 'observations', but also 'experiments' with 'phenomena') or substitutes one for another (e.g., "if no *Exception occur from Phaenomena*, the Conclusion may be pronounced generally. But if at any time afterwards any *Exception* shall *occur from Experiments*…" Newton, *Opticks*, 2nd English ed. [1718], 380 [italics mine]). There are however, authors who argue that Newton deliberately uses certain terms in certain passages, see for example, Guicciardini, *Mathematical Certainty and Method*, 317; Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 617.

[37] General Scholium, in Newton, *Principia*, 2nd ed. (1713), 484 (trans. C&W, 943) (italics and additions mine).

[38] Query 31, in Newton, *Opticks*, 2nd English ed. (1718), 380 (italics and additions mine).

3. Interpretations of the method

We have considered the early development of Newton's method and a preliminary discussion of its four methodological components in the previous chapter. Now, we will discuss and compare the selected interpretations of Newton's method, by Garrison, Hintikka & Remes, Shapiro, and Bechler. Three aspects of Newton's methodological statements and their interpretations will be considered specifically: firstly, the meaning of the terms 'analysis', 'synthesis', 'induction', and 'deduction', secondly, the importance assigned to each of them and lastly, their connection to the other three terms.

3.1 James Garrison

As mentioned in the introduction, Garrison is not a specialist of Newtonian studies, but he has written a number of articles on, or relating to, Newton's method and offers a valuable contribution to our present comparison by his approach of the method from the perspective of analysis and synthesis. Garrison argues that Newton's method is "guided and prefigured entirely by the geometrical method of analysis and synthesis" and emphasises that it "remained unchanged throughout his career."[39] He explains that this method proceeded by deductive reasoning and that Newton employed the method of analysis and synthesis as described by Pappus, but that he made two changes to the geometrical analysis in order to adapt it to the domain of natural philosophy.[40]

With the first modification, Newton replaced the *geometrical* instantiation with which the analysis begins in the original method, by an *empirical* instantiation. To understand the role of the original instantiation and Newton's adaptation of it, we will review Pappus' outline of the method cited above (on page 11). In Pappus' description, the geometrical instantiation is part of the method's first step: the geometer postulates a theorem, for example concerning the relation between the angles of a figure and the placement of its lines. Hereby he geometrically instantiates the figure, that is, he constitutes the drawn figure as a representative of its abstract form – the abstraction of a triangle, for example. Newton's modification consisted in adapting the type of representative and abstract form: instead of a particular *figure* functioning as the geometrical instantiation of an abstract *figure*, in natural philosophy an *experiment* functioned as the empirical instantiation of a *theorem*.[41]

The second modification concerns the implementation of inductive reasoning into the analysis. Garrison explains Newton's meaning of induction in relation to his concept of the

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[39] Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 620, 622.

[40] James W. Garrison, "Hintikka, Laudan and Newton: An Interrogative Model of Scientific Inquiry," *Synthese* 74, no. 2 (1988): 167; Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 620.

[41] Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 620-21.

experimentum crucis, which is a single, well-conceived experiment that by itself suffices to prove a theorem. To understand this relation, we will consider Newton's optical paper of 1672 in which he proved by means of his *experimentum crucis* that "Light it self is a *Heterogeneous mixture of differently refrangible Rays.*"[42] Newton described the experiment as follows: he took two boards, with a small hole in each of them, and placed the first board close behind a prism that he held in front of the window of an otherwise darkened room. The light refracted by this prism was projected through the hole and onto the second board behind which another prism was placed. Newton then slowly turned the first prism around its axis, so that the different colours of the refracted light projected on, and through the second board, where they would be refracted again by the second prism. While the light projected through the first prism appeared as a spectrum of colours on the second board, only that colour that proceeded through the hole and second prism was visible on the wall, and remained the same shade when projected onto different parts of the wall.[43] The significance of this result becomes apparent when we consider how light and colours were understood at this time. It was assumed that the colours made visible by a prism were either added *by* the prism, or appeared because the rays of light – which were thought to be simple, uniform, homogeneous and directed straightforward – were skewed by reflecting off of the prism and only perceived as different colours based on which part of the eye they came upon (red for the bottom of the eye, blue for the top of the eye, or a mixture of those for the parts in between).[44] The result of Newton's *experimentum crucis* was incompatible with these views, and demonstrated according to Newton that white light is compounded of a mixture of differently refrangible rays which are dispersed in different directions.[45]

Newton argued that this experiment sufficed to prove the heterogeneity of white light and that hence this result could be generalised to all similar experiments, without the need for it to be repeated in order to prove it. In this sense, Garrison asserts, can the role of induction be understood: the generalisation of the result of a single crucial experiment.

> For Newton induction was neither elimination or enumeration [i.e., generalisation of a collection of phenomena on the basis of their differences or similarities, respectively]. Newton explicitly defends his *experimentum crucis* against the "many hundreds of

[42] (cursive in original) Newton to Oldenburg, 6 February 1671/72, *Correspondence*, 1:95.
[43] Ibid., 94-95.
[44] Newton to Oldenburg, 6 February 1671/72, *Correspondence*, 1:95. This last theory is offered by Robert Hooke in his *Micrographia*, as he discusses in his letter to Oldenburg, 15 February 1671/72, Newton, *Correspondence*, 1:112.
[45] Hooke to Oldenburg, 15 February 1671/72, Newton, *Correspondence*, 1:112.

trials" of Hooke. All induction meant for Newton was the generalizing step, pronouncing the universal contained in the experimental particular generally.[46]

In other words, the result of a single experiment – an "experimental particular"– is generalised by induction into a "universal [result]".[47] The outcome of a crucial experiment therefore 'contains' the universal, as Garrison states. Hence, "[r]epetitions of an experiment were no more necessary for Newton than repetitions of a geometrical configuration would be for the geometrician."[48] If a single, crucial experiment is constructed its result can be generalised by induction to arrive at a universal outcome, without the need to repeat the experiment or perform similar experiments. This modification was needed in natural philosophy but not in geometry, because, Garrison explains, Newton held a constructivist view of geometry. Hence, for Newton, geometrical knowledge differed from natural philosophical knowledge, because the latter "is obtained from nature or God, and hence is beyond our privileged knowledge."[49] We *are* able to obtain certain geometrical knowledge, because this knowledge is the product of our own construction.[50]

To summarise, Garrison explains Newton's method is a modified form of the geometrical method of analysis and synthesis in which an empirical instantiation and inductive reasoning are incorporated to adapt it to the domain of natural philosophy.

3.2 Jaakko Hintikka & Unto Remes

A similar interpretation of Newton's methodological terms is offered by Jaakko Hintikka & Unto Remes, who discuss Newton's method in the concluding chapter of their book on the geometrical method of analysis and synthesis. Both authors are specialised in other fields than Newtonian scholarship. Jaakko Hintikka has made important contributions to logic and published numerous works on analytical philosophy. Unto Remes has written several books and chapters on the geometrical method of analysis and synthesis and its use in Antiquity.

Hintikka & Remes argue that the geometrical method of analysis and synthesis was used by the ancient geometers (and as such described by Pappus) to identify interdependencies in geometrical configurations – for example, a relation between the degrees of angles in a triangle.[51] The function of the method is to discover and prove

[46] Garrison, "Hintikka, Laudan and Newton: An Interrogative Model of Scientific Inquiry," 168.
[47] Ibid. (addition added).
[48] Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 621.
[49] Ibid.
[50] Ibid., 619.
[51] In their book, Hintikka & Remes distinguish two interpretations or uses of the original method: 'analysis of configurations' and 'directional analysis'. The first type is used to 'disentangle' the simpler aspects and components of a complex figure or situation until established truths or axioms are reached. They argue that this is how the ancient geometers and Pappus use the method, as well as Galileo and

geometrical theorems. In the same manner, they explain, is the method used by Newton: to identify interdependencies in and between natural phenomena: “Newton was trying to analyse an experimental situation in the same way as a Greek geometer like Pappus was trying to analyse a figure in the sense of trying to establish the interrelations of its several parts.”[52]

Like Garrison, Hintikka & Remes explain induction as an addition to the method of analysis, by which the outcome of the experiment or phenomenon examined is generalised: “for Newton induction meant just the generalizing step. Experimental conclusions are ‘rendered general by induction’.”[53] Hintikka & Remes also draw a connection between the function of induction and the concept of the *experimentum crucis*, arguing that the outcome of a single, crucial experiment can be generalised without the need for repetitions.[54] In this manner, both Garrison and Hintikka & Remes describe Newton’s concept of induction as “more anti-inductivist than inductivist”.[55] By which they mean that it is not used in a ‘traditional’ sense of generalising a collection of phenomena by means of induction by enumeration or elimination.[56] Additionally, Hintikka & Remes emphasise that it has a minor role in the method: “[i]nduction is but one of the steps in the Newtonian scheme, and a comparatively trivial one at that.”[57] Their interpretation of the relation between analysis, synthesis and induction can be schematised as follows:[58]

Analysis	Identifying the parts and factors of an experiment or phenomenon.
	Examining the relations of interdependencies among these parts.
	Generalising the relationships to all similar phenomena (i.e., induction).
Synthesis	Explaining and predicting other phenomena by means of the general laws uncovered in the analysis.

Besides the implementation of inductive reasoning into the analysis, Hintikka & Remes argue that Newton also modified the synthesis: “[f]or Newton, synthesis meant simply putting together new, often more complex configurations by means of the general laws which had been uncovered in the analysis and generalized in the induction.”[59] Hence, the function of the

Newton. The second interpretation is the *directional or propositional analysis*: an analysis of the successive steps between the postulated premise and the established truths or axioms. Hintikka & Remes argue that this is how Aristotle used the method and how it was employed in the medieval scholastic tradition. Hintikka and Remes, *The Method of Analysis*, 31-32, 106-07.

[52] Ibid., 106.

[53] Ibid., 111.

[54] Ibid., 110-11.

[55] Hintikka and Remes, *The Method of Analysis*, 111; Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 621.

[56] Hintikka and Remes, *The Method of Analysis*, 110-11.

[57] Ibid., 110.

[58] Ibid., 109-10.

[59] Ibid., 109.

synthesis in the geometrical method – proving the result of the analysis by retracing its steps – is replaced by a "deductive application of these general laws [the outcome of the analysis] to explain and to predict other situations."[60] Hintikka & Remes describe Newton's concept of deduction in general terms: "inferred, derived, or, as he sometimes put it, 'deduced' from phenomena."[61]

To summarise, Hintikka & Remes argue that Newton used the method of analysis and synthesis to identify relations of interdependencies in experiments or phenomena and generalised these relations by means of inductive reasoning. The outcome of the analysis is then used in the synthesis – in contrast to its role in the geometrical method – to explain and predict other phenomena by means of deduction.

3.3 Alan Shapiro

Our third interpretation, Alan Shapiro's, provides an insightful contrast to that of both Garrison and Hintikka & Remes, particularly with regard to his interpretation of the relation between the terms 'analysis', 'synthesis', 'induction', and 'deduction'. Shapiro has studied Newton's natural philosophy over the last half century and has published various books and articles on, among others, Newton's optical works and experimental method. In the article that we will primarily focus on, "Newton's 'Experimental Philosophy'," Shapiro explores the meaning of Newton's methodological terms and, particularly, his description of natural philosophy in terms of 'experimental philosophy'.

