

# Radboud University



ARTIFICIAL INTELLIGENCE

BACHELOR THESIS

---

## Using particle filters for short-term groundwater level predictions

---

*Author:*

Janne Bartels (s1011086),  
Radboud University

*Supervisors:*

Nils Donselaar, MSc, Donders Centre for Cognition, Radboud University  
dr. ir. Johan Kwisthout, Donders Centre for Cognition, Radboud University

June 18, 2020

## Abstract

Particle filter is a Bayesian inference method and provides a way to make predictions in dynamic systems where no assumption of linearity or a Gaussian distribution of uncertainty can be made. For these reasons, it was considered a suitable method for making a short-term prediction of groundwater levels. Groundwater level predictions allow farmers to preemptively water or drain their land and thus improve crop production. In the process of developing the particle filter six steps were elaborated: 1) developing a weather model; 2) defining the initial distribution; 3) calculating the prediction; 4) updating the prediction; 5) resampling for the next prediction and 6) calculating the outcome. Three methods were developed for the prediction step to account for applying the particle filter to a 14-days prediction. These methods were compared with a conventional hydrological model to assess prediction performance for a 48-days period. Upon testing these three methods, the characteristics of each method are described. Overall, there is not one method that outperforms the hydrological model that was taken as a baseline. Combining the methods and in this way complementing the individual strengths of each method is of interest for future research.

## 1 Introduction

Groundwater, which refers to all water that is stored in the ground, supplies sixty percent of drinking water in the Netherlands (RIVM, 2018) and is necessary for sustaining plants, crops and wetland habitats (Taylor et al., 2013). If groundwater levels are too low, crops may fail, and if too high, cities and crops may flood, which can cause lasting damage and expensive repairs.

Because of this importance, every day, everywhere, groundwater levels are monitored. This is done using monitoring wells, which are tubes placed in the ground that let water go through. The groundwater level is defined as the level the water reaches in these tubes when placed in the upper layer of groundwater. Changes in groundwater level can be explained in retrospect, however it is difficult to predict groundwater levels because influencing factors and their interactions with groundwater are complex and not fully understood. Yet, having accurate forecasts allows local authorities and farmers to take anticipatory measures, reducing the impact of extreme groundwater levels.

Models like Menyanthes (KWR, 2020) explain groundwater fluctuations using factors such as precipitation, evaporation and drainage data (Von Asmuth, 2012). Precipitation and evaporation in particular are often used as these explain much of the change in groundwater level close to the surface and are easier to obtain than drainage data. The models estimate the relative effect of these different factors on the groundwater level in the long run. This is done, for example, to identify trends or show the impact of urban developments. Results of these models are used to ensure long-term stable groundwater levels.

Depending on soil-type and depth of the groundwater level, changes in level may be fast or slow. When the groundwater level is close to the surface and the soil takes up rainwater easily, levels can change up to ten centimeters a day, which can result in flooding in these areas. In such situations, short-term predictions provide valuable information.

Contemporary research describes some short-term predictive models. These models often use a neural-network-based approach and make a single prediction per month for a period of at least one year (Shirmohammadi et al., 2013), (Sun, 2013), (Daliakopoulos et al., 2005). Although most

models seem to rely solely on groundwater level measurements, some models add precipitation and evaporation data, specifically in the form of monthly averages (Kasiviswanathan et al., 2016), (Fang et al., 2019). Interestingly, none seem to use weather forecasts. This might be a missed opportunity as precipitation and evaporation are the most influential factors on groundwater level close to the surface and weather forecasts could give more accurate information on the expected influence than averages.

Another method to make a prediction given observations, is Bayesian modelling. Bayesian inference, a subset of Bayesian modelling methods, in particular is used to find the posterior probability distribution, that is the probability of  $x$  (groundwater level) given observations  $y$  (weather forecast). Some reasons for using a Bayesian inference method instead of a neural network, are that less data is required for training the model parameters and the connections between factors can be better interpreted (Zhang and Bivens, 2007). Particularly, the latter is interesting as it could lead to new insights on the relative influence of different factors on groundwater.

A specific Bayesian inference method is the particle filter. With this method, no assumption of linearity or a Gaussian distribution of uncertainty has to be made (Chen et al., 2003), which makes it useful for describing complex or unpredictable systems. Samples are used to approximate the posterior probability distribution. Because it is an approximation, there is no need for an exact mathematical solution which can only be achieved by making assumptions such as the aforementioned.

In this study the aim was to develop a particle filter model to make daily fourteen-day prediction of the groundwater level with associated uncertainty. Specific measuring wells were selected based on highly fluctuating groundwater levels and weather forecast information was added as observations. The particle filter predictions were defined as successful if the uncertainty was less than the average change in groundwater per day. The particle filter was chosen as it allowed inclusion of weather forecast information as well as training with a relatively small dataset and because linearity and Gaussian distribution of uncertainty cannot be assumed to hold in a complex system such as groundwater.

In sections 2.1 and 2.2 more background on the groundwater system and particle filters is given. The development of the initial model is described in section 3, initial results (section 3.4.1) showed that some adjustments to the model were required. These adjustments are detailed in sections 3.4.2-3.4.4. Section 4 gives the results of testing the adjusted model and a discussion with conclusions on results can be found in section 5.

## 2 Theoretical background

### 2.1 The groundwater system

Every type of soil has pores and fractures which can be filled with water or air. Groundwater is defined as the water in saturated layers of the soil. Above the saturated layer there might still be water in some of the pores or fractures, this is the unsaturated layer and not considered part of the groundwater.

Soil consists of different layers of sediment, each of these layers has a characteristic permeability (how easily water flows through the layer) and storage space (how much water can be taken up in the layer). The less permeable a layer, the longer it will take water to pass through. This can

create several bodies of groundwater, so-called aquifers. Separated by impermeable layers, these aquifers have little interaction. As a rule of thumb it holds that the lower the aquifer, the slower the changes in groundwater level. Only in the uppermost aquifer, the phreatic aquifer, there are groundwater level fluctuations fast enough that short-term predictions, such as described in this thesis, are relevant. Therefore, only these aquifers will be considered. For a more in-depth explanation, see (Bear, 2012).

Groundwater levels are measured using monitoring wells, see (Barcelona et al., 1983). Which are tubes that are placed in the ground. These can have lengths varying between four and one hundred meters depending on what layer of soil is assessed. A part of the tube is perforated, this lets the groundwater through which rises to the level of saturated soil around the tube, or even higher in lower aquifers where pressure can build up. This level is recorded and, if measured at the phreatic aquifer, used as a groundwater level measurement.

Groundwater levels generally have a wave-like shape throughout the year with the highest level during winter and the lowest level during summer. This fluctuation is influenced by precipitation, evaporation, soil-type and surrounding water levels. Besides these, human interventions such as pumping may have a strong effect. The factors that generally have the strongest effect are precipitation and evaporation. Precipitation adds to the groundwater level, while evaporation detracts from it.

Groundwater models such as Menyanthes or Pastas (Collenteur et al., 2019) are used to quantify the effect precipitation and evaporation have as well as the delay in their effect, which is caused by the time it takes to traverse the unsaturated soil between the surface and the groundwater body. Often, the effects of precipitation and evaporation are combined by calculating the recharge, which is the amount of precipitation ( $p$ ) minus the amount of water that will evaporate ( $e$ ) multiplied by some factor ( $f$ ):  $R = p - f \cdot e$

The model used in this thesis is based on this approach.

## 2.2 Particle Filtering

Particle filters have been around since the 1950s, though they were only first named such in a seminal work by Gordon et al. (Gordon et al., 1995). Around the same time Pierre del Moral (Del Moral, 1997) provided the first rigorous proof of the theory. Many contributions and expansions to the field have been made since, amongst which the proof by Hu et al. (Hu et al., 2008) which shows convergence for unbounded functions, such as groundwater levels.

During this time particle filters have been applied to a variety of problems, such as determining the 3D orientation of spacecrafts in videos (Bazik et al., 2019), (indoor) location of people based on WiFi-signals (Nurminen et al., 2013), and parameter estimation of a hydrological function (Berg et al., 2019).

As mentioned, particle filters are a method to approximate the posterior distribution,  $p(x | y)$  where  $x$ , the particles, represent the possible states of the system and  $y$ , the observations, represent measurements about the system. To make the approximation a Monte Carlo approach is used. The

particle filter is a first order Markov process for which the following distributions must be known:

$$p(x_0) - \text{the initial distribution} \quad (1)$$

$$p(x_{t+1} | x_t) - \text{the prior distribution} \quad (2)$$

$$p(y_t | x_t) - \text{the likelihood distribution} \quad (3)$$

Where  $x_t$  represents a set of particles at time  $t$ ,  $y_t$  the associated observations.  $x_0$  is a special case, describing the particles at time 0, the first time step of this model. The first distribution,  $p(x_0)$ , is the initial spread of particles. The second distribution,  $p(x_{t+1} | x_t)$ , describes the relation between the current ( $x_t$ ) and the next ( $x_{t+1}$ ) particles without any prior knowledge of the observations. Finally,  $p(y_t | x_t)$  describes the likelihood of the observation  $y$  at time  $t$  given the particle  $x$  at time  $t$ .

The basic concept of the particle filter is that the posterior can be approximated at all times by a weighted sum over all particles, specifically as follows:

$$p(x_t | y_t) = \sum_{i=1}^N (w_t(i) \cdot \delta(x_t - x_t(i))) \quad (4)$$

Here  $N$  indicates the total number of particles,  $x_t(i)$  is the value of particle  $i$  at time  $t$  and  $w_t(i)$  is the associated weight and  $\delta()$  is the Dirac delta function (Salamon and Feyen, 2009).

The process of particle filtering consists of three steps: prediction, update and resampling.

Prediction is done each new time step to go from  $x_t$  to  $x_{t+1}$ . The prediction distribution should describe how much a system could change in the time step. This is done using a proposal distribution  $q$ :

$$q(x_{t+1} | x_t, y_t) \quad (5)$$

The proposal distribution should approximate the posterior distribution as closely as possible; in practice it is often defined as the prior distribution.

$$q(x_{t+1} | x_t, y_t) = p(x_{t+1} | x_t) \quad (6)$$

(Salamon and Feyen, 2009), (Nurminen et al., 2013).

The update step assigns weights to the newly distributed particles, based on new observations.

$$w(i)_t = w(i)_{t-1} \cdot \frac{p(y_t | x(i)_t) \cdot p(x(i)_t | x(i)_{t-1})}{q(x(i)_t | x(i)_{t-1}, y_t)} \quad (7)$$

Because of the approach described by (6), this becomes:

$$w(i)_t \propto w(i)_{t-1} \cdot p(y_t | x(i)_t) \quad (8)$$

The weights are then normalized through division by the sum of all weights. Using this formula, weights are assigned inversely proportional to deviation from the observed value.