Shapiro argues that Newton had two natural philosophical methods: the method of the *Opticks*, which was characterised by the terms 'analysis' and 'synthesis', and the method of the *Principia*, which was characterised by 'induction' and 'deduction'. In the second English edition of the *Opticks* (1717/18), Newton combined the descriptions of these separate methods and used all four terms to describe them as a unified, single method of natural philosophy.

We will begin our consideration of Shapiro's reading of the method with his discussion of the statements from the early, separate methods, for which he considers the following two methodological accounts from the early editions of the *Opticks* and *Principia*:

> As Mathematicians have two Methods of doing things which they call Composition [synthesis] & Resolution [analysis] & in all difficulties have recourse to their method of resolution before they compound so in explaining the Phaenomena of nature the like methods are to be used.[62] (*Opticks*, 1704)

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[60] Ibid., 110 (addition added).
[61] Ibid.
[62] This passage is from the draft preface for the 1st English edition of the *Opticks* (1704). It is later published – in slightly changed form – in the 1st Latin edition (1706) as part of query 23 (which is later,

> In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method.[63] (*Principia*, 1713)

In line with both Garrison and Hintikka & Remes, Shapiro explains that Newton derived his method of the *Opticks* from the ancient geometers. He argues that the function of analysis is to "uncover elements or causes" and that of the synthesis to "explain the causes of other phenomena" by means of "conclusions drawn from the observations."[64] Although more general, this interpretation is compatible with that of Hintikka & Remes – except that Shapiro separates induction from analysis and synthesis.

The method of the *Principia* is characterised by 'deduction' and 'induction'. According to Shapiro, Newton used these terms without a specific, technical meaning in mind.[65] To demonstrate this, he considers Newton's use of the method in deriving the properties of bodies, the laws of motion and the law of gravity – which Newton mentions in the passage of the General Scholium that is quoted above. According to Shapiro, these properties and laws are so heterogeneous that it is impossible to individuate a single method by which they were found. Hence, Shapiro argues, "it is apparent that he [i.e., Newton] had to be using 'induction' and 'deduction from the phenomena' in a general way."[66] Shapiro further explains that by the

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in the 2nd English edition, renumbered as query 31). Portsmouth Collection in the University Library, Cambridge, MS Add. 3970, fol. 480v, quoted in Shapiro, "Newton's 'Experimental Philosophy'," 195-96 (square brackets added).

[63] Newton, *Principia*, 2nd ed. (1713), 484 (trans. C&W, 943). Earlier we have considered another methodological statement from the 2nd edition of the *Principia* (1713) – from Roger Cotes' Editor's Preface – in which 'analysis' and 'synthesis' were mentioned. Shapiro considers this Editor's Preface, and Newton's correspondence about it with Cotes, but not its apparent contradiction to his claim that the methods – analysis and synthesis on the one hand, and induction and deduction on the other – were only merged in 1717/18. He does note elsewhere that "Newton played no direct role in the composition of the preface other than providing some general guidelines to Cotes." Shapiro, "Newton's 'Experimental Philosophy'," 208. Indeed, Newton writes at the end of his answer to Cotes' questions about the preface: "If you write any further Preface, I must not see it. for [*sic*] I find that I shall be examined about it." Isaac Newton, *The Correspondence of Isaac Newton, 1709-1713*, 8 vols., vol. 5, ed. A. Rupert Hall and Laura Tilling (Cambridge: Cambridge University Press, 1975), 400. We will consider this apparent contradiction further in the conclusion.

[64] Shapiro, "Newton's 'Experimental Philosophy'," 194, 198.

[65] Ibid., 211.

[66] Ibid., 213 (addition added). Briefly summarised, Shapiro's comparison of the methods of deduction and induction by which these three outcomes are concluded is as follows: (1) he argues that the universal characteristics of bodies (impenetrability, mobility and impetus) are the result of 'transdiction' or 'transduction', which is the extension of the observable properties and laws of bodies, to their unobservable parts; (2) he argues that Newton's argumentation for the law of gravity is "so complex that contemporary philosophers of science cannot agree on the method that Newton used," but that it is different from (3) the method by which he deduced the laws of motion: Shapiro explains that in Newton's argumentation for the first law of motion, "Newton is not really presenting evidence, but examples or phenomena that this law could explain." Ibid., 213-15.

phrase 'deduction from the phenomena', Newton "meant only to set forth the general idea that his conclusions were grounded on or argued from phenomena" and hence that they do not depend on hypotheses (i.e., non-empirical propositions).[67] In addition, Shapiro argues that Newton's term of 'induction' referred to the generalisation of empirical propositions. He explains that Newton indicated in two unpublished passages that he used inductive reasoning to derive a general result from a selection of experiments or phenomena. The first passage is from a draft letter Newton addressed to his editor, Roger Cotes, in which he discusses the passage in the General Scholium quoted above. Newton used a slightly different phrasing here to describe induction: instead of "made general by induction" he writes "draws general conclusions from the consent of phaenomena."[68] Shapiro explains that by the 'consent of phenomena' Newton "presumably means that the more phenomena that agree with or can be explained by the conclusion (property, principle, proposition, etc.) the more general it will be."[69] He argues that this is confirmed in another passage: a draft preface to the *Principia* written between 1716 and 1718, where Newton gave the following clarification of induction: "an argument by induction […] the more numerous the experiments or phenomena from which it is deduced the stronger it proves to be."[70] Hence, Shapiro argues, Newton's induction is a generalisation of multiple phenomena or experiments.

Shapiro's explanation of induction directly opposes that of Garrison and Hintikka & Remes, who defined it as the generalisation of a *single* experiment or phenomenon. While the two alternative descriptions of the concept, to which Shapiro refers, appear quite unambiguous, it must be taken into account that they are both from draft material and do not appear in any published works or sent correspondence. It is significant that Newton was so consistent in describing induction, using the same phrasing throughout the different works and editions, and that these clarifying alternative descriptions do not appear in any of these. We will consider this further in the conclusion.

According to Shapiro, it was only in the second English edition of the *Opticks* (1717/18) that Newton merged the methodological statements of the *Opticks* and the *Principia* and presented them as one unified natural philosophical method:[71]

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[67] Ibid., 215.

[68] Newton, *Principia*, 2nd ed. (1713), 484 (trans. C&W, 943); Newton to Cotes (draft), ca. 28 March 1713, in *Correspondence*, 5:399.

[69] Shapiro, "Newton's 'Experimental Philosophy'," 212.

[70] This draft was intended for the third edition of the *Principia* (1726), but was eventually not used. It is dated by Whiteside. Isaac Newton, The Mathematical Papers of Isaac Newton, 8 vols., vol. 8, ed. Derek Thomas Whiteside (Cambridge: Cambridge University Press, 1981), 452n34, quoted in (and translated by) Shapiro, "Newton's 'Experimental Philosophy'," 197n21 (ellipsis in original).

[71] Even though this development takes place in 1717/18, Shapiro does discuss connections between the methods before this time. He states that Newton describes analysis and synthesis (without using the terms) in his preface to the first edition of the *Principia* (1687), in the following passage: "the basic problem of philosophy seems to be to *discover the forces of nature from the phenomena of motions* [analysis] and then to *demonstrate the other phenomena from these forces* [synthesis]." Author's

As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction [...]. And the Synthesis consists in assuming the Causes discover'd, and establish'd as Principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanations.[72] (*Opticks*, 1717/18)

Shapiro emphasises that Newton merely merged his methodological statements, while the methods themselves remain only related to each other in very general terms.[73] He implies that there was a need for Newton to present his methods as unified at this point. And while he does not specifically identify a reason for this unification, he indicates two factors that could have influenced Newton in this regard.

The first of these concerns Newton's priority dispute with Leibniz.[74] Shapiro explains that "the Leibnizians accused Newton of not having written the *Principia* in analytic form, but rather synthetic, because he did not command analytic calculus before Leibniz."[75] Hence, "Newton had to engage in a lot of fancy footwork to get around his use (or lack of use) of analysis in the Principia, while claiming that he had discovered it many years earlier."[76] Shapiro implies that Newton's merging of his methodological statements was, at least in part, a response to the accusations of the Leibnizians. In presenting his natural philosophy as relying on a single method in both the *Opticks* and the *Principia* – in which analysis, synthesis, induction, and deduction were combined – Newton could defend his position of priority in the invention of calculus more easily.

The second factor concerns the development of Newton's method – which is also considered by Walsh and Bechler. Shapiro argues that Newton changed his views on the certainty of his method: "after a long and productive career, Newton, I am sure, recognized

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Preface, in Newton, *Principia*, 1st ed. (1687), [5] (trans. C&W, 382) (italics and brackets mine). In addition, Shapiro argues that in the draft preface to the first edition of the *Opticks* (1704), Newton describes induction by means of the following two phrases: "proceeding alternately from experiments to conclusions" and "considering all the Phaenomena of nature relating to the subject in hand"). He states here: "this is also probably the procedure that he [Newton] had in mind for rendering principles 'general by induction'." Shapiro does not elaborate on the implication of these passages for his interpretations that the methods were separate at this time. Shapiro, "Newton's 'Experimental Philosophy'," 196n17, 212-13 (square brackets mine).

[72] Query 31, in Newton, *Opticks*, 2nd English ed. (1718), 380-81.

[73] Shapiro, "Newton's 'Experimental Philosophy'," 198.

[74] The priority dispute – the controversy between Leibniz and Newton over the invention of calculus – is examined by, for example, Alfred Rupert Hall, *Philosophers at War: The Quarrel between Newton and Leibniz* (Cambridge, United Kingdom: Cambridge University Press, 1980).

[75] Shapiro, "Newton's 'Experimental Philosophy'," 191.

[76] Ibid.

that the program that he espoused in his youth of establishing a more certain science founded on mathematics was unrealistic."[77] He explains that Newton's view of certainty "shifted through his long career from one of certainty to probabilism."[78] Shapiro demonstrates this shift by contrasting two of Newton's statements on certainty: one from 1672 and the other from 1717.[79]

> A naturalist would scearce expect to see ye science of those [i.e., colours] become mathematicall, & yet I dare affirm that there is as much certainty in it as in any other part of Opticks. For what I shall tell concerning them is not an Hypothesis but most rigid consequence, not conjectured by barely inferring 'tis thus because not otherwise or because it satisfies all phaenomena (the Philosophers universall Topick,) but evinced by ye mediation of experiments concluding directly & wthout any suspicion of doubt.[80] (Newton to Oldenburg, February 1672)

> And although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of, and may be looked upon as so much the stronger, by how much the Induction is more general.[81] (*Opticks*, 1717/18)

In the first passage, in 1672, Newton presents his method as certain and secure. He argues that his conclusions are not obtained by means of hypotheses or conjectures, but rather derived directly from experiments. His remarks on certainty in the latter passage, of 1717, are much more reserved. Newton states here that his method is not a demonstration of general conclusions, but within the limitations of natural philosophy, it is still the best procedure. Shapiro argues that in this second passage, in 1717, Newton acknowledges the method's limitations and "is no longer even hinting at the infallibility of his method as he had over forty years earlier."[82]

To summarise, Shapiro argues that Newton had two natural philosophical methods – one for the *Opticks* and another one for the *Principia* – but merges the descriptions of these methods in 1717/18 where he presents them as a single, unified method in which, furthermore, not a rigorous mathematical view of certainty is described, but a more probabilistic one.

[77] Ibid., 216.
[78] Alan E. Shapiro, "Newton did change his Views from Certainty to Probability," *Philosophy of Science* 88, no. 1 (2021): 169, https://doi.org/10.1086/710056.
[79] Shapiro, "Newton's 'Experimental Philosophy'," 216-17.
[80] Newton to Oldenburg, 6 February 1671/72, in *Correspondence*, 1:96-97 (addition added).
[81] Newton, *Opticks*, 2nd English ed. (1718), 380.
[82] Shapiro, "Newton's 'Experimental Philosophy'," 197.

Let us add here that Kirsten Walsh has strongly objected to Shapiro's argument. In her paper, "Newton: From Certainty to Probability?," she has responded to Shapiro's interpretation of the development of Newton's view of certainty into a probabilistic one, disputing the claim that it underwent a development of any kind.[83]

Walsh argues that the first passage Shapiro discusses – which is from a letter from Newton to Oldenburg in February 1672 – is not necessarily a representation of Newton's perspective on certainty at that time. She considers another passage from the same period in which Newton expressed a more moderate view on certainty in a letter he wrote in June 1672 to Robert Hooke. Hooke had criticised Newton's views on certainty that were described in the letter to Oldenburg written in February. In this response to Hooke, Newton clarified these statements as follows:

> I should take notice of a casuall expression wch intimates a greater certainty in these things then I ever promised, viz: The certainty of *Mathematical Demonstrations*. I said indeed that the *Science of Colours was Mathematicall & as certain as any other part of Optiques;* but who knows not that Optiques & many other Mathematicall Sciences depend as well on Physicall Principles as on Mathematicall Demonstrations: And the absolute certainty of a Science cannot exceed the certainty of its Principles. Now the evidence by wch I asserted the Propositions of colours is in the next words expressed to be from *Experiments* & so but *Physicall*: Whence the Propositions themselves can be esteemed no more then *Physical Principles* of a Science. [84] (Newton to Oldenburg, June 1672)

Walsh explains that in this passage, Newton distinguished between two types of certainty: "mathematical certainty" and "certainty offered by the 'mathematical sciences'."[85] The first type is achieved when reasoning from mathematical principles to mathematical propositions, and the second type when reasoning *mathematically* from physical principles to physical propositions. While mathematical principles are true by definition, physical principles are not. Walsh emphasises that Newton stated here that his certainty was of the second type: "the evidence by wch I asserted the Propositions of colours is […] expressed to be from *Experiments* & so but [i.e., merely] *Physicall*."[86] She argues that this corresponds to what Newton had stated five months earlier: "[a] naturalist would scearce expect to see ye science of those [i.e., colours] become mathematicall, & yet I dare affirm that there is as much certainty

[83] Kirsten Walsh, "Newton: From Certainty to Probability?," *Philosophy of Science* 84, no. 5 (2017), https://doi.org/10.1086/693963.
[84] Newton to Oldenburg, 11 June 1672, in *Correspondence*, 1:187 (italics in original).
[85] Walsh, "Newton: From Certainty to Probability?," 874.
[86] Newton to Oldenburg, 11 June 1672, in *Correspondence*, 1:187 (italics in original; addition mine).

in it as in any other part of Opticks."[87] Newton did not assert that his use of mathematical reasoning could overcome the certainty limitations of the physical proposition in either June or February. In the latter passage, Newton had stated that there would be "as much certainty in it as in any other part of Opticks."[88] Hence, according to Walsh, Newton already acknowledged in 1672 that the certainty of his natural philosophical method was restricted to the limitations of the physical domain. The passage from 1717 which Shapiro compares it to, is therefore not a new insight, according to Walsh, but a reiteration of his earlier position. Bechler – to which we will turn now – discusses these same passages, but draws yet another conclusion by considering more of the context of Hooke's criticism.

3.4 Zev Bechler

The final interpretation we will discuss is that of Zev Bechler. Bechler researches philosophy of science from the 16th to the 18th century – among which that of Newton – and ancient Greek philosophy. In his book, *Newton's Physics*, Bechler presents an elaborate consideration of Newton's natural philosophy against the background of the scientific revolution. This work evoked controversial reactions and is consequently mostly overlooked in recent Newtonian literature.[89] Nevertheless, I will argue, Bechler offers the most historically reliable interpretation that effectively resolves some of the discrepancies that other readings of Newton's method entail – which we will encounter in our discussion of Bechler, and in the comparison of the interpretations following it.

Bechler argues that Newton used a single natural philosophical method for both the *Opticks* and the *Principia*. Before 1713, this method is characterised by 'analysis' and 'synthesis', as Newton had described it in the first Latin edition of the *Opticks* (1706) and – implicitly – in the first edition of the *Principia* (1687).[90] Bechler explains that this method

[87] Newton to Oldenburg, 6 February 1671/72, in *Correspondence*, 1:96-97 (addition added); Walsh, "Newton: From Certainty to Probability?," 874.

[88] Newton to Oldenburg, 6 February 1671/72, in *Correspondence*, 1:96-97 (addition added).

[89] These objections (from reviews) concentrate mostly on Bechler's division of the structures or conceptions of scientific thought in modern philosophy into Aristotelian vs. Platonic (see for example Casini, Suchting and Meyer, although Orstrom and Kerszberg regard this division as a useful one). Guerlac's evaluation that "[i]t is a confident book, marked by sweeping affirmations" seems to be shared by several authors, although mostly with a negative connotation. In addition, although there are some contemporary Newton scholars who consider Bechler's arguments, such as Cohen and Ducheyne, he is not at all mentioned by most other authors of notable works on Newton, such as Harper's *Isaac Newton's Scientific Method*, McGuire's *Tradition and Innovation: Newton's metaphysics of Nature* and Janiak's *Newton as Philosopher*, or edited books, such as *Newton and Newtonianism* edited by Force and Hutton (the references to these reviews and works are included in the bibliography only).

[90] Bechler argues that Newton described the method of analysis and synthesis in the first edition of the *Prinicpia* – though without mentioning the terms – in the following passage: "the basic problem of philosophy ⟨*lit*. whole difficulty⟩ seems to be to *discover the forces of nature from the phenomena of motions* [analysis] and then to *demonstrate the other phenomena from these forces* [synthesis]." Shapiro offers the same interpretation, as discussed in note 71. Author's Preface, in Newton, *Principia*,

functioned by identifying "the true, unobservable, causal reality hidden beneath the phenomena" by means of the "rigorous, logical and mathematical deduction of physical causes."[91] However, the method was repeatedly challenged and objections of one particular deficiency of the method recur throughout Newton's career.

> [A] constant argument in all the critiques against Newton's science was the hypothetical nature of his theories, in contrast to his pretensions. Ever since Hooke's critique of Newton's optics in the early 1670's, […] up to the 1713 Cotes' dispute, their critical analysis always came up with some tacit assumptions which play major roles in the theories but are neither declared nor proved or even suggested by any further argument in the theory.[92]

In 1713, in response to the objection of his editor, Roger Cotes, Newton saw himself forced to modify his method. He introduced a new methodological component – induction – resulting, according to Bechler, in "a less glorious but a much better defensible method – science by generalisation of observed facts."[93] Bechler's consideration of Newton's method is structured by a discussion of these challenges and Newton's responses to them. We will consider the objections of Newton's method leading up to this methodological development, starting with the passage from Newton's letter to Oldenburg from 6 February 1672, discussed by both Shapiro and Walsh.

As mentioned by Walsh, in his letter to Oldenburg, Newton clarified his certainty claims in response to Robert Hooke's criticism. Bechler's analysis of this exchange is focussed on the purpose of the text – which is to respond to Hooke's objections. He argues that Newton was conspicuously unsuccessful in resolving these: he evaded an important aspect of the objection, which is that even though Hooke expressed his disagreement with some of Newton's conclusions – such as those derived from the *experimentum crucis* discussed above – his objections did not concern the correctness of the individual propositions, on which Newton focussed his response.[94]

> I can in my supposition conceive the white or uniform motion of light to be compounded […] but I see no *necessity* of it. If Mr Newton had any argument that he supposeth an

1st ed. (1687), [5] (trans. C&W, 382) (italics and square brackets mine; angle brackets in translated original); Newton, *Optice*, 1st Latin ed. (1706), 347.

91 Bechler, *Newton's Physics*, 347-48.

92 Ibid., 433.

93 Ibid., 361.

94 Ibid., 348-49.

absolute Demonstration of his theory, I should be very glad to be convinced by it.[95] (Hooke to Oldenburg, 15 February 1672)

Hooke had criticised the absence of a justification in Newton's argument regarding the nature of light, not the certainty of his conclusions. As Bechler explains, for Hooke "the uncertainty of the basic facts was, actually, an irrelevant side issue […]. [T]he whole dispute was about the logical status of the theory."[96] Newton's clarification of the certainty status of mathematical versus physical propositions in June 1672 – which we have already encountered as outlined by Walsh – does not resolve Hooke's criticism, and even deflects his emphasis on the absence of a demonstration of the propositions' necessity. What in Walsh' eyes had looked like a solid rejoinder is depicted as an evasive move by Bechler.[97]

Irrespective of whether he purposefully evaded Hooke's criticism, as Bechler argues, Newton employed a different strategy to defend the certainty of his method when he published the first edition of his *Principia* fifteen years later.[98] In his preface, Newton explained that he applied the same approach as 'the ancients', who divided mechanics into two parts: 'rational mechanics' (or 'geometry') and 'practical mechanics' (or just 'mechanics'). The two differ with regard to their level of exactness, Newton explained: "the whole subject of mechanics is distinguished from geometry by the attribution of exactness to geometry and of anything less than exactness to mechanics. Yet the errors do not come from the art but from those who practice the art."[99] Geometry is the part of mechanics that is not subject to the inexactness caused by the artificer: "geometry is founded on mechanical practice and is nothing other than that part of universal mechanics which reduces the art of measuring to exact propositions and demonstrations."[100] Newton explains that this separation of inexact mechanics on the one hand and exact geometry on the other, is also used in the *Principia*, which reasons only from exact, geometrical (mathematical) principles – hence its title, the *Mathematical Principles of Natural Philosophy*.[101] Therefore, Newton clarifies, all terms that are commonly used to refer to physical forces or phenomena should be understood in a mathematical sense:

[95] Hooke to Oldenburg, 15 February 1671/72, in Newton, *Correspondence*, 1:114 (italics mine).
[96] Bechler, *Newton's Physics*, 348.
[97] This does not necessarily mean that Walsh' interpretation is incorrect. However, following Bechler's reasoning, Newton wrote the clarification of his earlier remarks on certainty with Hooke's objection in mind that he had failed to demonstrate the necessity of his conclusions. If Newton primarily tried to evade confrontation (purposefully or not) and used this revision merely to circumvent resolving Hooke's actual objections, then this passage might not actually represent Newton's views on certainty at that time. And although it is not the only argument Walsh discusses, it would complicate her refutation of Shapiro's claims.
[98] Bechler, *Newton's Physics*, 348.
[99] Author's Preface, in Newton, *Principia*, 1st ed. (1687), [4] (trans. C&W, 381).
[100] Ibid., [4] (trans. C&W, 382).
[101] Ibid., [4-5] (trans. C&W, 382). By this defence of the certainty of his method, Newton rendered his earlier justification of certainty, in response to Hooke's objections, irrelevant. He had argued in 1672 that even though his conclusions are drawn by means of a certain, mathematical method, they are

I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view. Therefore, let the reader beware of thinking that by words of this kind I am anywhere defining a species or mode of action or a physical cause or reason, or that I am attributing forces in a true and physical sense.[102] (*Principia*, 1687)

While Newton's *Principia* received some neutral and laudatory reviews, an anonymous review from 1688 in the *Journal de Sçavans* demonstrates that not all of Newton's critics were convinced by this defence of the method's certainty:[103]

[T]he author recognizes himself […] that he has not considered their Principles as a Physicist, but as a mere Geometer. He confesses the same thing at the beginning of the third book, where he endeavors nevertheless to explain the System of the World. But it is 〈done〉 only by hypotheses that are, most of them, arbitrary, and that, consequently, can serve as foundation only to a treatise of pure [rational] mechanics.[104]

The reviewer criticises Newton for first restricting his natural philosophy to the domain of rational mechanics (geometry), while subsequently using it to "explain the System of the World," which belongs to practical mechanics.[105] Hence, at least according to some, Newton had again failed to demonstrate the necessity of his conclusions, and, in the absence of such a demonstration, his theorems were to be considered merely good hypotheses.[106]

Let us turn to the third episode singled out by Bechler, which takes place in 1713 when Newton undertook a new approach to demonstrate the certainty and validity of his method in the second edition of the *Principia*. This was prompted, Bechler explains, by a difficulty Newton's editor, Roger Cotes, raised in relation to his task of writing a preface for this new

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derived "from *Experiments* & so but *Physicall*." Newton to Oldenburg, 11 June 1672, in *Correspondence*, 1:187 (italics in original). By contrast, he asserted here, in 1687, that he concerned himself only with the 'mathematical principles of natural philosophy', hence the physical propositions could no longer be the cause of the uncertainty.