In the third step of the process, the particles are resampled according to weight such that there are more particles in areas with a higher weight. In this way particle degeneracy is avoided, which is a problem where particles are not properly representing the probability distribution (Snyder et al., 2008). There are many methods for resampling (Kozierski et al., 2013), the one most commonly used being redistribution according to weight.

## 3 Methods

### 3.1 Groundwater measurements

In this study, data on groundwater levels was used from two wells in Losser and two wells in Oldenzaal both in The Netherlands. These wells were selected as daily uninterrupted groundwater levels were recorded for at least one and a half years and water levels in these areas reacted quickly to precipitation. The wells are part of a larger network of wells monitored by Tauw, an environmental consultancy agency that initiated this research.

On average, the groundwater level was 3700 centimetres above Amsterdam Ordance Datum (Nationaal Amsterdamse Peil or NAP) with an average change of about one centimetre a day. With extreme amounts of rain, a reaction can be seen within two days. The groundwater data was split up in a training and a testing set. For testing, a period of 48 days was selected based on the weather forecast data available, the remaining full years were selected as training data. Testing was done by comparing the particle predictions against the actual groundwater measurements.

### 3.2 Meteorological data

In the Netherlands, the Dutch meteorological institute (KNMI) records and publishes daily weather measurements. These recordings are used by experts at Tauw to explain groundwater level changes. The KNMI also provides weather forecasts, however these were not stored in a format that was practical for this application. The stored forecast consisted of a general text but did not give any numerical estimate and had no mention of the expected evaporation.

Further search showed that no other weather institute stored weather forecast data either. It was possible to create a smaller testing dataset of weather forecasts by downloading daily forecasts from WorldWeatherOnline.com. WorldWeatherOnline.com provided an easy to use API to request the weather forecast for a specific period and with the required variables, which was called daily using a Python script <sup>1</sup>. The script ran 48 days - from the 10th of April until the 28th of May 2020. A longer period of forecast data would have been preferable but was not possible because of deadlines for this project. Therefore, weather measurements in Twente as recorded by the KNMI were used for training instead, specifically daily precipitation in millimetres and evaporation in centimetre squared.

In the forecast data, evaporation was not given. Instead, it was calculated using the provided radiation and temperature in the Makkink formula described by the KNMI guidebook (KNMI, 2005), see Appendix 7.1.

### 3.3 Implementation of the particle filter

The pseudo code below shows the implementation of a generic particle filter. Before going over the implementation, time indexing needs to be addressed. In most particle filtering problems, predictions are given for one time step only. In the loop,  $x_t$  goes to  $x_{t+1}$ , the weights get updated and resampling happens. Then  $x_{t+1}$  changes into the next  $x_t$  and the loop starts over (see Figure 1).

In this study, that would translate to a one-day ahead prediction, instead the goal was to make a fourteen-days ahead prediction. In order to indicate this, the following notation is introduced.

$$x_t^{dn}(i) \tag{9}$$

---

<sup>1</sup>code can be found at <https://gitlab.socsci.ru.nl/J.Bartels/particlefilter>

---

**Pseudo-code**

---

```

1:  $N_p \leftarrow$  number of particles to be used
    $N_d \leftarrow$  number of days to make predictions about (here 14)
    $t \leftarrow 0$ 
2: for  $dn \in 1 \dots N_d$  do
3:   for  $i \in 1 \dots N_p$  do
      $x_0^{dn}(i) \leftarrow p(x_0)$  initialize particles
      $w_0^{dn}(i) \leftarrow 0$  initialize weights
4:   end for
5: end for
6: for each day  $t$  do
7:   for each  $d_n \in N_d$  do
8:      $x_{t+1}^{dn} \leftarrow$  model of system movement prediction
9:      $y_{t+1}^{dn} \leftarrow$  model(weather predictions) get observations
10:    for each  $i \in N_p$  do
       $w_{t+1}^{dn}(i) \leftarrow w_t^{dn}(i) \cdot \text{weighting function}(y_{t+1}^{dn}, x_{t+1}^{dn}(i))$  update weights
11:    end for
12:     $x_{t+1}^{dn} \leftarrow$  weighted choice( $x_{t+1}^{dn}, w_{t+1}^{dn}$ ) resample
13:     $t \leftarrow t + 1$  go to next day
14:  end for
15: end for

```

---

This indicates particle  $x(i)$  at day  $t$  making a prediction about  $dn$  days ahead, where  $t$  is the current day. As an example, for particle 20 making a prediction seven days ahead from yesterday, it is noted as:  $x_{t-1}^{d7}(20)$ . This is one particle in a total of  $Np$  particles at  $dn$ . The weighted sum all  $Np$  particles approximates the probability  $p(x_t^{dn} | y_t^{dn})$ , so the posterior probability for prediction day  $dn$  at time  $t$ .

In order to develop the particle filter, six steps needed to be specified:

- A weather model
- The initial distribution -  $p(x_0)$
- The prediction step -  $p(x_{t+1} | x_t)$
- The update step -  $p(y_t | x_t)$
- The resampling step
- A function to calculate the outcome

### 3.3.1 Weather model

The weather model is a model used to translate weather data into predictions of groundwater levels, as the observations need to be comparable to the particle predictions. The model used a formula where parameters were optimized using a training set of groundwater measurements and weather data. Using these parameters and new weather data, the model simulated the expected groundwater level. This was used as an observation ( $y$ ) to weigh a particle prediction ( $x$ ).

As the model, the Python package Pastas was used. Pastas is "... an open-source framework for hydrological analysis" (Collenteur et al., 2019). The model was trained using as many full years of

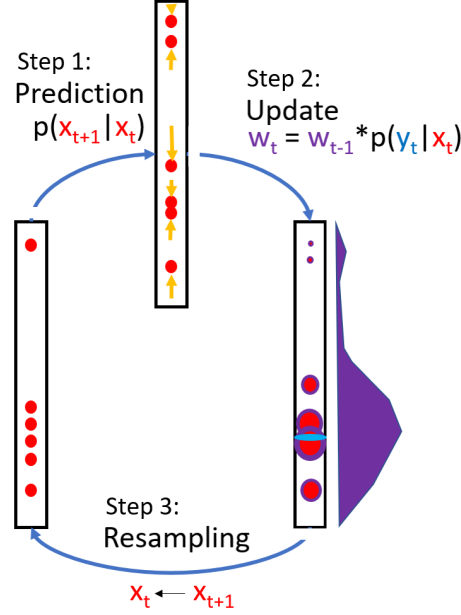


Figure 1: Particle filtering loop.

Individual particles ( $x$ , red dots) change position through the prediction step (yellow arrows) and are weighted in the update step (purple lining of dots) based on an observation (blue oval), resulting in an approximation of the posterior distribution (purple graph next to particles of update step). Finally, the particles get resampled based on weight, after which the next round of particle filtering begins.

groundwater measurements and weather data as were available after exempting the testing data. For the wells in Losser this was two years of measurements, for the wells in Oldenzaal this was one year. Full years of training were needed to accommodate for the seasonality of the groundwater system. Forecast data was added to the model each day and used to simulate the groundwater movement for the next fourteen days<sup>2</sup>.

### 3.3.2 Initialization

In order to start the particle filter, an initial, uniform distribution of particles,  $p(x_0)$ , was defined to allow particles to spread over the range of groundwater levels found in the training data:

$$p(x_0) = \mathcal{U}(\max(\text{train}), \min(\text{train})) \quad (10)$$

### 3.3.3 Prediction step

In the prediction step, the system goes from time  $t$  to time  $t + 1$  using the probability function  $p(x_{t+1} | p_t)$  (6). First, it needed to be defined how the system changed in one time step and then the change needed to be translated into a probability function.

The function did not use  $y$ , so nothing about the environment was taken into account. Instead it described fluctuation in groundwater levels disregarding environmental factors. Here, fluctuation

<sup>2</sup>For the full specifications, see <https://gitlab.socsci.ru.nl/J.Bartels/particlefilter>



was defined as the difference in groundwater level between two days. When plotting, the histogram approximated a normal distribution (Figure 2). For this reason, it was decided to model the

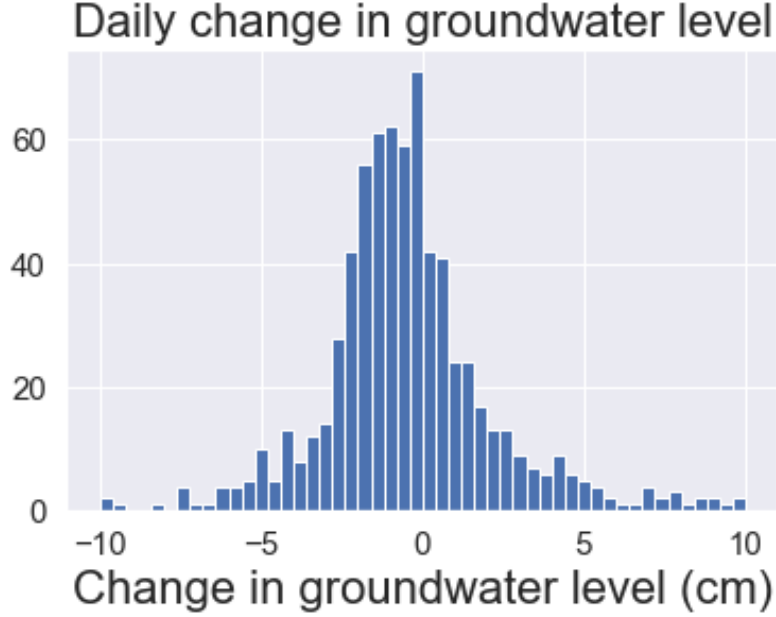


Figure 2: Histogram of differences in groundwater levels between days.

groundwater movement using a normal distribution with the mean change of groundwater level per day in the training data and the associated standard deviation.  $p(x_{t+1} | x_t)$  is then defined as:

$$x_{t+1} = x_t + \mathcal{N}(\mu, \sigma^2) \quad (11)$$

### 3.3.4 Update

The weights were assigned according to (8) in the update step. The question was how to calculate  $p(y_t | x_t)$ . In general it should hold that particles further away from the observation should get a lower weight. The exact implementation differs per domain, often taking into account constraints specific to that domain.

The constraints in this case were unknown, therefore it was decided to base the update function on approaches used in other groundwater related papers. In many cases the specification of the update function was not given. When it was given, for example in (Salamon and Feyen, 2009) and (Moradkhani et al., 2005), the following function was used:

$$w_{t+1}^{dn}(i) = w_t^{dn} \cdot \frac{\exp(-\frac{1}{2} \cdot (y_{t+1}^{dn} - x_{t+1}^{dn}(i))^2)}{\sum_i^{N_p} \exp(-\frac{1}{2} \cdot (y_{t+1}^{dn} - x_{t+1}^{dn}(i))^2)} \quad (12)$$

Here  $w$  are the weights,  $x$  the particles and  $y$  the observations.

The function gave an exponentially lower weight when the difference between the observation and the particle was larger. This resulted in a distribution approaching normal around the observation. As the weather-based predictions were expected to be somewhere around the actual groundwater measurements, such a distribution seemed appropriate. This function was therefore chosen as the update function.

### 3.3.5 Resampling

For resampling particles were redistributed by weighted choice. The probability of a particle getting selected was proportional to its weight. This is a relatively simple method but it seemed appropriate because the problem was also relatively simple, as it had only one dimension.

### 3.3.6 Outcome

The outcome of the particle filter should be one value per prediction day along with the uncertainty of this value. The mean value over all particles after the resampling step was taken as the outcome value, representing the best estimate of the groundwater level. The standard deviation was calculated to indicate the uncertainty of the prediction. The weighted sum over particles is not used as this describes a probability of a state, not the state itself.

## 3.4 Second implementation based on initial results

### 3.4.1 Initial model assessment

For developing the particle filter, groundwater level measurements and weather measurements for a third well in Losser were used. Training was a period of one year from June 2018 until June 2019 and testing was two months, June and July 2019. This approach was chosen as the other data was not available at the time. A first run of this trial data, gave the following output<sup>3</sup> (Figure 3). The groundwater level (blue dots) remained quite stable, moving two centimeters up or down each day and remaining around 3530 cm above NAP. The weather-based predictions (yellow lines) did not exactly follow the fluctuations of the groundwater level but remained within ten centimetres. The particle filter predictions (coloured lines) centered around 3550 centimetres with a fluctuation of about five centimeters a day. The uncertainty in the particle filter predictions (vertical lines) remained twenty centimetre at all times.

The following issues were observed:

**Observation 1:** The uncertainty started and remained large.

The uncertainty was expected to be not much more than the average change in groundwater level per day and decrease over time as particles approached to the posterior probability distribution. Yet, while the average change in groundwater level was around one centimetre, the uncertainty was nearly thirty and it did not decrease.

**Observation 2:** The particles were not influenced by the weather-based predictions.

Initially the particles were far apart but over time they were expected to move towards the weather-based predictions. As Figure 3 showed, the particles started and stayed much above the weather predictions.

**Observation 3:** There was no coherence between particle predictions on the same day.

It was expected that every prediction about the same day was similar. For example, it was expected that all predictions for June the 6th predicted a low groundwater level. Instead, there was a ten centimeter difference between the different predictions. The prediction made at the last time step (orange line) predicted low levels, the one before (brown line) predicted high levels and the prediction made at the first time step (light blue line) predicted something in between.

The first two issues could be caused at all steps, the latter only at the prediction step.

---

<sup>3</sup>The figure below shows a small selection of the trial period that suffices for the current purposes. For results for the full trial period, see Appendix 7.2

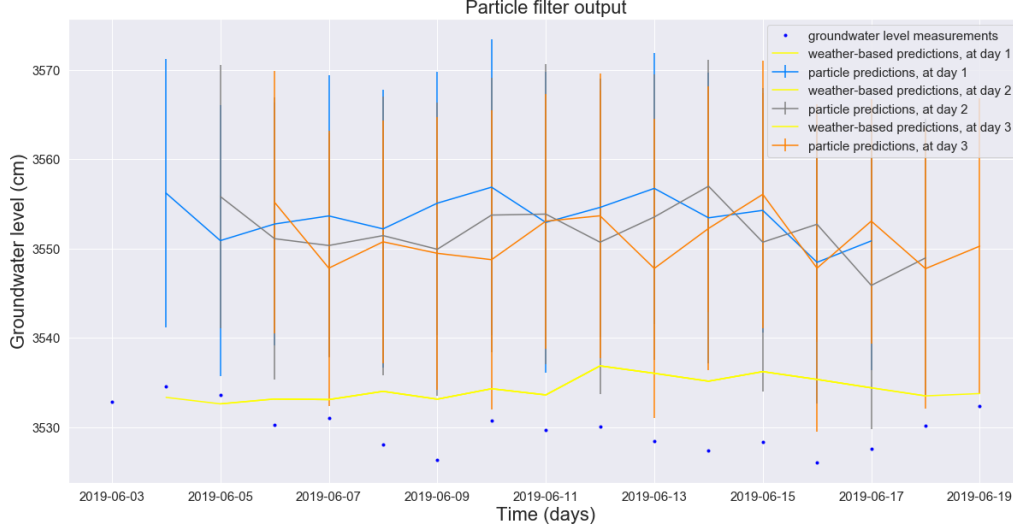


Figure 3: Result of first implementation of the particle filter for a period of three days. Groundwater levels in centimeters ( $y$ -axis) is set against dates ( $x$ -axis, 3 to 19 June 2019) for groundwater level measurements (blue dots), three weather-based predictions (yellow lines) and three particle predictions (coloured lines) with associated uncertainty (vertical lines). Predictions were made, 4 June (blue line), 5 June (brown line) and 6 June (orange line) each for a period of 14 days.

The weather model was not seen as the cause of these problems, as the uncertainty in the outcome was larger than the difference between the measured and the weather-based predicted groundwater levels.

Resampling was considered as a cause of the issues. There is an addition to the chosen resampling method, which is to resample only after a certain threshold has been reached (Chen et al., 2003). Resampling increases variance, so decreasing the number of times that resampling happens might decrease variance. However, although the large variance was a problem, a larger problem was that the particles did not move towards the weather-based predictions. By resampling less often, this would happen even less and therefore this method was not applied.

After further investigation, it was found that problems with resampling were primarily caused by extremely small weight differences (see Figure 4). It was expected that the distribution of weights before resampling (red dots) and the distribution of particles after resampling (blue bins) had the same form, both centering around the weather-based prediction (green line). As the figure shows, this is not the case. The spread of particles after resampling seemed to depend more on where there were more particles originally than on the weight of the particles. This was explained by the small difference in weights (right  $y$ -axis, difference is only 0.01). The weight differences would need to get fixed before changing the resampling method.

### 3.4.2 Adjusting the initialization

The easiest method to minimize spread in the particles, was to not spread them widely initially. The initial distribution had a range of two or more metres. Per time step the particles could at most move ten centimetres, but would on average move one cm. So even if everything else worked

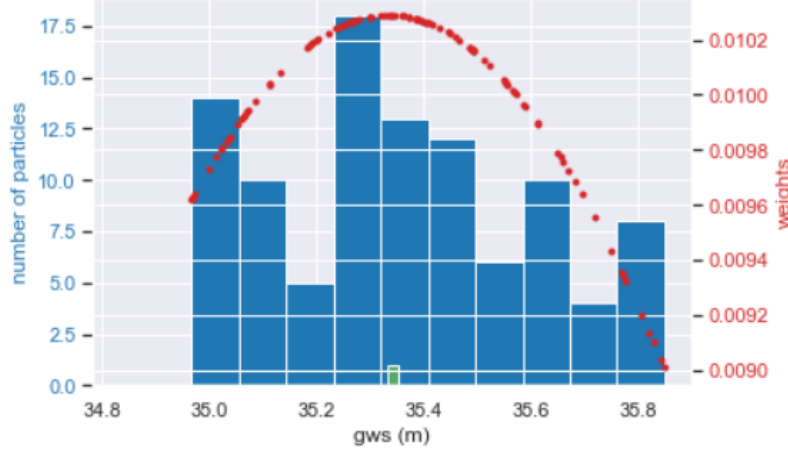


Figure 4: Resampling

The distribution of weights before resampling (red dots) and the division of particles after resampling (blue bars). The weather-based prediction is visualized as a small green bar. The x-axis shows the groundwater level value in metres of the particles and weather-based prediction. The right y-axis shows the weights of the particles before resampling, the left y-axis shows the number of particles that have a specific groundwater level value after resampling.

correctly, it would take at least twenty time steps for the particles to reach the right state. The testing period consisted of 48 days, or 48 time steps, so the particles might never reach the right state in time. For that reason, it was decided to only use the last groundwater measurement in the training data in combination with the normal distribution of change of groundwater levels in a day. This did decrease the spread in particles, but the uncertainty remained large and predictions about the same day were still incoherent.

### 3.4.3 Adjusting the update function

To minimize uncertainty and further minimize spread, changing the weighting of particles was considered. This was done by introducing a factor ( $F$ ) into the update function.

$$w_{t+1}^{dn}(i) \leftarrow \frac{\exp(\mathbf{F} \cdot -\frac{1}{2} \cdot (y_{t+1}^{dn} - x_{t+1}^{dn}(i))^2)}{\sum_i^{N_p} \exp(\mathbf{F} \cdot -\frac{1}{2} \cdot (y_{t+1}^{dn} - x_{t+1}^{dn}(i))^2)} \quad (13)$$

As a result, particles closer to the weather prediction were more heavily weighted and consequently more often resampled. As is shown in Figure 5, this reduced spread and uncertainty of the particles<sup>4</sup>. The difference between predictions was now around five cm and the uncertainty around fifteen cm. However, it also pushed the particles towards the, often incorrect, weather-based groundwater level predictions. The predictions (coloured lines) followed the weather-based predictions (yellow lines) but the fluctuations of the predictions had nothing to do with the groundwater measurements (dots). Therefore, this approach was discarded.

<sup>4</sup>Again, a selection of the output was made, the full results can be found in appendix 7.2

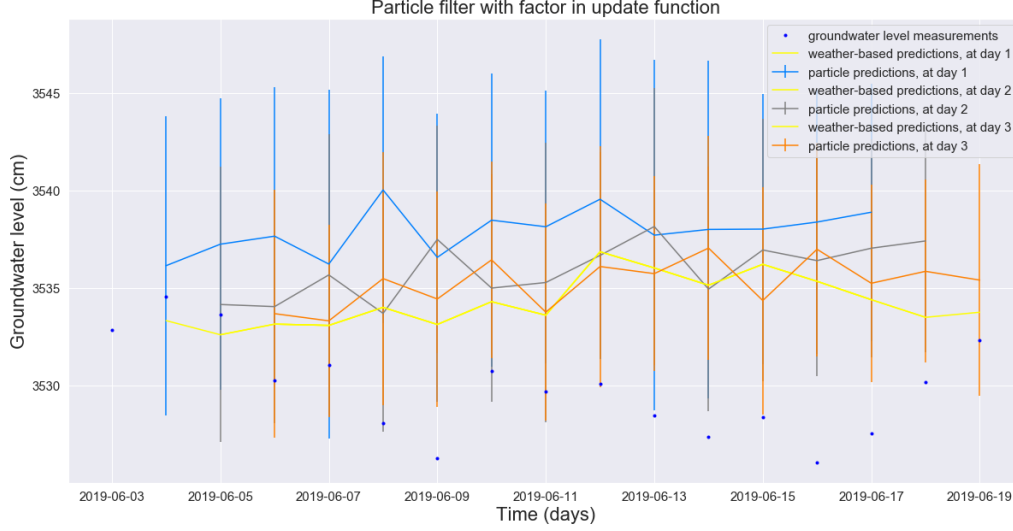


Figure 5: Particle filtering with added factor in update function

The  $y$ -axis is the groundwater level in centimetres and the  $x$ -axis shows the time in days. The dots are groundwater measurements. The yellow lines represent the weather-based predictions of the groundwater, for each day there is a 14-day prediction period. The coloured lines are the particle filter predictions, where each colour is the prediction made at a different day. The vertical lines are the associated uncertainty of the prediction

### 3.4.4 Adjusting the prediction step

Before delving into what improvements of the prediction step have been considered, something needs to be addressed. The prediction step was described by (6), what needs to be defined is which particles were chosen as  $x_t$ . These should be the particles that gave most information about the next state,  $x_{t+1}$ . In most particle filters, there is only option, because the only time-index is  $t$ . In this particle filter there are two time-indices,  $t$  and  $dn$ . As such, there are three methods to choose the previous particles (see Figure 6).