[102] Definitions, in Newton, *Principia*, 1st ed. (1687), 4-5 (trans. C&W, 408).

[103] Bechler, *Newton's Physics*, 356, 547n6.

[104] The review is attributed to Pierre-Sylvain Régis by Paul Mouy. *Journal de Sçavans* (2 August 1688), 153, quoted in Paul Mouy, *Le Développement de la Physique Cartésienne* (Paris, 1934), 256. Quoted in, and translated by, Alexandre Koyré, *Newtonian Studies* (Chicago: The University of Chicago Press, 1965), 115 (square brackets added; angle brackets in original).

[105] Ibid.

[106] Bechler also discusses the objections from Leibniz on this first edition of the *Principia*, according to whom, Bechler explains, Newton's conclusions were hypotheses and no actual proof or demonstration was provided, as was argued by Hooke. Bechler, *Newton's Physics*, 346-47, 386.

edition of the *Principia*. In March 1713, Cotes wrote to Newton about his plan for the preface, which was to describe the method by means of a number of examples drawn from the *Principia*, among which the argument for the mutuality of gravitational attraction. Here, Cotes was confronted with a problem that he presented to Newton in his letter. This problem – 'Cotes' difficulty', as it is usually referred to – is often viewed as a misunderstanding of Newton's argument by Cotes. Bechler, however, argues that Cotes raised an objection that is very similar to that of Hooke and the anonymous reviewer discussed above.

The difficulty is described as follows. In the fifth proposition of the third book, Newton had argued that the curvilinear motion of the planets was caused by their attraction to a central body.[107] For example: the orbital motion of the circumsolar planets (i.e., the planets orbiting the sun) is caused by their attraction to the sun. In the first corollary to this proposition, Newton had invoked the third law of motion – "[t]o any action there is always an opposite and equal reaction" – in order to assert that attraction is mutual, so that the circumsolar planets are not only attracted to the sun, but that the sun is also attracted to the circumsolar planets.[108] This argument dissatisfied Cotes, as he wrote to Newton:

> [I]n the first Corollary of the 5th [proposition of Book III] I meet with a difficulty, it lyes in these words *Et cum Attractio omnis mutua sit* [and since all attraction is mutual] I am persuaded they are then true when the Attraction may properly be so call'd, otherwise they may be false.[109] (Cotes to Newton, 1713)

Cotes argued here that the *mutuality of attraction* is a valid inference if it is indeed *attraction* that causes the centripetal motion. He demonstrates this by means of the following example:

> Suppose two Globes *A* & *B* placed at a distance from each other upon a Table, & that whilst *A* remains at rest *B* is moved towards it by an invisible Hand. A by-stander who observes this motion but not the cause of it, will say that *B* does certainly tend to the centre of *A*, & thereupon he may call the force of the invisible Hand the Centripetal force of *B*, or the Attraction of *A* since ye effect appears the same as if it did truly proceed from a proper & real Attraction of *A*. But then I think he cannot by virtue of the Axiom ⟨Attractio omnis mutua est⟩ conclude contrary to his Sense & Observation, that the Globe *A* does also move towards the Globe *B* & will meet it at the common centre of Gravity of both Bodies. This is what stops me in the train of reasoning. […] For 'till this Objection be cleared I would not undertake to answer any one who should assert

[107] Newton, *Principia*, 2nd ed. (1713), 365 (trans. C&W, 806).
[108] Ibid., 13 (trans. C&W, 417).
[109] Cotes to Newton, 18 March 1713, in Newton, *Correspondence*, 5:392 (additions added).

> You do Hypothesim fingere [invent a hypothesis] I think You seem tacitly to make this Supposition that the Attractive force resides in the Central Body.[110] (Cotes to Newton, 1713)

Cotes objects to Newton's derivation of the mutuality of attraction on the basis that its premise – the explanation that the orbital motion of the planets is caused by attraction – is not sufficiently justified. Attraction is a possible explanation for these motions, and is in agreement with the phenomena, but this is also true for other explanations, for example, that the planets are moved by an invisible hand.[111] In the absence of a justification that "Attraction may properly be so call'd," the derivation of the mutuality of this attraction is not a valid conclusion, but merely a hypothesis.[112] Even more so since this mutuality is not confirmed by the phenomena, even if the third law of motion is invoked, as Cotes explained: "I think he cannot by virtue of the Axiom ⟨Attractio omnis mutua est⟩ conclude contrary to his Sense & Observation, that the Globe *A* does also move towards the Globe *B*."[113] In other words, there is no observational confirmation of bodies moving toward each other, only of them moving around each other, which, again, can be explained in other ways.[114]

Newton answered that Cotes' difficulty would be removed by considering the characteristics of his method, among which the distinction between axioms and hypotheses.[115] Newton had defined the laws of motion as 'axioms' or '(first) principles' and explained in his letter that "[t]hese Principles are deduced from Phaenomena & made general by Induction."[116] By contrast, the term 'hypothesis' signifies "only such a Proposition as is not a Phaenomenon

[110] Ibid. (angle brackets in [Cotes'] original; square brackets mine).

[111] Bechler argues that Cotes' scenario of the invisible hand is (intentionally) equivalent to the vortex theory of Descartes, according to which the motions of the planets are caused by vortices that exert a centrifugal force on bodies and thereby bring about their orbital motion. Bechler, *Newton's Physics*, 357.

[112] Cotes to Newton, 18 March 1713, in Newton, *Correspondence*, 5:392.

[113] Ibid. (angle brackets in [Cotes'] original).

[114] Bechler argues that Cotes was already aware of this lacuna in Newton's argumentation before he started preparing his preface (contrary to how Cotes phrases it in his letter to Newton) and that it represents the 'continental criticism' of Newton's method, as raised by, among others Leibniz. Bechler, *Newton's Physics*, 353.

[115] This distinction is already in place when Cotes prepared his preface, but when the Principia was first published in 1687, 'hypothesis' was used in a more general sense, for example to describe the preliminaries at the beginning of the third book, which were consequently divided into 'rules of reasoning' and 'phenomena' in the second edition. Cohen argues that Newton changed this in response to the criticism of the anonymous reviewer in 1688, who objected that Newton explained the system of the world only by hypotheses. Furthermore, Koyré argues that, "[i]n the first edition of the Principia, this term is taken in its classical sense, as a fundamental proposition of a theory. In the second edition, on the contrary, a hypothesis is taken to be a fiction, and mostly a false one, or, at the very least, an unproved assertion." I. Bernard Cohen, *Introduction to Newton's 'Principia'* (Cambridge: Cambridge University Press, 1971), 157; I. Bernard Cohen, "A Guide to Newton's Principia," in *The Principia: Mathematical Principles of Natural Philosophy* (Berkeley: University of California Press, 1999), 23-24; Koyré, *Newtonian Studies*, 40.

[116] Newton to Cotes, 28 March 1713, in *Correspondence*, 5:397.

nor deduced from any Phaenomena but assumed or supposed wthout any experimental proof."[117] Accordingly, the third law of motion – an axiom – is based on phenomena and is therefore not a hypothesis. Newton then explained that this law included mutual attraction: "the mutual & mutually equal attraction of bodies is a branch of the third Law of motion."[118] It seems that this inclusion was already implied before Cotes wrote to Newton about his difficulty, but Newton included it explicitly in his description of the third law of motion of this edition of the *Principia*, where he added the following sentence to the – otherwise unchanged – passage: "This law is valid also for attractions."[119] Newton thereby resolved Cotes' objection that the mutuality of attraction is not based on phenomena, since it is part of an axiom and is therefore, per definition, "deduced *from Phaenomena* & made general by Induction."[120] Hence, Cotes' difficulty is removed by this extension of the third law of motion to attraction, since the mutuality of attraction is thereby not a hypothesis but an axiom (or rather, the branch of an axiom) that is deduced from phenomena and generalised by induction.

Hall and Tilling, the editors of Newton's correspondence, note that "Cotes seems naively to have misunderstood the implications of the third law here."[121] Since, as Newton explains in his reply, this axiom resolves Cotes' objection that the mutuality of attraction is a hypothesis. In an earlier edition of Newton's correspondence published in 1830, Edleston stated more bluntly: "[t]he difficulty raised by Cotes here affords an instance of the temporary haze which may occasionally obscure the brightest intellects."[122] Bechler, on the other hand, argues that Cotes made a valid objection: "Cotes denied that mutuality can be derived from Law III *without any further assumption*."[123] Hence, he did not object that the third law of motion is invoked, but argued that it required a further justification to affirm that it is indeed attraction that causes the centripetal motion as opposed to another mechanism. As Cotes stated in his letter: "I am persuaded they are then true when the Attraction may properly be so call'd, otherwise they may be false."[124] However, Bechler explains, Newton tacitly assumed that attraction was the actual cause of the centripetal motion of the planets and that, for this explanation to function, this attraction must be mutual. Newton did therefore not provide a

[117] Ibid.
[118] Ibid.
[119] Newton, *Principia*, 2nd ed. (1713), 13 (trans. C&W, 417).
[120] Newton to Cotes, 28 March 1713, in *Correspondence*, 5:397 (italics mine).
[121] Cotes to Newton, 18 March 1713, in Newton, *Correspondence*, 5:393n5.
[122] Isaac Newton, *Correspondence of Sir Isaac Newton and Professor Cotes*, ed. J. Edleston (Cambridge: Cambridge University Press, 1850), 152.
[123] Bechler, *Newton's Physics*, 358 (italics in original). Bechler is not the only one who views Cotes' difficulty as valid. Alexander Koyré, for example, explains that Cotes accurately identified the shortcomings of Newton's argumentation, but he states that Cotes' assertion that attraction was a supposition was incorrect and that the difficulty would be resolved if Newton expressed himself more clearly. Koyré, *Newtonian Studies*, 280-81. Bechler, as we will see, argues that Newton's argumentation is invalid at a more fundamental level, as, he argues, Cotes correctly identifies.
[124] Cotes to Newton, 18 March 1713, in Newton, *Correspondence*, 5:392; Bechler, *Newton's Physics*, 357.

justification "that the attraction may properly be so called," neither in the first corollary of the fifth proposition, nor in his answer to Cotes' difficulty.[125]

Moreover, Bechler states that Newton had fully understood Cotes' difficulty. He argues that this is demonstrated by the existence of an unfinished draft that preceded Newton's actual letter sent off to Cotes.[126] The draft contains a similar but much more elaborate consideration of Cotes' difficulty, and an admission of Newton's inability to refute Cotes' arguments. The letter can be summarised as follows: Newton explained by which propositions and axioms he had concluded that orbital motion is caused by a centripetal force.[127] He then discussed two reasons why the argument of the 'invisible hand' could not refute the mutuality of attraction. Firstly, Newton explained that Cotes' scenario of the invisible hand is a hypothesis – which he defined, as discussed above, as a proposition that is not derived from phenomena – and that it therefore cannot be used to disprove a phenomenon-based proposition.[128] Secondly, he stated that if attraction were not mutual, an absurd situation would necessarily follow: "a body attracted by another body without mutually attracting it would go to the other body & drive it away before it with an accelerated motion in infinitum, contrary to ye first law of Motion."[129] Newton consequently emphasised that his experimental philosophy is based on phenomena and proceeds via induction and deduction. He then repeated the two arguments against Cotes' explanation of the invisible hand and formulated a definition of 'principles', 'propositions' and 'axioms'.[130] The draft concludes with an unfinished sentence in which Newton reflects on Cotes' example of the invisible hand moving the planets. In this sentence, Newton conceded that he did not actually succeed in refuting Cotes' alternative explanation (i.e., planet *B* being

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[125] Cotes to Newton, 18 March 1713, in Newton, *Correspondence*, 5:392; Bechler, *Newton's Physics*, 357-58.

[126] Newton's reply consisted of two letters, written a few days apart. The second letter is announced at the end of the first one with the reason that Newton had no time to finish his reply at the time. The discussion of Cotes' difficulty in the second letter is mostly a summary of the arguments Newton had presented in the first one – with some added emphasis on the limitations of the role of hypotheses in his philosophy: Newton gives a description of his method in terms of deduction and induction and adds that hypotheses cannot be used to refute a proposition which is deduced from phenomena and generalised by induction. There is also a draft of the second letter, but this does not differ significantly from the actual letter. Newton to Cotes, 31 March 1713, in *Correspondence*, 5:400-1.

[127] Newton to Cotes (draft), ca. 28 March 1713, in Newton, *Correspondence*, 5:398.

[128] This rejection of the use of hypotheses in rebutting empirical propositions – which Newton phrased as follows in his draft: "I regard not Hypotheses in explaining the Phenomena of nature so I regard them not in opposition to arguments founded upon Phaenomena by Induction or to Principles setled upon such arguments" – is added in the third edition of the *Principia* (1726) as a new rule of reasoning in philosophy. Newton explains there that the rule is added "so that arguments based on induction may not be nullified by hypotheses." Newton to Cotes (draft), ca. 28 March 1713, in *Correspondence*, 5:398; Newton, *Principia*, 3rd ed. (1726), 796.

[129] Newton to Cotes (draft), ca. 28 March 1713, in *Correspondence*, 5:398. This argument – which Newton also included in the actual letter – is problematic however, because it presupposes that attraction is the correct explanation for the motions of the planets in order to prove that this attraction is mutual. However, this is exactly what Cotes objects to: since the explanation of attraction is not justified and might not be correct, it cannot be used to derive the mutuality of attraction.

[130] Newton to Cotes (draft), ca. 28 March 1713, in *Correspondence*, 5:399.

moved by an invisible hand toward planet *A*, instead of by *B*'s attraction to *A*) by the arguments he had presented.

> What has been said, doth not hinder the body *B* from being moved by an invisible hand towards the resting body *A*:[131] (Newton to Cotes (draft), 1713)

Bechler argues that this sentence demonstrates that Newton had not just understood Cotes' difficulty, but that he had explicitly recognised his inability to resolve it. Nevertheless, Newton rewrote the letter, in a much more self-assured tone, condensed his argumentation of the draft and sent it off to Cotes, even though he was aware of its inadequacy. Again responding to the objections on his method in an evasive manner.[132]

Nevertheless, Newton did integrate part of his answer to the difficulty in a passage that was to be added, as he instructed Cotes, to the General Scholium (which had been completed earlier that month).[133] We have already encountered this passage in the discussion of Shapiro's interpretation.

> In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method.[134] (*Principia*, 1713)

Hall and Tilling note that "[i]t is curious that these famous sentences were written by Newton as an afterthought to an addendum, merely in response to Cotes' doubts."[135] Indeed, when Cotes' difficulty is interpreted as a misunderstanding that is resolved by Newton's answer, an addition of this kind seems excessive. For Bechler, on the other hand, it is a confirmation of Newton's acknowledgement of the difficulty in the draft. Even though he did not disclose it in his actual answer to Cotes, Newton had recognised that the difficulty was a problem that required a response. According to Bechler, this response represented a methodological shift, of which the added passage to the General Scholium was the first demonstration. In this passage, Newton introduced a new methodological component – induction – which he had not mentioned before in any of his published or unpublished works on natural philosophy.[136]

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[131] Ibid.
[132] Bechler, *Newton's Physics*, 360.
[133] Newton to Cotes (draft), ca. 28 March 1713, in *Correspondence*, 5:399.
[134] Newton to Cotes, 2 March 1713, in *Correspondence*, 5:384; Newton to Cotes, 28 March 1713, in *Correspondence*, 5:397.
[135] Newton, *Correspondence*, 5:399n4.
[136] Shapiro also mentions this late introduction of the term. Shapiro, "Newton's 'Experimental Philosophy'," 196.

Thus, Bechler argues, "generalisation and induction as the exhaustive method of the *Principia* came to replace Newton's previous conception of analysis and synthesis *as direct response to Cotes' 'difficulty', and as a central answer to it*."[137] He explains that Cotes' difficulty could not be solved under the method of analysis and synthesis – which proceeded according to Bechler, by deriving unobservable causes from observable effects.[138] Newton's new method offered a new kind of certainty through its use of induction. By the generalisation of deductions from experiments and observed phenomena, Newton's arguments remained within the observable domain.[139] He could therefore validly deduce and generalise propositions even if they could not be experimentally rendered visible.

"From then onward," Bechler demonstrates, "Newton made it a point to introduce the concept and term 'induction' into all his scientific publications."[140] Hence, following the implementation of the term in the General Scholium in 1713, it was introduced into the second English edition of the *Opticks* published in 1717/18. In this edition, Newton expands his methodological statements of the preceding edition (1st Latin ed., 1706), by adding a passage on induction (and hypotheses), that is placed in between the existing statements (italicised words are added in 1717/18).

> As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition [synthesis]. This Analysis consists in making Experiments and Observations *and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For Hypotheses are not to be regarded in experimental Philosophy. And although the arguing from Experiments and Observations by Induction be no Demonstration of*

[137] Bechler, Newton's Physics, 364, 361 (italics in original). Even though Bechler introduces Newton's implementation of induction here as a solution to Cotes' difficulty, he explains that it did not precisely solve it. Instead, he argues, Newton "diverts attention from Cotes' problem by replacing it by another, new problem, namely the problem of induction." Bechler explains that because Newton was both unable to prove that attraction was the cause of the planetary motions, and disprove that other explanations – such as an invisible hand – were not the cause (as he had admitted in the draft letter to Cotes), he circumvents this need for a justification by introducing induction. Since, as an axiom, the mutuality of attraction is deduced from phenomena and generalised by induction and, as Newton asserts later, "although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of." (Newton, *Opticks*, 2nd English ed. [1718], 380.) Hence, Newton simply concedes that the mutuality of attraction is indeed not absolutely certain, but that this is a shortcoming of the method. He thereby trivialised the problem of induction as a necessary evil of the method as a whole and not a particular failure of the derivation of the mutuality of attraction which, Bechler explains, "enabled him to brush off the criticism – for since all of the involved uncertainty allegedly arises only from this inductive procedure, surely his theory is merely trivially uncertain." Bechler, *Newton's Physics*, 364-66.

[138] Bechler, *Newton's Physics*, 361, 364.

[139] Ibid., 361.

[140] Ibid., 351.

general Conclusions; yet it is the best way of arguing which the Nature of Things admits of […]. And the Synthesis consists in assuming the Causes discover'd and establish'd as Principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanations.[141] (*Opticks*, 1717/18)

This passage remained unchanged in all later editions of the *Opticks*. It is the version we have first considered in the introduction, and which served as the basis for Garrison's and Hintikka & Remes' interpretations of the method.

To summarise, Bechler argues that Newton used the method of analysis and synthesis to derive unobservable causes from observable effects by means of a rigorous mathematical method. However, this method was repeatedly challenged. Critics objected that Newton did not provide sufficient justification for his arguments, and hence failed to reach his own standard of certainty. Newton employed several strategies to defend the certainty and validity of the method, though without much success. Then, in 1713, as Bechler demonstrates, Newton sees himself forced to respond in a more drastic manner, to an objection raised by Cotes. Newton implements a new component into his method, induction, and revises the methodological statements of both the *Principia* and *Opticks* in the editions published thereafter.

3.5 Conclusion

Bechler has convincingly demonstrated that Newton introduced the concept of induction into his methodological statements from 1713 onward, in response to the objections of his contemporaries, specifically that of Roger Cotes. The revised methodological statements – the passage from the General Scholium added in 1713 in the *Principia*, and the passage from query 31 added in 1717/18 in the *Opticks* – remain unchanged thereafter. As the most recent descriptions of the method, they are the primary focus for interpreters of Newton's method. Among these are Garrison and Hintikka & Remes, who argued that Newton had added induction in the analysis to adapt the method of analysis and synthesis to the domain of natural philosophy. This interpretation corresponds with Newton's description of induction quoted above: "Analysis consists in making Experiments and Observations *and in drawing general Conclusions from them by Induction*."[142] And when the earlier editions are not taken into account, this explanation of induction as a modification of the method of analysis and synthesis

[141] The first and last part of this passage (in roman) is from Newton's English drafts that were translated by Samuel Clarke and published in the first Latin edition of the *Opticks* (1706). Portsmouth Collection in the University Library, Cambridge, MS Add. 3970, fol. 286r, quoted in Shapiro, "Newton's 'Experimental Philosophy'," 196-97 (addition added). And the italicised passage is from Newton, *Opticks*, 2nd English ed. (1718), 380-381.

[142] Ibid.

is a very plausible one. However, Bechler's demonstration of the connection between Newton's use of induction, and his response to Cotes' difficulty complicate the interpretation of Garrison and Hintikka & Remes. It furthermore problematises the interpretation of Shapiro, who argues that only the terms 'induction' and 'deduction' characterised the method of the *Principia*, and 'analysis' and 'synthesis' that of the *Opticks*, and that the combined use of the terms in the methodological statement of the *Opticks* in 1717/18 quoted above is an attempt by Newton to conceal the difference between the methods. While this interpretation is not necessarily incompatible with the late implementation of induction, it does make it much less plausible that the use of this term in 1717/18 indicates a merging of the methodological descriptions, while its use in the Principia, four years earlier, in 1713 is merely a description of the *Principia*'s sole method – especially since Shapiro argues that the method of analysis and synthesis appears in the *Principia*'s first edition.[143] We will further consider the differences between the interpretations and their implications in the conclusion.