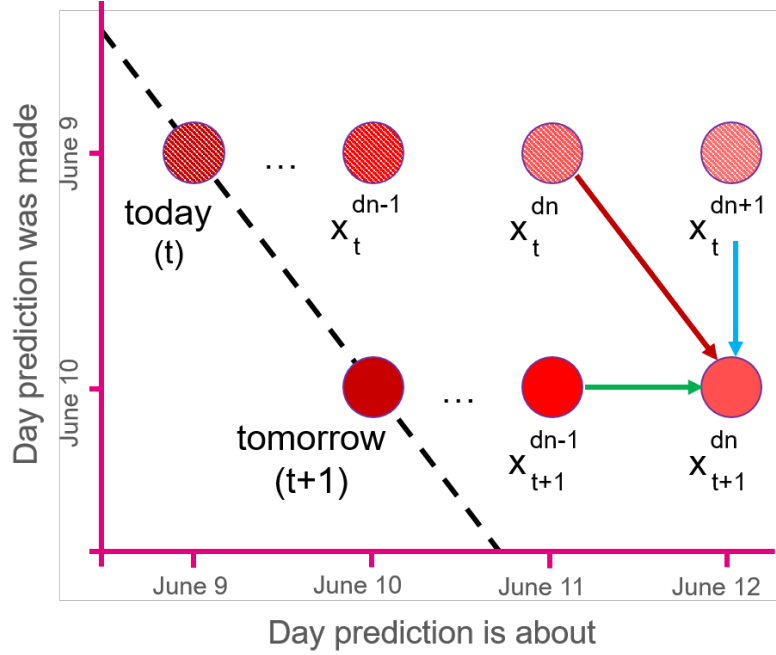


Figure 6: Different particles with one time step difference.

The y-axis indicates the time step  $t$  a prediction was made at, each row of circles are at the same time step. The x-axis indicates the day the prediction is about, so circles in the same column are predictions about the same day. The black line indicates the current time, everything to the left is in the past and everything to the right is in the future. Every circle represents a different prediction day ( $dn$ ), with the leftmost (dark red) circles showing  $d0$  or the current day. The original prediction method of ‘moving time steps’ (red diagonal arrow) is compared with the method of ‘moving prediction steps’ (horizontal green arrow) and ‘moving day-to-day’ method (vertical blue line).

**Moving time steps**, the diagonal red line in Figure 6.

This method is commonly used in particle filters and connects time steps, as particles at time step  $t$  for prediction day  $dn$  are used to predict particles at time step  $t + 1$  for prediction day  $dn$ . In other words, the prediction made at the 9th of June ( $t$ ) about the 11th of June ( $d2$ ) was used to make a prediction at the 10th of June ( $t + 1$ ) about the 12th of June ( $d2$ ). In this way, time steps are connected but prediction days are not (Figure 7a), what was predicted at time step  $t$  for prediction day  $dn$  had no interaction with the prediction at time step  $t$  for prediction day  $dn + 1$  or  $dn - 1$ .

**Moving prediction days**, the horizontal green line in Figure 6.

This method connects particles with a different  $dn$ , it uses the prediction at time step  $t + 1$  about  $dn$  days ahead to make a prediction at time step  $t + 1$  about  $dn + 1$  days ahead. In other words, the prediction made at the 10th of June ( $t + 1$ ) about the 11th of June ( $d1$ ) is used to make a prediction at the 10th of June ( $t$ ) about the 12th of June ( $d2$ ). Using this method, the groundwater measures that were recorded everyday can also be incorporated. Because  $x_t^{d0}$ , or the particles ‘predicting’ today, were assigned the groundwater value that was measured, which was then used to predict  $x_t^{d1}$ . The groundwater measurement can be assumed to be correct, and is therefore the most trustworthy

'prediction'. Incorporating it might increase reliability of the other predictions. However, in this method the different time steps are not connected, so what was predicted at time step  $t$  had no influence on the predictions at time step  $t + 1$ . (Figure 7b).

**Moving day-to-day**, the vertical blue line in Figure 6.

This method connects the particles making predictions about the same day. A prediction made at time step  $t$  about prediction day  $dn + 1$  concerns the same day as a prediction made at time step  $t + 1$  about prediction day  $dn$ , or the prediction made at the 10th of June ( $t$ ) about the 12th of June ( $d3$ ) concerns the same day as the prediction made at the 10th of June ( $t + 1$ ) about the 12th of ( $d2$ ). Using this property, both the different time steps,  $t$ , and prediction days,  $dn$ , are connected. However, this is technically not a change in time, as nothing about a previous or next day is taken into account. (Figure 7c)

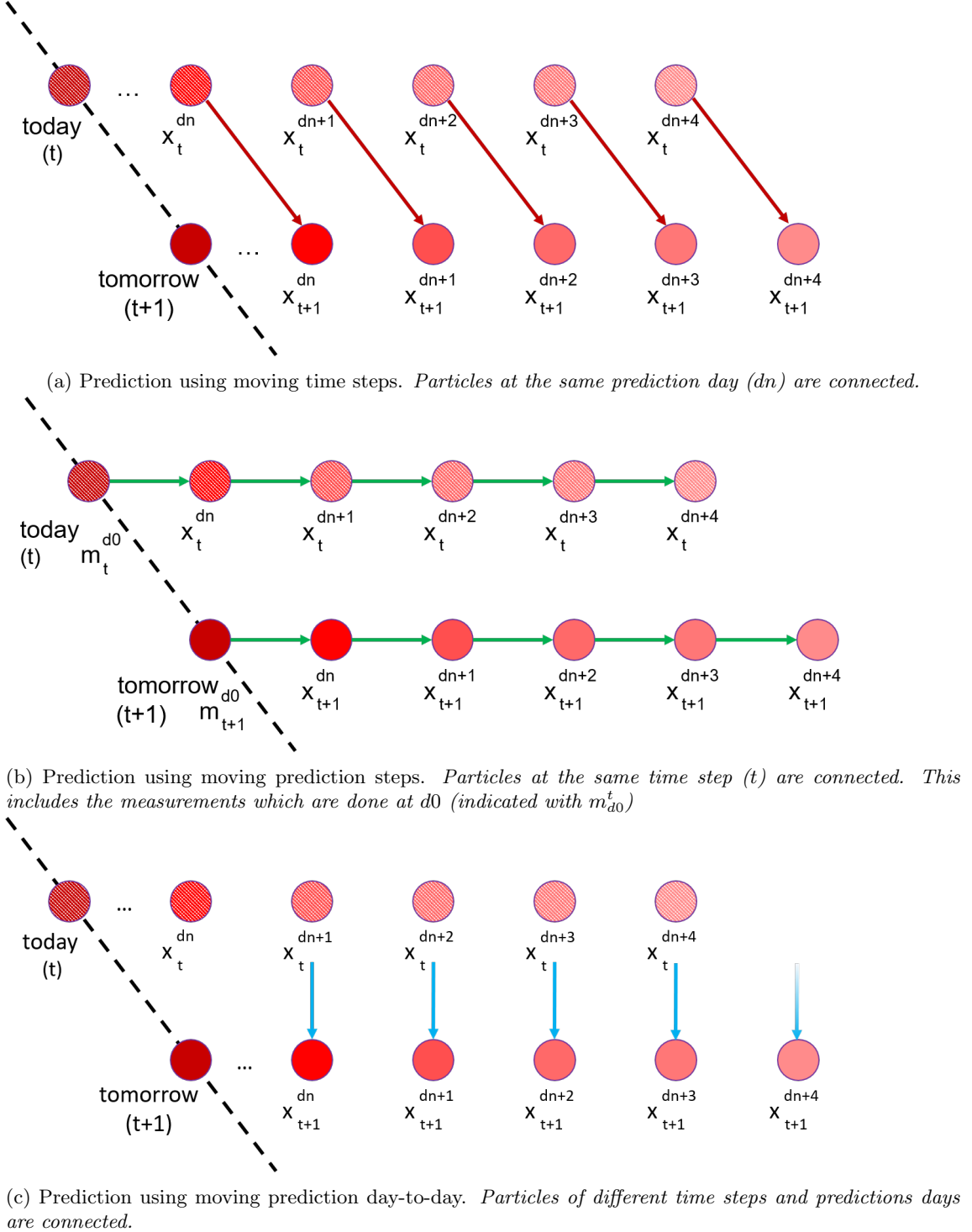


Figure 7: Representation of three different methods to particle-filter predictions . Shows which particles are connected in the prediction step. Each row of circles are at the same time step ( $t$ ). Each column of circles are predictions about the same day. Every circle is a different prediction day ( $dn$ ), the first (dark red) circle is the  $d_0$  or the current day. The black line shows the current time, so everything to the left of the line is in the past and everything to the right is in the future.



These three methods were each used as a predictive method. Their predictive power was compared calculating the Root Mean Squared Error (RMSE) for the difference between the predicted value and the actual groundwater measurement. An overall RMSE was taken as the average of the RMSE for every prediction day for every time step, resulting in one value summarizing all fourteen predictions made for each of the 48 days in the testing period. The particle filter algorithm was run for all four wells and for each the overall RMSE value for each method was calculated. In addition, a baseline RMSE for each well was calculated using the weather-based predictions.