[143] See note 71.

4. CONCLUSION

In this article, we have considered Newton's methodological statements in the *Principia* and *Opticks* through a discussion and comparison of the interpretations of four important, contemporary Newton scholars. Three aspects of these statements have been examined specifically: (a) the meaning of the key terms Newton uses to describe his method: 'analysis', 'synthesis', 'induction', and 'deduction'; (b) the interrelation between these components; and (c) their relative importance in the method. By way of conclusion, I will provide a brief summary of the central elements of the interpretations, compare them and discuss a number of relevant differences and inconsistencies.

The first two interpretations, Garrison's and Hintikka & Remes', are largely compatible. Both regard the four key terms as part of the same method: a modified form of the ancient Greek geometrical method of analysis and synthesis. They regard deduction as a part of this method – analysis and synthesis proceed via deductive reasoning – but do not consider its meaning specifically. Hintikka & Remes argue that Newton used analysis to identify interdependencies within natural phenomena, and induction to consequently generalise these interdependencies to equivalent phenomena. Garrison explains the role of induction in a compatible manner, arguing that it is added to analysis and synthesis to supplement the function of generalisation inherent in the original, geometrical method. By the implementation of induction, the method of analysis and synthesis is adapted to the domain of natural philosophy. In both interpretations, Newton's concept of induction is distinguished from that of 'traditional inductivists', according to whom induction is a generalisation by means of the identification of shared similarities (induction by enumeration) or differences (induction by elimination) of a collection of phenomena. Instead, Garrison and Hintikka & Remes argue, Newton's induction is the generalisation of a *single* phenomenon to all similar phenomena. In their explanation of induction and its role in analysis, they refer to Newton's concept of the *experimentum crucis* – a single well-conceived experiment that alone suffices to prove a theorem without the need for (numerous) repetitions.

Shapiro offers a different interpretation, specifically with regard to the relation between the methodological components. He argues that Newton used separate methods in the *Opticks* and *Principia*, as he had described them in the editions that were published before 1717. Shapiro explains that the method of the *Opticks* – analysis and synthesis – proceeds by uncovering elements or causes and consequently explaining the causes of other phenomena. By contrast, the method of the *Principia* – induction and deduction from the phenomena – proceeds by the generalisation of a collection of propositions derived from phenomena. The methods first appear together in the second English edition of the *Opticks* (1717/18). Shapiro argues that Newton merely merged the methodological statements in this edition and actually continued to view and use them as separate methods. He reasons that this reformulation of the statements was influenced by a development in Newton's views on certainty, which, Shapiro asserts, had gradually changed over the course of Newton's career from a rigorous mathematical, to a moderate and probabilistic view on certainty. He demonstrates this development by means of a comparison of Newton's early and later statements on certainty. Walsh responds to Shapiro's conclusion and refutes his claim that such a development occurred. She argues that Newton had already expressed a more moderate view of certainty, early in his career, in another passage.

Bechler considers these same passages compared by Shapiro and by Walsh – though not in response to their discussion – and offers another interpretation. He explains that Newton had formulated the earlier of his more moderate statements on certainty in response to Hooke's objections to his method. However, Bechler argues that Newton is conspicuously unsuccessful in resolving these objections and his discussion of certainty is more an evasion of Hooke's criticism, than an actual attempt to clarify his views. Bechler considers two additional objections of Newton's method that address similar issues, and to which Newton responds in a likewise evasive manner. The most important of these is the criticism of his editor, Roger Cotes. Cotes wrote to Newton about his task of composing a preface for the *Principia*'s second edition, where he encountered a difficulty in outlining Newton's argument for the mutuality of attraction. He explained that it seemed that the argumentation relied on a tacit supposition. Newton answered that no supposition was involved and explained that his argumentation was based on phenomena alone. However, the draft that preceded this letter demonstrates that Newton had recognised that he was not able to resolve the issue. And, even though he did not acknowledge it in his answer to Cotes, Newton addressed the objection and implemented a new concept into his methodological statements of the *Principia*: induction. Bechler argues that this implementation represented a methodological development that is further demonstrated by Newton's consequent introduction of the concept into his methodological statements of the *Opticks*. Hence, Bechler argues, Newton's implementation of induction indicates a methodological shift evoked by the repeated confrontation of the shortcomings of the method with regard to the justification of his conclusions.

This comparison yields two key questions: was there a methodological shift, and what did Newton mean by 'induction'? According to Garrison and Hintikka & Remes, no methodological shift takes place. And since they regard the method as it was described in the second English edition of the *Opticks* (1717/18) – where induction and deduction are explained in relation to the method of analysis and synthesis – they interpret the four components as interdependent. Accordingly, Garrison and Hintikka & Remes regard induction as a step within the analysis. They cite, to this effect, the aforementioned edition of the *Opticks* (1717/18): "This Analysis consists in making Experiments and Observations and in drawing general

Conclusions from them by Induction."[144] They argue that Newton's concept of induction is related to that of the *experimentum crucis*, and on this basis the term can be understood as the generalisation of a single phenomenon or experiment.[145] Shapiro convincingly refutes this interpretation of induction. He demonstrates that Newton describes induction between 1713 and 1718 as being stronger when more experiments or phenomena are used, and phrases it in terms of the 'consent of phenomena'. Although both these passages are from draft material and a description of this sort is not found in any published works or correspondence, it clearly challenges the interpretation of induction provided by Garrison and Hintikka & Remes. Bechler's assertion that induction is only introduced into Newton's method in 1713 further contradicts their interpretation. If induction had been implemented in the method of analysis and synthesis in order to adapt this originally geometrical method to the domain of natural philosophy, and especially if, as Garrison asserts, the method consequently remained unchanged, it would be expected that these components are introduced around the same time. This also challenges Garrison's and Hintikka & Remes' argument that the meaning of induction can be derived from its relation to the *experimentum crucis*. This term is only discussed around 1672 and not mentioned by Newton in relation to induction after he introduced it forty years later. To summarise, Garrison's and Hintikka & Remes' interpretation of induction in terms of the generalisation of a single phenomenon or experiment is contradicted by the explanation of the term Newton gives in the draft passages quoted by Shapiro. In addition, their assertion that induction can be understood in reference to the *experimentum crucis* is refuted by the late introduction of the term demonstrated by Bechler. We may therefore conclude that Bechler and Shapiro convincingly demonstrate that a methodological development takes place and that Newton's induction is a generalisation of multiple phenomena as opposed to the generalisation of a single phenomenon – contradicting these central aspects of the interpretations of Garrison and Hintikka & Remes.

However, the interpretations of Bechler and Shapiro are not fully compatible either. Bechler's demonstration that induction was added only in 1713 challenges Shapiro's claim that Newton used separate methods in the *Opticks* and the *Principia,* which were merged only in 1717. Shapiro's interpretation appears less convincing once we realise that induction was only introduced four years before its alleged merge with the method of analysis and synthesis. Since it was only introduced into the *Principia* in the 1713 edition and subsequently into the *Opticks* in 1717/18, it seems more plausible that its introduction signals, as Bechler suggests, a shift in Newton's thinking which takes place at this time. As for Shapiro's claim that Newton's objective of certainty shifted from a rigorous conception of mathematical certainty in his early

[144] Newton, *Opticks*, 2nd English ed. (1718), 380.

[145] It could also be an analysis of the interrelations between two phenomena, but I want to emphasise the contrast with the interpretation of induction by Shapiro, in terms of a collection of phenomena.

career to a probabilistic one later in his life, we may conclude that it is comparable to the shift that Bechler describes. He argued that in 1713, Newton “decided to retreat back to a less glorious but a much better defensible method – science by generalisation of observed facts,” in response to the repeated objections from Hooke up to Cotes.[146] However, I do not think that we can conclude, as Shapiro suggests, on the basis of the revised methodological statements, that Newton’s objective of certainty shifts to a probabilistic one. I therefore endorse Bechler’s reading, which states that a development of Newton’s method and objective of certainty took place in response to the repeated criticism of this method.

By means of a consideration of four contemporary scholarly interpretations of Newton’s methodological statements, we have obtained more insight into the relation between the meaning, role, and importance of Newton’s methodological components as they are portrayed in the different editions of the *Opticks* and *Principia*. This has yielded a productive evaluation of the individual claims. And we have on that basis reached the conclusion that Bechler's account is the most convincing reading of Newton’s methodological statements. We have also acquired a better understanding of the source of these differences and may conclude that the controversies in the literature do not merely *appear* very different, but are actually are fundamentally different and incompatible readings of Newton’s statements and terms. While this is not unexpected on the basis of the ambiguity of Newton’s statements, it does raise the question why the terms are so elusive. Hence, by means of this comparison, we have obtained a more comprehensive overview of Newton’s methodological statements, but there still remain a number of questions that require a more extensive consideration – perhaps an examination of the background against which interpreters consider Newton’s terms and statements might further clarify the controversies in the literature.

[146] Bechler, *Newton's Physics*, 361.

5. APPENDIX: CHRONOLOGICAL OVERVIEW OF NEWTON'S METHODOLOGICAL STATEMENTS

Principia

1st ed. (1687) Author's Preface	[O]ur present work sets forth mathematical principles of natural philosophy. For the basic problem ⟨*lit.* whole difficulty⟩ of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces […]. [W]e derive from celestial phenomena the gravitational forces by which bodies tend toward the sun and toward the individual planets. Then the motions from the planets, the comets, the moon, and the sea are deduced from these forces.[147]
1st ed. (1687) Definitions	I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view. Therefore, let the reader beware of thinking that by words of this kind I am anywhere defining a species or mode of action or a physical cause or reason, or that I am attributing forces in a true and physical sense.[148]
2nd ed. (1713) Editor's Preface	[T]hose whose natural philosophy is based on experiment [...]. [T]hey proceed by a twofold method, analytic and synthetic. From certain selected phenomena they deduce by analysis the forces of nature and the simpler laws of those forces, from which they then give the constitution of the rest of the phenomena by synthesis.[149]
2nd ed. (1713) General Scholium	In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method.[150]

[147] Newton, *Principia*, 1st ed. (1687), [5] (trans. C&W, 382) (angle brackets in original).
[148] Newton, *Principia*, 1st ed. (1687), 4-5 (trans. C&W, 408).
[149] Newton, *Principia*, 2nd ed. (1713), [9] (trans. C&W, 386).
[150] Newton, *Principia*, 2nd ed. (1713), 484 (trans. C&W, 943).

Opticks

1st English ed. (1704) Draft preface	As Mathematicians have two Methods of doing things which they call Composition & Resolution & in all difficulties have recourse to their method of resolution before they compound so in explaining the Phaenomena of nature the like methods are to be used.[151]
2nd English ed. (1717/18) Query 20/28	[T]he main Business of Natural Philosophy is to argue from Phaenomena without feigning hypotheses, and to deduce causes from effects.[152]
2nd English ed. (1717/18) Query 23/31	As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For Hypotheses are not to be regarded in experimental Philosophy. And although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of, and may be looked upon as so much the stronger, by how much the Induction is more general. And if no Exception occur from Phaenomena, the Conclusion may be pronounced generally. But if at any time afterwards any Exception shall occur from Experiments, it may then begin to be pronounced with such Exceptions as occur. By this way of Analysis we may proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general from Effects to their Causes, and from particular Causes to more general ones, till the Argument end in the most general. This is the Method of Analysis: And the Synthesis consists in assuming the Causes discover'd and establish'd as Principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanations. [153]

[151] Portsmouth Collection in the University Library, Cambridge, MS Add. 3970, fol. 480v, in Shapiro, "Newton's 'Experimental Philosophy'," 195-96.
[152] Query 28, in Newton, *Opticks*, 2nd English ed. (1718), 344.
[153] Query 31, in Newton, *Opticks*, 2nd English ed. (1718), 380-81.