---

**Pseudo-code final version Particle filter**


---

```

1:  $N_p \leftarrow$  : number of particles to be used
    $N_d \leftarrow$  number of days to make predictions about (here 14)
    $t \leftarrow 0$ 
2: for  $dn \in 1 \dots N_d$  do
3:   for  $i \in 1 \dots N_p$  do
      $x_0^{dn}(i) \leftarrow m_{t-1} + \mathcal{N}(\mu, \sigma^2)$    initialize particles
      $w_0^{dn}(i) \leftarrow 0$    initialize weights
4:   end for
5: end for
6: for each day  $t$  do
7:   for each  $d_n \in N_d$  do
8:      $x_{t+1}^{dn} \leftarrow \begin{cases} x_{t+1}^{dn} + \mathcal{N}(\mu, \sigma^2), & \text{if moving time steps} \\ x_t^{dn-1} + \mathcal{N}(\mu, \sigma^2), & \text{if moving prediction days} \\ x_t^{dn+1} + \mathcal{N}(\mu, \sigma^2)3, & \text{if moving day - to - day} \end{cases}$    prediction
9:      $y_{t+1}^{dn} \leftarrow \text{model}(\text{weather predictions})$    get observations
10:    for each  $i \in N_p$  do
       $w_{t+1}^{dn}(i) \leftarrow \frac{\exp(F \cdot \frac{-1}{2} \cdot (y_{t+1}^{dn} - x_{t+1}^{dn}(i))^2)}{\sum_i^{N_p} \exp(F \cdot \frac{-1}{2} \cdot (y_{t+1}^{dn} - x_{t+1}^{dn}(i))^2)}$    update weights
11:    end for
12:     $x_{t+1}^{dn} \leftarrow \text{weighted choice}(x_{t+1}^{dn}, w_{t+1}^{dn})$    resample
13:     $t \leftarrow t + 1$    go to next day
14:   end for
15: end for

```

---

## 4 Results

Groundwater levels for the testing period differed between the four wells (Figure 8).

Figure 9 shows the trained model for measuring well 1003 in Oldenzaal for the period of April 2019 to April 2020. The groundwater level showed the expected seasonal movement; in summer, groundwater levels went down, followed by an upward trend during autumn and winter with a peak around February and another drop in levels by April. The weather-based model followed this pattern and had an r-squared value of 95.9%. However, as mentioned in paragraph 3.2 this model was trained using weather measurements. For testing, weather predictions were used which gave different results as there were more inconsistencies in values between days.

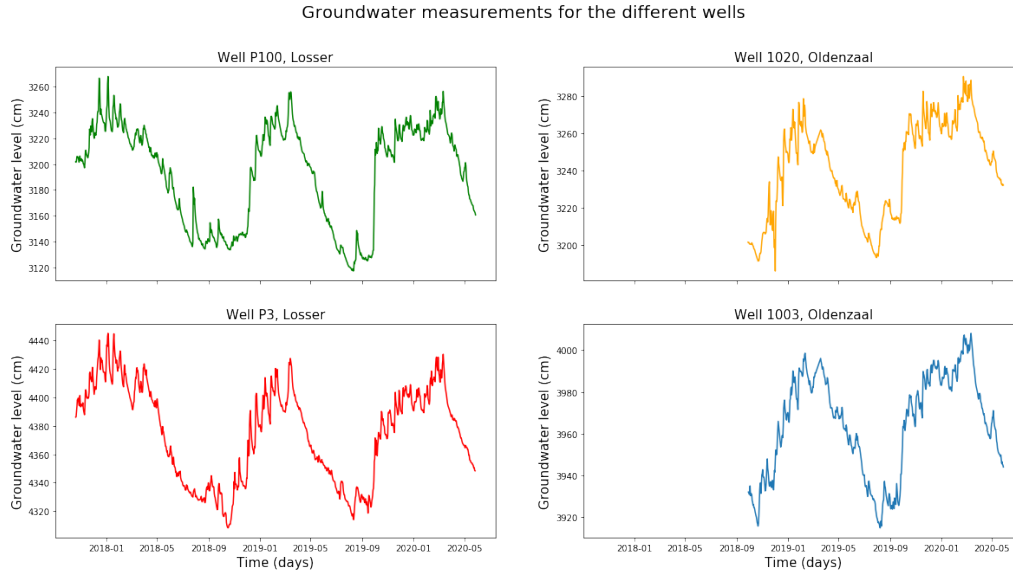


Figure 8: The groundwater level measurement data available for all the wells. The groundwater level in cm ( $y$ -axis) is set against the time in days ( $x$ -axis). The two graphs on the left show measurements for the wells located in Losser where there was two and a half year worth of data, the graphs on the right show the wells located in Oldenzaal for which there was one and a half year of data

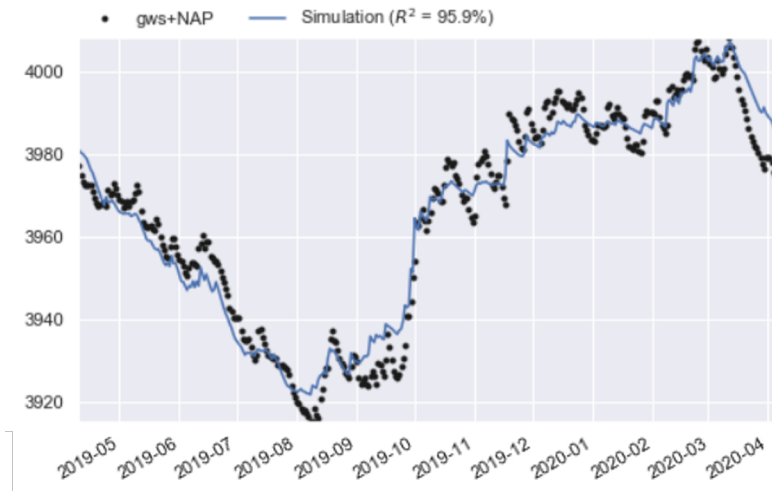


Figure 9: Weather model for measuring well 1003 in Oldenzaal. Groundwater measurements in centimeters ( $y$ -axis) against time of training the model (from 4-10-2019 until 4-10-2020). Comparison of the actual groundwater measurements (black dots) with prections (blue line) using the weather-model for measuring well 1003 in Oldenzaal.

Using the parameters obtained by this model, the weather-based prediction and the three methods

of the adjusted particle filter resulted in the predictions for the same well (1003) shown in Figure 10 .

As can be seen from the figure, each prediction method gave a different pattern.

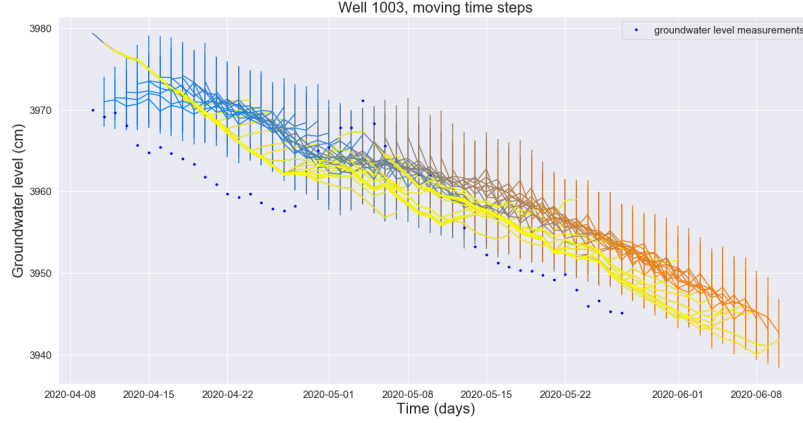
The groundwater level (dots) decreased steadily over time with an exception in early May when there was a sharp increase for around five days. The weather-based predictions (yellow lines) deviated more from the observed measurements than during training period because weather-forecast data was used instead of the actual measurements. The weather-based predictions captured the downward trend reasonably well, but almost completely missed the brief increase in groundwater levels occurring in early May.

Using the **moving time steps** method, Figure 10a, particle filter predictions (coloured lines) showed a downward trend, however the peak in groundwater level was not predicted. There was also an angular pattern in the predictions, with prediction days at consecutive time steps having similar predictions while consecutive prediction days at the same time step had quite large differences. As mentioned, this was dependent on the prediction method. The particles used as previous time ( $x_t$ ) have the largest influence on the next time step ( $x_{t+1}$ ), resulting in correlation between prediction days at consecutive time steps.

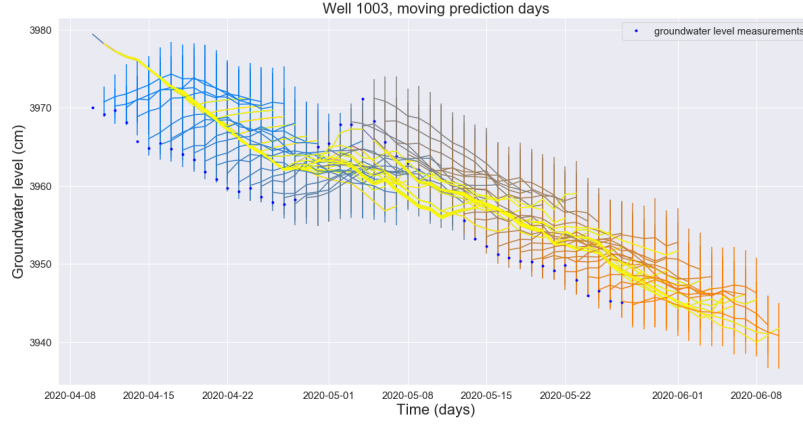
The method of **moving prediction days**, Figure 10b, gave smoother lines that were more separate compared to the other two methods. The smoothness was the result of the dependency of a prediction day on the previous prediction day for that same time step. The larger distance between the lines was caused by the lack of interaction between predictions at different time steps. The individual lines had an almost upwards trend in contrast to the downward movement of the groundwater level. The peak was not predicted but was incorporated in predictions after the actual peak.

The **moving day-by-day** method, Figure 10c, did show a downward trend and again did not predict the peak in groundwater level. Using this method resulted in grouped peaks, because predictions were dependent on previous predictions made about the same days. There were also quite large differences between days, an effect of there being no connection between consecutive days.

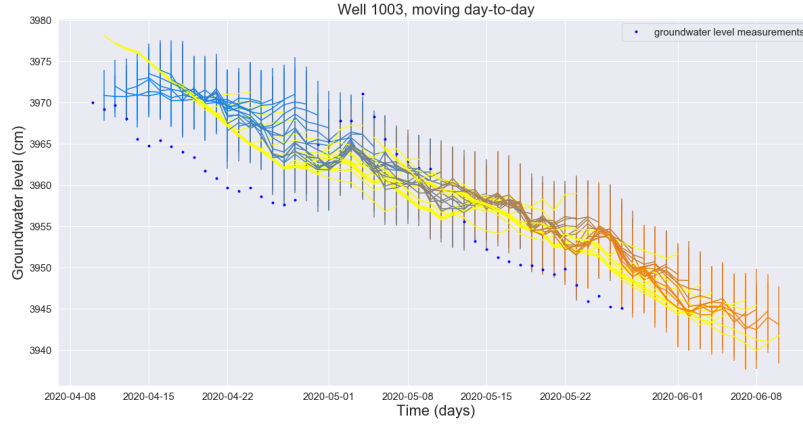
The moving time steps and moving day-to-day methods had better coherence between particle predictions (Appendix 7.3 shows the output for the other three wells). However, these also tended to drift further from the actual measurements. Between wells, there were differences to the extent of which the particle filter prediction was influence by the weather-based predictions. For all wells the uncertainty was and remained larger than one centimeter (approximately the average change in groundwater level per day).



(a) Prediction using moving time steps



(b) Prediction using moving prediction steps



(c) Prediction using moving prediction day-to-day

Figure 10: Results of measuring well 1003 in Oldenzaal.

Groundwater measurements in centimeters ( $y$ -axis) against time of testing the prediction models. Comparison of actual groundwater level measurements (blue dots) with weather-model based predictions (yellow lines, each day predicting 14 days) and the particle-filter based predictions (predictions per day are represented by a different colour starting with blue and changing into orange) for the method of ‘moving time steps’ (A), ‘moving prediction steps’ (B) and ‘moving prediction day-to-day’ (C).

Between wells, there were considerable disparities for the RMSE values (Figure 11). Specifically the RMSE of well 1020 in Oldenzaal was larger for all methods. The mean RMSE values over all wells, indicated by a purple cross, were between a value of six and eight.

It was lowest for the moving-prediction days method, based on comparatively low RMSE values for all wells. However, except for well 1020, the weather-based RMSE was lower than all of the particle filter methods.

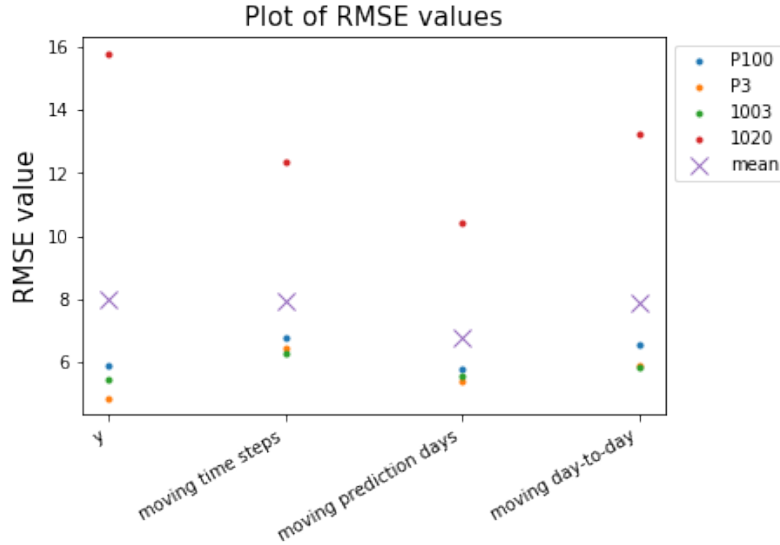


Figure 11: RMSE of all methods for each measuring well.

*The value of RMSE (y-axis) for weather-based model (Y), moving time steps, moving prediction steps and moving day-to-day.*

## 5 Discussion and conclusion

In this thesis a particle filter to make short-term predictions of groundwater levels was developed. Weather forecast data in combination with a hydrological model were used as observations. The particle filter was tested on four monitoring wells located in the Netherlands.

In the implementation process, three different methods (moving time steps, moving prediction days and moving day-to-day) to implement the particle filter were developed. In particular, methods differed regarding the prediction step. When tested and compared with the weather-based predictions, the following characteristics came to light:

The moving time steps and moving day-to-day method both showed a downward trend and were quite far from actual groundwater measurements. The moving time steps method had a more angular pattern, with predictions made at different time steps peaking at consecutive days. The moving day-to-day method had more grouped peaks, with the predictions at different time steps peaking at the same day. Both of these patterns can be explained by the functioning of the method, namely predictions are most coherent with the particles that were chosen as the base for the new prediction. The moving prediction days method had less of an overall trend but stayed closer to

actual groundwater level measurements. These predictions were also smoother, with only minor differences between predictions made at the same time step, as the predictions in this method were based on particles at the same time step.

Looking at the mean RMSE, moving prediction days showed the best predictions. However, with exception of one well, the weather-based predictions had lower RMSE values for individual wells. The different results for well 1020 were likely caused by the training data not representing the pattern found during the testing period. The trained hydrological model for this well had an  $r$ -squared value of 96% and therefore reliable simulations were expected, see Appendix 7.3.3 for the trained hydrological model of well 1020. Overall, none of the methods had the accuracy that was anticipated. The use of particle filters to predict groundwater levels, seemed to be appropriate as observations are uncertain and the system as a whole is complex, however the results are not ready for use.

There are some points that can be addressed as these can provide insight into future improvements of the methods.

The particle filter required the assumption that the system was first-order Markovian. In other words, it was required that a current state could fully be explained based on the previous state and that any earlier states provided no additional information. However, every factor that influences groundwater level has a delayed effect, so this assumption may not hold. Neural-networks do not require the Markovian assumption and might provide a good alternative. This approach was not taken as papers describing neural-network-based approaches often have a ten year training period, and only two and a half years of data were available.

This brings up a second point, the model could benefit from more data. Groundwater level measurements and weather measurements for a longer time period would allow a longer training period. This would result in more robust parameters for the weather model, which might avoid issues as addressed for well 1020. Weather forecast data for a whole year and therefore a testing period of a year, would give a more complete idea of the functioning of the particle filter. Weather forecast data for both the training and testing period would allow to optimize parameters towards the weather forecast instead of weather measurements and thus improve performance. Data on more monitoring wells could allow for more definitive conclusions on the functioning of the different methods.

Including data on more factors such as pumping or soil type might also improve predictions. One factor that could be added to the model, is a seasonal factor. As can be seen in the particle filter output (Appendix 7.3) the particle predictions tend to stay above the measured groundwater level. This could be caused by using a distribution based on the change in groundwater level per day over the period of a year as  $p(x_{t+1} | x_t)$ . Towards summer, the groundwater level decreases more than average and towards winter the opposite happens, a seasonal factor could describe this property and might therefore improve the model.

It is also worth considering the quality of the weather forecasts. WorldWeatherOnline.com is not a national weather institute such as the KNMI and might therefore have a less accurate weather model. Using weather forecasts from a different source or combining forecasts from different sources could improve predictive accuracy, which in turn improves particle predictions.

Model specifications could also influence model performance and require further investigation. One

such specification is the number of particles. In this study, a thousand particles were used. Research is needed to determine the effect of using more or less particles. In addition, it is worth to change the weight of the weather predictions, as described in this thesis but not implemented. Results showed that the particle predictions were influenced by the weather-based predictions, sometimes strongly. Reducing the weight would make particle predictions less dependent on the weather-based predictions, and might give better results. Further work is needed to evaluate the effect of changing the weighting. However, of special interest is to combine the different methods of choosing particles in the prediction step as developed and described in this thesis. Each method described had its own strengths and weaknesses, combining these might complement the strengths and result in better predictions.

Finally, there is need to find different methods to evaluate predictive performance. This study only used visual comparison and a rough overall model-fit parameter in the form of the mean RMSE. Additional statistical methods are needed to further compare methods and get more insights in the differences of their predictive performance.

As mentioned, particle filters have not been applied to this type of problems before and might not be the best approach. Neural-networks offer a good alternative, provided that there is enough data for training. It is also worth considering to apply the particle filter in a different way. Here a hydrological model and a Python package were used to train the model, resulting in the need to make a fourteen day prediction at each time step. Instead the particle filter could be used to train the parameters of a hydrological model, where for each of the fourteen days a different set of parameters could be trained. In this way, the particle filter would not be used to make a forecast for multiple days as at each time step the predicted parameters could be checked against groundwater level measurements, at least during a training period. Such a hydrological model is provided in the thesis by Jos von Asmuth (Von Asmuth, 2012), it has four different parameters which does make the problem more complex. This was also the reason why this approach was not used, but it is interesting to explore in future research.

A last suggestion for future research, would be to apply the methods of particle filtering developed in this thesis to other domains to see if the characteristics of the methods are similar. This could provide further insight into both the functioning of a particle filter and the functioning of the methods described in this thesis.

In conclusion, this thesis described the application of a particle filter to make short-term predictions of groundwater levels. Three different methods of particle filtering were developed and tested. All three had different characteristics and not one method excelled in providing predictions that were accurate enough for practical application. As these methods have not been described previously, there is ample opportunity for in-depth research both in improving the current model and applying the methods to different domains.

## 6 Acknowledgements

Henriëtte Keijzer and Erwin Stamsnijder from Tauw for their help with all groundwater related data and questions and Dr. Johan Kwisthout and Nils Donselaar, MsC for their advice on methodology.

## References

- Barcelona, M. J., Gibb, J. B., and Miller, R. A. (1983). *A guide to the selection of materials for monitoring well construction and ground-water sampling, Section 5*, volume 327, page 13–26. Illinois State Water Survey.
- Bazik, M., Flewelling, B., Majji, M., and Mundy, J. (2019). Bayesian inference of spacecraft pose using particle filtering. *arXiv preprint arXiv:1906.11182*.
- Bear, J. (2012). *Hydraulics of groundwater, Chapter 2*, pages 19–31. Courier Corporation.
- Berg, D., Bauser, H. H., and Roth, K. (2019). Covariance resampling for particle filter–state and parameter estimation for soil hydrology. *Hydrology & Earth System Sciences*, 23(2).
- Chen, Z. et al. (2003). Bayesian filtering: From kalman filters to particle filters, and beyond. *Statistics*, 182(1):1–69.
- Collenteur, R. A., Bakker, M., Caljé, R., Klop, S. A., and Schaars, F. (2019). Pastas: open source software for the analysis of groundwater time series. *Groundwater*, 57(6):877–885.
- Daliakopoulos, I. N., Coulibaly, P., and Tsanis, I. K. (2005). Groundwater level forecasting using artificial neural networks. *Journal of hydrology*, 309(1-4):229–240.
- Del Moral, P. (1997). Nonlinear filtering: Interacting particle resolution. *Comptes Rendus de l’Académie des Sciences-Series I-Mathematics*, 325(6):653–658.
- Fang, H.-T., Jhong, B.-C., Tan, Y.-C., Ke, K.-Y., and Chuang, M.-H. (2019). A two-stage approach integrating som-and moga-svm-based algorithms to forecast spatial-temporal groundwater level with meteorological factors. *Water resources management*, 33(2):797–818.
- Gordon, N., Salmond, D., and Ewing, C. (1995). Bayesian state estimation for tracking and guidance using the bootstrap filter. *Journal of Guidance, Control, and Dynamics*, 18(6):1434–1443.
- Hu, X.-L., Schon, T. B., and Ljung, L. (2008). A basic convergence result for particle filtering. *IEEE Transactions on Signal Processing*, 56(4):1337–1348.
- Kasiviswanathan, K., Saravanan, S., Balamurugan, M., and Saravanan, K. (2016). Genetic programming based monthly groundwater level forecast models with uncertainty quantification. *Modeling Earth Systems and Environment*, 2(1):27.
- KNMI (April 2005). *Handboek Waarnemingen, Hoofdstuk 10*, page 10.9. KNMI.
- Kozierski, P., Lis, M., and Zietkiewicz, J. (2013). Resampling in particle filtering-comparison.
- KWR (2020). Menyanthes, <https://www.kwrwater.nl/en/tools-producten/menyanthes/>.
- Moradkhani, H., Hsu, K.-L., Gupta, H., and Sorooshian, S. (2005). Uncertainty assessment of hydrologic model states and parameters: Sequential data assimilation using the particle filter. *Water resources research*, 41(5).
- Nurminen, H., Ristimäki, A., Ali-Löytty, S., and Piché, R. (2013). Particle filter and smoother for indoor localization. In *International Conference on Indoor Positioning and Indoor Navigation*, pages 1–10. IEEE.
- RIVM (2018). Drinkwaterkwaliteit, <https://www.rivm.nl/drinkwater/drinkwaterkwaliteit>.