Correspondence

Newton to Oldenburg, 6 February 1672	A naturalist would scearce expect to see ye science of those [i.e., colours] become mathematicall, & yet I dare affirm that there is as much certainty in it as in any other part of Opticks. For what I shall tell concerning them is not an Hypothesis but most rigid consequence, not conjectured by barely inferring 'tis thus because not otherwise or because it satisfies all phaenomena (the Philosophers universall Topick,) but evinced by ye mediation of experiments concluding directly & wthout any suspicion of doubt.[154]
Newton to Oldenburg, 11 June 1672	I should take notice of a casuall expression wch intimates a greater certainty in these things then I ever promised, viz: The certainty of *Mathematical Demonstrations*. I said indeed that the *Science of Colours was Mathematicall & as certain as any other part of Optiques;* but who knows not that Optiques & many other Mathematicall Sciences depend as well on Physicall Principles as on Mathematicall Demonstrations: And the absolute certainty of a Science cannot exceed the certainty of its Principles. Now the evidence by wch I asserted the Propositions of colours is in the next words expressed to be from *Experiments* & so but *Physicall*: Whence the Propositions themselves can be esteemed no more then *Physical Principles* of a Science.[155]
Newton to Oldenburg, 6 July 1672	[T]he proper Method for inquiring after the properties of things is to deduce them from Experiments. And I told you that the Theory wch I propounded was evinced to me, not by inferring tis thus because not otherwise, that is not by deducing it onely from a confutation of contrary suppositions, but by deriving it from Experiments concluding positively & directly.[156]
Newton to Cotes (draft), ca. 28 March 1713	Experimental philosophy argues only from phaenomena, draws general conclusions from the consent of phaenomena, & looks upon the conclusion as general when ye consent is general without exception.[157]
Newton to Cotes, 28 March 1713	[T]he Difficulty you mention wch lies in these words ⟨Et cum Attractio omnis mutua sit⟩ is removed by considering that as in Geometry the word Hypothesis is not taken in so large a sense as to include the Axiomes & Postulates, so in experimental Philosophy it is not to be taken in so large a sense as to include the first Principles or Axiomes wch I call the laws of

[154] Newton to Oldenburg, 6 February 1671/72, in *Correspondence*, 1:96-97 (addition added).
[155] Newton to Oldenburg, 11 June 1672, in *Correspondence*, 1:187 (italics in original).
[156] Newton to Oldenburg, 6 July 1672, in *Correspondence*, 1:209.
[157] Newton to Cotes (draft), ca. 28 March 1713, in *Correspondence*, 5:396-97.

	motion. These Principles are deduced from Phaenomena & made general by Induction: wch is the highest evidence that a Proposition can have in this philosophy. And the word Hypothesis is here used by me to signify only such a Proposition as is not a Phaenomenon nor deduced from any Phaenomena but assumed or supposed wthout any experimental proof. Now the mutual & mutually equal attraction of bodies is a branch of the third Law of motion.[158]

Optical Lectures

Lecture 3, 1670-72	I hope to show – as it were, by my example – how valuable mathematics is in natural philosophy. I therefore urge geometers to investigate nature more rigorously, and those devoted to natural science to learn geometry first. […] But truly with the help of philosophical geometers and geometrical philosophers, instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a natural science supported by the greatest evidence.[159]

[158] Newton to Cotes, 28 March 1713, in *Correspondence*, 5:396-97 (angle brackets in original).
[159] Newton, *The Optical Papers of Isaac Newton*, 1:86-89.

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RESEARCH PROPOSAL

DAVID BREWSTER'S INTERPRETATION OF NEWTON'S SCIENTIFIC METHOD

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The contemporary Newtonian literature is divided with regard to the interpretation of a number of central aspects of Newton's methodological statements and terminology. These statements are few and concise and thereby challenging to interpret, especially since there is considerable disagreement on their historical background – for example, whether Newton's method of analysis and synthesis was influenced by the medieval scholastic tradition, or whether Newton's concept of 'induction' is based on the methodological views of Francis Bacon. I propose therefore to examine these elusive terms and components of Newton's method from the historical perspective of the 18th century Scottish natural philosopher David Brewster – a proponent of Newton's natural philosophy and method, and the first biographer of Newton's life and works. I will consider Brewster's interpretation of Newton's method by means of an examination of his biographical works, the adaptations of Newton's scientific method in his own optical research, and a comparison of the application of the methods of both Newton and Brewster. By means of the historical background hereby constructed, I will reflect on the controversies regarding Newton's method in contemporary literature.

Description of the proposed research

Sir Isaac Newton's scientific theories and method had a significant influence on the worldview of the 17th and the 18th century. Newton asserted that his method differed from those of his predecessors and contemporaries with regard to his rigorous standard of certainty. However, Newton was not particularly forthcoming in describing his method. The most complete account presents it as a twofold method, consisting of analysis, which proceeds by deducing propositions from experiments or observed phenomena and generalising them by means of induction, and synthesis, which proves the aforementioned propositions and uses them to explain other phenomena.[1] In the contemporary scholarly literature, Newton's methodological statements and terminology are interpreted in diverse and often incompatible terms. A comparison of these interpretations provides some insight into where the differences lie,

[1] Isaac Newton, *Opticks: Or, a Treatise of the Reflections, Refractions, Inflections and Colours of Light*, 2nd English ed. (London: W. and J. Innys, printers to the Royal Society, 1718), 380-81.

namely, in Newton's meaning of the term 'induction', its relation to analysis and synthesis, and the stability of this relation – that is, whether it undergoes a development. This comparison furthermore reveals some inconsistencies among the interpretations. For example, Bechler shows that Newton first mentions 'induction' relatively late in his career.[2] This contradicts the interpretation of Garrison, who argues that Newton's method remained unchanged and that induction had been a part of this method throughout his career.[3] In addition, Shapiro demonstrates by means of draft material that Newton's concept of induction refers to the generalisation of multiple phenomena or experiments.[4] This conflicts with Garrison's and Hintikka & Remes' explanation of induction as the generalisation of a *single* phenomenon or experiment.

In this manner, a comparison of interpretations facilitates a critical discussion of Newton's methodological statements. However, the differences between the interpretations become even more conspicuous, which raises the question why Newton's terminology and statements are so diversely interpreted. One difficulty is that Newton describes his method in very concise terms, and he often only elaborates when explicitly requested to do so. These succinct descriptions leave considerable room for interpretation and little possibility for correction – particularly with regard to the meaning and interrelation of the methodological components. This may be illustrated by an example: Garrison and Hintikka & Remes argue that Newton's concept of induction can be understood in relation to his concept of the *experimentum crucis* – a single, well-conceived experiment that by itself suffices to prove a theorem and needs only to be performed once. Newton's theory of the different refrangibility of light rays, for example, was established by such an *experimentum crucis*. In this experiment, Newton refracted white light by means of two prisms and thereby demonstrated that the colours thereby shown were contained in the white light. Garrison and Hintikka & Remes argue that induction, which Newton described as a generalisation of empirically deduced propositions, functions in this manner: the result of an *experimentum crucis* does not need to be repeated but may be regarded as sufficient proof and hence generalised to all similar experiments.[5] By itself, this reading is quite convincing. It is only because of Shapiro's thorough exploration of the concept of induction in draft material – whereby he confirms his

[2] Zev Bechler, *Newton's Physics and the Conceptual Structure of the Scientific Revolution* (Dordrecht: Kluwer Academic Publishers, 1991), 351.
[3] James W. Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," *Journal of the History of Ideas* 48, no. 4 (1987): 622, https://doi.org/10.2307/2709690.
[4] Alan E. Shapiro, "Newton's 'Experimental Philosophy'," *Early Science and Medicine* 9, no. 3 (2004): 212-13.
[5] Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 622; Jaakko Hintikka and Unto Remes, *The Method of Analysis: Its Geometrical Origin and its General Significance*, Boston studies in the philosophy of science, (Dordrecht: Reidel, 1974), 110-11.

own interpretation that the term refers to the generalisation of a selection of phenomena, that this explanation is contradicted.[6]

If Newton's (published) methodological statements allow for such different, and even contradicting interpretations as that of induction described above, why did he not specify his terminology? It may be reasoned that he did not realise that his terms were ambiguous, or that he purposefully gave vague descriptions to, for example, dissuade criticism. But a more congenial and, I think, more plausible explanation is that the terms were not as vague or ambiguous for Newton and his readers as they now appear to us. If this is the case, the difficulties in our contemporary interpretation of these terms could originate from a difference in context or background from which the terms are understood. This seems to be confirmed by some of the discussions in the Newtonian literature, as the following three examples demonstrate. The first example concerns the interpretation of Newton's method of analysis and synthesis. This method, which originates from the ancient geometers, had an important role in the medieval scholastic tradition. Newton's use of this method has been associated with this scholastic adaptation of it. However, both Levitin and Hintikka & Remes question this association. In the scholastic tradition, analysis and synthesis were used to derive causes from effects by means of linear deductions.[7] Instead, Hintikka & Remes argue, Newton's method of analysis and synthesis proceeded by means of the identification of relations of interdependencies, and hence did not have a linear structure.[8] Levitin explains that this connection between the scholastic tradition and Newton's use of analysis and synthesis has "a very major impact […] on our understanding of Newton's intellectual formation."[9] He explains for example that some of Newton's statements – that are interpreted as metaphysical statements on the basis of this association with the scholastic tradition – could otherwise be more consistently read as a criticism on this tradition.[10] Another example concerns the meaning of Newton's concept of induction. Our understanding of inductive reasoning in Modern Philosophy is generally associated with Francis Bacon's methodological tradition. It is argued, for example by Ducheyne, that Newton's use of induction is derived from this tradition, and that Newton's meaning of the term therefore may be understood as in similar terms.[11] Pérez-Ramos explains however, that Newton never mentioned Bacon by name and that this connection is only suggested by Newton's posthumous editors.[12] In addition, an early

[6] Shapiro, "Newton's 'Experimental Philosophy'," 212-13.
[7] Hintikka and Remes, *The Method of Analysis*, 39, 107.
[8] Ibid., 107-09.
[9] Dmitri Levitin, "Newton and scholastic philosophy," *The British Journal for the History of Science* 49, no. 1 (2016): 56, https://doi.org/10.1017/S0007087415000667.
[10] Ibid., 70-73.
[11] Steffen Ducheyne, "Bacon's Idea and Newton's Practice of Induction," *Philosophica* 76, no. 2 (2005): 116.
[12] Antonio Pérez-ramos, "Bacon's Legacy," in *The Cambridge Companion to Bacon*, ed. Markku Peltonen (Cambridge: Cambridge University Press, 1996), 319.

biographer of Newton explicitly states that there is no indication that Newton's term of 'induction' is derived from Bacon.[13] Lastly, Garrison argues that Newton's method of analysis and synthesis should be understood in the context of Newton's constructivist view of mathematics. However, he explains, "[s]ince constructivism is currently unfashionable, it is rarely taught or at least not taught sympathetically. This is one likely source for many misinterpretations of Newton's work."[14]

These examples demonstrate how the interpretation of Newton's methodological terms is influenced by the historical background associated with them. They thereby provide some additional insight into the source of the controversies, and in how our understanding of the method may be improved: a consideration of the historical background of Newton's methodological statements and terminology could provide a more transparent basis for the interpretation of the method. Such a background may for example be provided by an examination of Newton's natural philosophy and method in relation to that of his predecessors, contemporaries or followers – consider, for example, works as Hall's *From Galileo to Newton* and Butts & Davis' *The Methodological Heritage of Newton*.[15] Especially when such comparisons are considered together, they can provide a critical and comprehensive overview of the historical background. Another possibility, which offers a more comprehensive overview by itself, is to examine the material of a historical interpreter of Newton's method. Such an examination has not yet been employed as much, and can provide valuable insight into our consideration of Newton's methodology.