- Salamon, P. and Feyen, L. (2009). Assessing parameter, precipitation, and predictive uncertainty in a distributed hydrological model using sequential data assimilation with the particle filter. *Journal of Hydrology*, 376(3-4):428–442.
- Shirmohammadi, B., Vafakhah, M., Moosavi, V., and Moghaddamnia, A. (2013). Application of several data-driven techniques for predicting groundwater level. *Water Resources Management*, 27(2):419–432.
- Snyder, C., Bengtsson, T., Bickel, P., and Anderson, J. (2008). Obstacles to high-dimensional particle filtering. *Monthly Weather Review*, 136(12):4629–4640.
- Sun, A. Y. (2013). Predicting groundwater level changes using grace data. *Water Resources Research*, 49(9):5900–5912.
- Taylor, R. G., Scanlon, B., Döll, P., Rodell, M., Van Beek, R., Wada, Y., Longuevergne, L., Leblanc, M., Famiglietti, J. S., Edmunds, M., et al. (2013). Ground water and climate change. *Nature climate change*, 3(4):322–329.
- Von Asmuth, J.-R. (2012). Groundwater system identification through time series analysis.
- Zhang, R. and Bivens, A. J. (2007). Comparing the use of bayesian networks and neural networks in response time modeling for service-oriented systems. In *Proceedings of the 2007 workshop on Service-oriented computing performance: aspects, issues, and approaches*, pages 67–74.

## 7 Appendices

### 7.1 Makkink formula

- 1) Verzadigde dampspanning t.o.v. water:

$$e_s(T) = 6,107 \cdot 10^{7,5 \cdot \frac{T}{237,3+T}} \text{ [hPa]}$$

- 2) Verzadigde dampspanningsgradiënt t.o.v. water:

$$\delta(T) = \frac{7,5 \cdot 237,3}{(237,3 + T)^2} \cdot \ln(10) \cdot e_s(T) \text{ [hPa/°C]}$$

- 3) Psychrometerconstante:

$$\gamma(T) = 0,646 + 0,0006 \cdot T \text{ [hPa/°C]}$$

- 4) Verdampingswarmte van water:

$$\lambda(T) = 1000 \cdot (2501 - 2,38 \cdot T) \text{ [J/kg]}$$

- 5) Soortelijke massa van water:

$$\rho = 1000 \text{ [kg/m}^3\text{]}$$

- 6) Dagsom globale straling:

$$Q \text{ [J/m}^2\text{]}$$

- 7) Etmaalgemiddelde luchttemperatuur:

$$T \text{ [°C]}$$

- 8) Verdamping:

$$E_r = \frac{1000 \cdot 0,65 \cdot \delta(T)}{\{\delta(T) + \gamma(T)\} \cdot \rho \cdot \lambda(T)} \cdot Q \text{ [mm/etm]}$$


---

Figure 12: Makkink formulas as described by the KNMI, T is the temperatur [°C] and Q is the radiation [ $m^2/Jperday$ ]

## 7.2 Supplementary graphs

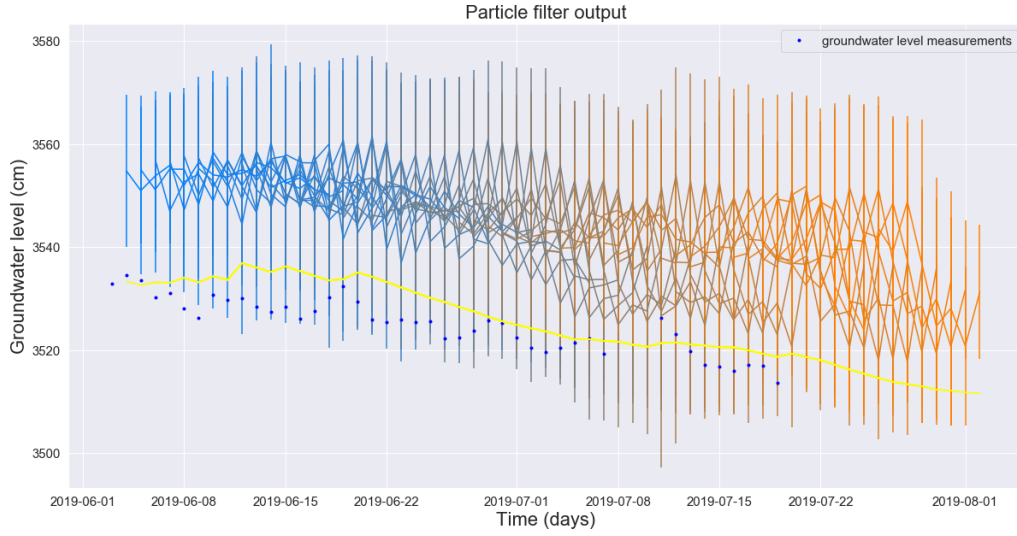


Figure 13: Complete output of initial run of particle filter.

Groundwater measurements in centimeters ( $y$ -axis) against time in days ( $x$ -axis). Comparison of actual groundwater level measurements (blue dots) with weather-model based predictions (yellow lines, each day predicting 14 days) and the particle-filter based predictions (predictions per day are represented by a different colour starting with blue and changing into orange).

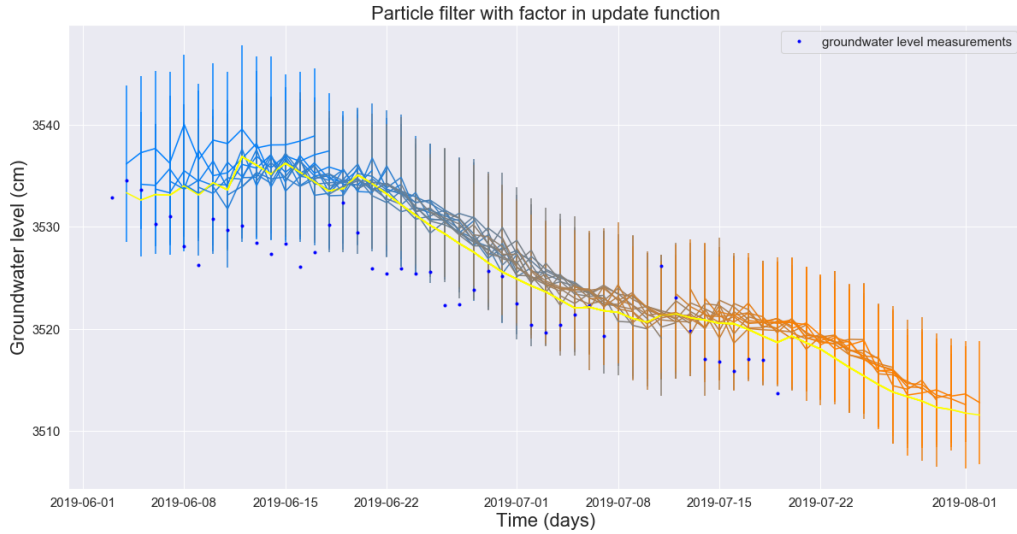


Figure 14: Complete output of particle filter with factor in update function.

Groundwater measurements in centimeters ( $y$ -axis) against time in days ( $x$ -axis). Comparison of actual groundwater level measurements (blue dots) with weather-model based predictions (yellow lines, each day predicting 14 days) and the particle-filter based predictions (predictions per day are represented by a different colour starting with blue and changing into orange).

7.3 Output of other wells

7.3.1 Well P3, Losser

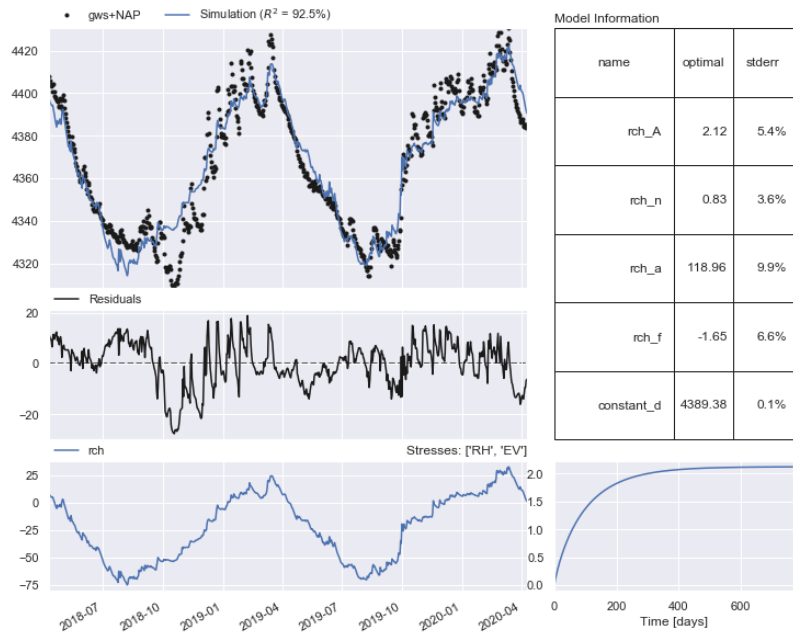
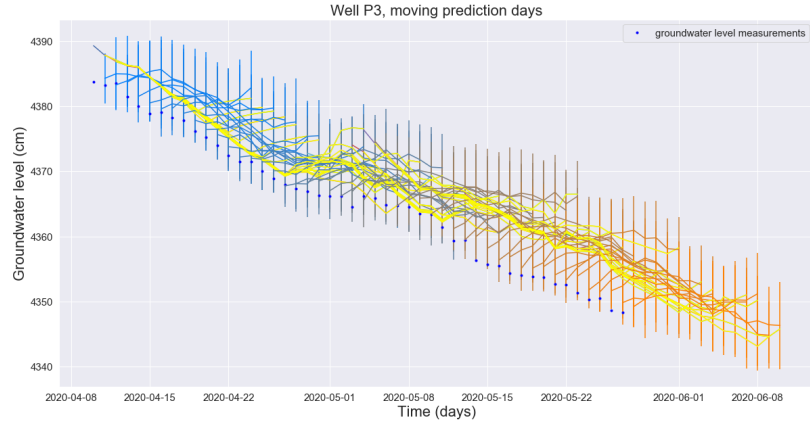


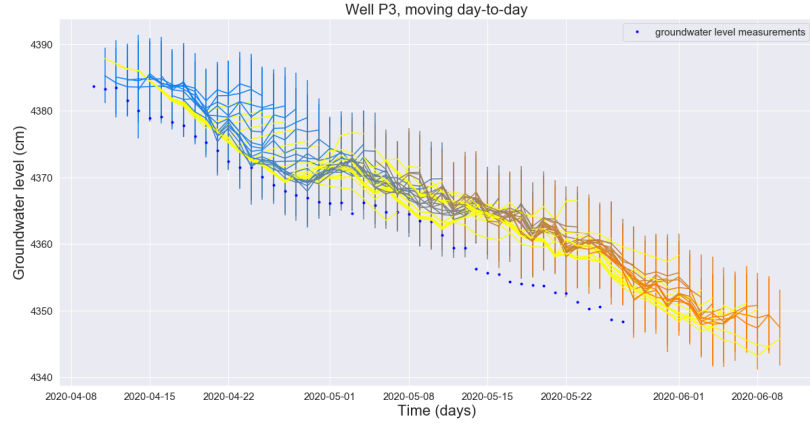
Figure 15: Weather model for measuring well P3 in Losser.