I propose that the Scottish natural philosopher Sir David Brewster (1781-1868) would be a suitable candidate for this role of historical intermediary. Brewster was the first to examine the collection of Newton's correspondence, notebooks, drafts and other unpublished materials that were available at the time. He wrote two substantial biographies on Newton's life, his contributions to natural philosophy and his scientific method. And although these biographies are quoted now and again, there has not been a specific study of Brewster's portrayal of Newton's method. He furthermore wrote about the works and methods of Galileo, Brahe and Keppler, which had influenced Newton's studies on celestial mechanics. In addition, Brewster studied and lectured physics, geometry and optics. In his optical works he examined many of the subjects Newton had considered in his *Opticks* (1704). Brewster was therefore in a position of intimate familiarity with, and understanding of, Newton's natural philosophy and method. His interpretation of Newton's method could therefore provide a valuable reference point for our contemporary interpretations. On the basis of the background knowledge he had – or did

[13] David Brewster, *The life of Sir Isaac Newton* (London: Gall and Inglis, 1831), 330-32.
[14] Garrison, "Newton and the Relation of Mathematics to Natural Philosophy," 609.
[15] Robert E. Butts and John Whitney Davis, eds., *The Methodological Heritage of Newton* (Oxford: Basil Blackwell, 1970); A.R. Hall, *From Galileo to Newton* (Dover Publications, 2012).

not have, and hence which we might erroneously connect with Newton – Brewster could provide insight into Newton’s methodological statements and terminology.

Aim and method

The aim of my research is to examine Newton’s scientific method by means of David Brewster’s biographical and scientific works, and on this basis to evaluate the contemporary interpretations of Newton’s method, among which particularly the interpretations of those aspects that appear to us ambiguous. In addition, I aim to provide an outline of Brewster’s scientific method, since there has not yet been a study to this effect.

Three aspects of Brewster’s interpretation of Newton will be considered. Firstly, Newton’s and Brewster’s natural philosophy and method will be outlined. This will include a discussion of their methodological statements – that is, the description or portrayal of their methods – as well as an examination of the methods they use in their scientific works. Secondly, I will study Brewster’s interpretation of Newton’s scientific method, on the basis of both Brewster’s biographical materials on Newton, and his own scientific works and lectures. In these latter, non-biographical texts, Brewster regularly reflects on Newton’s methodology and compares it with his own method and that of a number of his contemporaries and predecessors. Thirdly, the application of Newton’s and Brewster’s methods will be compared by means of a side-by-side consideration of some of the prominent experiments and arguments described by both authors. This comparison will consist primarily of a selection of Newton’s optical experiments that Brewster replicates. The historical context and background that is constructed by this exploration of Brewster’s examination of Newton’s natural philosophy and method will, lastly, enable us to evaluate on the controversial aspects of the contemporary interpretations of Newton’s method.

Relevance

The scientific relevance of my project is twofold. As described above, there are controversial and incompatible views of a number of central elements of Newton’s scientific method in the contemporary literature. A consideration of these controversies suggests that a better understanding of the historical context of Newton’s terminology and methodological statements could provide a useful context for their interpretation. Through an examination of Brewster’s interpretation of the method I will provide such a historical background, by means of which the controversial aspects in contemporary interpretations may be evaluated. This will, secondly, provide a comprehensive exploration of Brewster’s views, reflection and adaptation of Newton’s scientific method and thereby supplement the modest number of studies on David Brewster in general and, particularly, those regarding his views on Newton’s natural philosophy and method.

The social relevance of my project may be viewed in terms of a contribution to our understanding of a prominent and influential figure in the history of science. A more complete insight into Newton's methodological statements would enable a better understanding of how he viewed and presented his own approach to science, especially since Newton's method differed considerably from those of his contemporaries and predecessors. This may furthermore be compared with the views from other consequential historical thinkers, and from which we could draw lessons on how to facilitate the development of such influential ideas. In addition, the examination of Newton's method by David Brewster could enhance our insight into the role of historical biographers. It would also be a possibility to increase the availability of Brewster's letters and possibly other material that is not yet digitalised.

Key words

Isaac Newton, David Brewster, Optics, Natural Philosophy, Philosophy of Science, Scientific Methodology, Scottish Enlightenment

Timetable

<table>
<tr><th></th><th>1st quarter</th><th>2nd quarter</th><th>3rd quarter</th><th>4th quarter</th><th>Output</th></tr>
<tr><td rowspan="4">1</td><td>Start reading secondary literature on Newton and Brewster</td><td>Start reading Newton's scientific works (focussed on optics)</td><td colspan="2">Write a description of Newton's application of his method (focussed on optics)</td><td rowspan="4">* Part of the first chapter(s) on Newton's description ad use of his method complete
* Outline comparison of Brewster's and Newton's methods
*Published reviews</td></tr>
<tr><td rowspan="2">Write and publish a review of secondary literature on Newton's method</td><td>Write a description of Newton's methodological statements and terms</td><td>Write and publish a review of secondary literature on Brewster</td><td>Start reading Brewster's scientific works and lectures</td></tr>
<tr><td>Prepare course to be taught in next semester</td><td colspan="2">Teach a course to BA Philosophy students</td></tr>
<tr><td colspan="4">Attend a Philosophy of Science Association event</td></tr>
</table>

<table>
<tr><td rowspan="3">2</td><td colspan="2">Write a description of Brewster’s methodological statements</td><td>Select optical experiments for the side-by-side comparison</td><td>Start writing an overview of Brewster’s interpretation of Newton’s method</td><td rowspan="3">* First chapters on the description and application of Newton’s and Brewster’s method are complete (but not finalised)

*Chapter(s) on Brewster’s discussion of Newton’s method are mostly complete

* Outline of difficulties in contemporary literature that I will reflect on later</td></tr>
<tr><td>Start reading Brewster’s biographies of Newton</td><td colspan="2">Write a description of Brewster’s application of his method (focussed on optics)</td><td>Select controversies in contemporary literature that are considered by Brewster (e.g., the origin and meaning of Newton’s induction)</td></tr>
<tr><td colspan="4">Study physical letters of Brewster at the universities of Edinburgh and St. Andrews and digitalise a selection of relevant letters</td></tr>
<tr><td rowspan="4">3</td><td colspan="2">Compare the selected optical experiments</td><td colspan="2">Write an article on Brewster’s optical method</td><td rowspan="4">* Chapter(s) on Brewster’s discussion of Newton’s method are complete

*Chapter(s) on the side-by-side-comparison of Newton’s and Brewster’s optical experiments is complete

* introduction is complete</td></tr>
<tr><td>Continue writing an overview of Brewster’s interpretation of Newton’s method</td><td colspan="2" rowspan="2">Write chapter on the comparison of Newton and Brewster’s experiments</td><td rowspan="2">Start writing chapter difficulties in interpreting Newton in recent literature and their evaluation by means of Brewster’s interpretation.</td></tr>
<tr><td>Write an introduction</td></tr>
<tr><td colspan="4">Give a reading on Brewster’s portrayal of Newton’s natural philosophy and method</td></tr>
<tr><td>4</td><td>Continue writing chapter on the</td><td>Finalise chapters and write conclusion</td><td>Continue finalising the dissertation</td><td>Implement feedback</td><td>* Chapter(s) on difficulties in interpreting</td></tr>
</table>

	difficulties in interpreting Newton in recent literature and their evaluation by means of Brewster's interpretation		and implement feedback		Newton in recent literature and their evaluation by means of Brewster's interpretation
		Hand in mostly complete dissertation	Hand in finalised thesis for last feedback	Prepare for the defence	
				Start preparing for the presentation of my results at a history of philosophy conference	* Finalised dissertation

Summary for non-specialists

Sir Isaac Newton's scientific theories and method had a significant influence on the worldview of the 17th and the 18th century. Newton asserted that his method differed from those of his predecessors and contemporaries with regard to his rigorous standard of certainty. However, he was not particularly forthcoming in describing this method: Newton's methodological statements are few and concise and thereby leave considerable room for interpretation, and little possibility for the correction of erroneous explanations. As a result, the interpretations of the method in contemporary scholarly literature differ significantly from each other and are partly incompatible. This raises the question why Newton did not provide a less ambiguous description of his method. It may be reasoned that he did not realise that his terms were ambiguous, or that he purposefully gave vague descriptions to, for example, dissuade criticism. But a more congenial and, I think, more plausible explanation is that the terms were not as vague or ambiguous for Newton and his readers as they now appear to us. But that we interpret the terms from such a different context that the explanations Newton gave are insufficient for us to apprehend his meaning. The interference of the difference in context is also discussed in the contemporary Newtonian literature. For example, authors argue that Newton's meaning of 'induction' can be interpreted in various manners based on whether it is considered from the perspective of one historical tradition or another. Hence, if we could obtain an overview of the historical context by means of which Newton used and explained his methodological terms, we could use the same context as a reference point to evaluate our interpretations, particularly those aspects that are perceived as ambiguous by contemporary scholars. Such a background may be derived from the interpretation of Newton's scientific work and method by the Scottish natural philosopher Sir David Brewster (1781-1868). Brewster was a proponent of Newton's scientific views and method, and was the first to examine the collection of Newton's correspondence, notebooks, drafts and other unpublished materials. He wrote two substantial biographies on Newton's life, his contributions to natural philosophy and his scientific method. And although these biographies are quoted now and again, there has not been a specific study of them. Hence, Brewster's works could provide a valuable reference point for our contemporary interpretations. On the basis of the background knowledge he had – or did not have, and hence which we might erroneously connect with Newton – Brewster's discussions could provide insight into Newton's methodological statements and terminology. I propose therefore to examine Newton's scientific method by means of David Brewster's biographical and scientific works, and, on this basis, to evaluate the contemporary interpretations of Newton's method, among which particularly the interpretations of those aspects that appear to us ambiguous. In addition, I aim to provide an outline of Brewster's scientific method, since there has not yet been a study to this effect.

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MA Philosophy	2021
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LANGUAGE SKILLS

Modern languages

Dutch	native proficiency
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German	professional working proficiency

Ancient languages

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EDUCATION

Research Master History of Philosophy - Radboud University 2020-current

Master Philosophy and Science - Radboud University 2019-2021

- **Thesis** "The Citizen's Pursuit of Happiness: On the Social and Political Philosophy of Epicurus"

Premaster Philosophy - Radboud University 2017-2019

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- **Minor** Applied Psychology at Thomas More, Antwerp
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