(a) Prediction using moving time steps



(b) Prediction using moving prediction steps



(c) Prediction using moving prediction day-to-day

Figure 16: Results of measuring well P3 in Losser.

7.3.2 Well P100, Losser

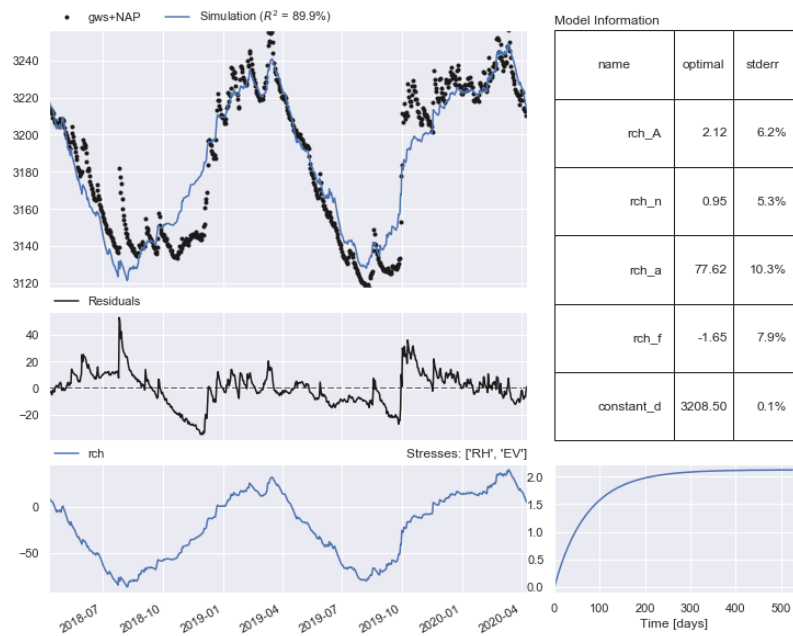
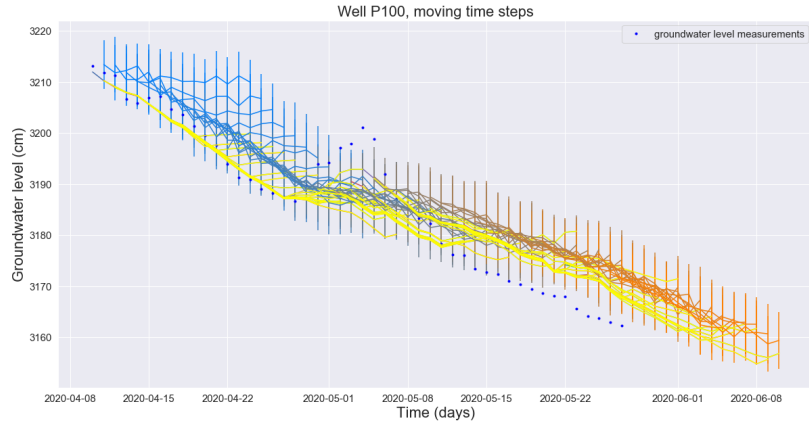
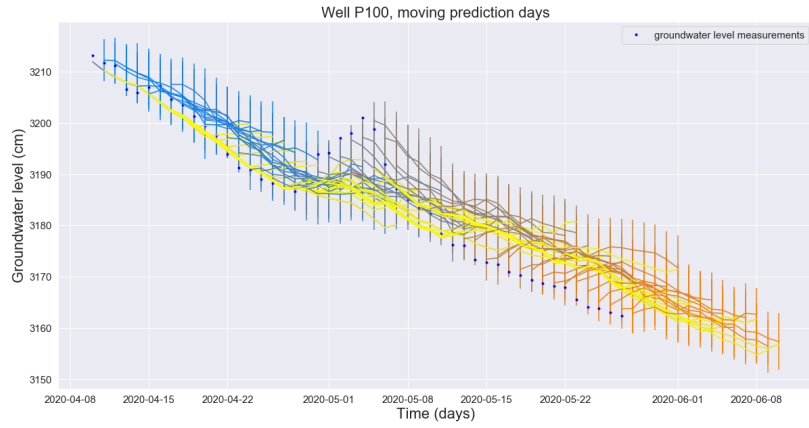


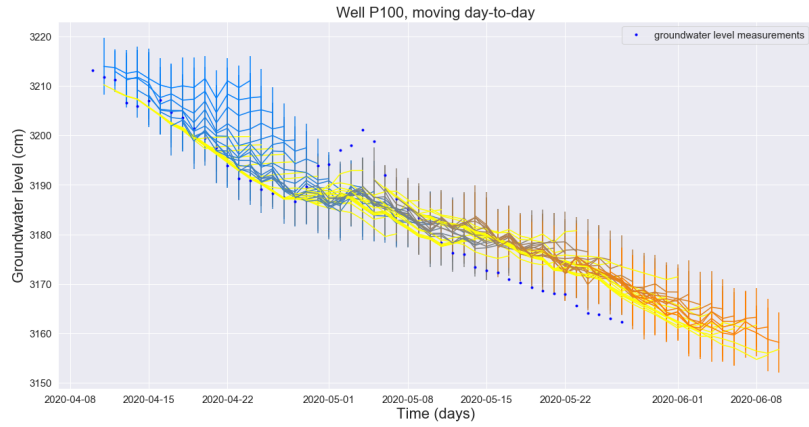
Figure 17: Weather model for measuring well P100 in Losser.



(a) Prediction using moving time steps



(b) Prediction using moving prediction steps



(c) Prediction using moving prediction day-to-day

Figure 18: Results of measuring well P100 in Losser.

7.3.3 Well 1020, Oldenzaal

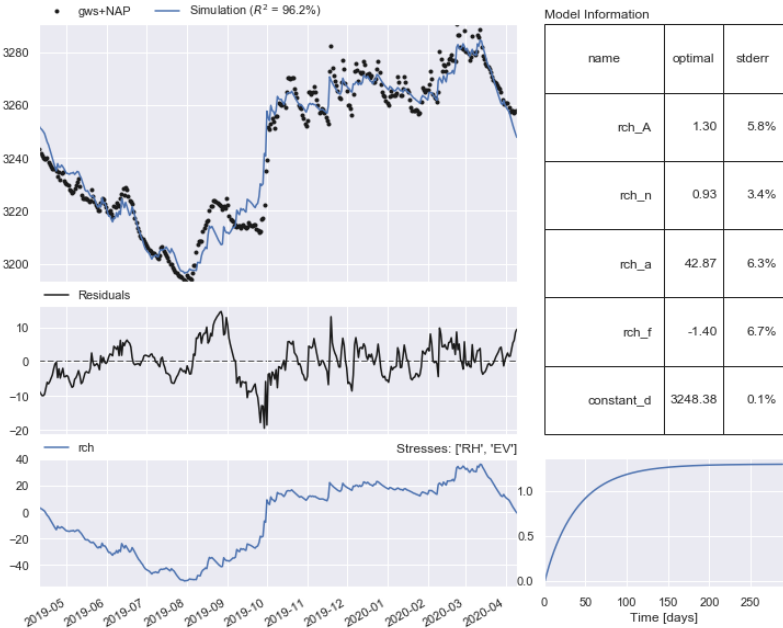
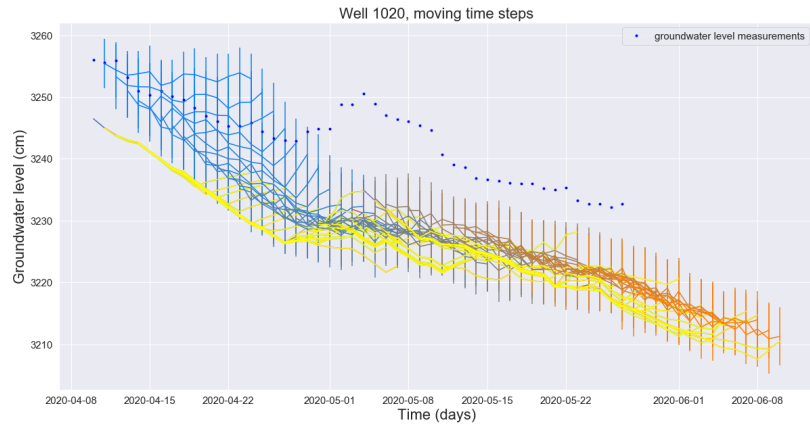
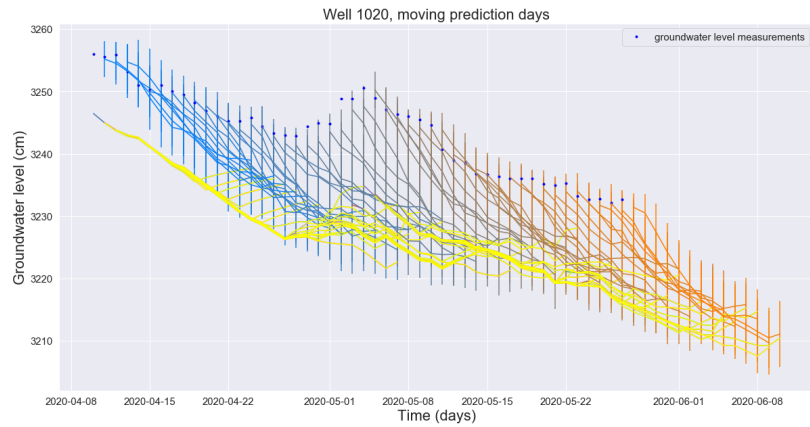


Figure 19: Weather model for measuring well 1020 in Oldenzaal.

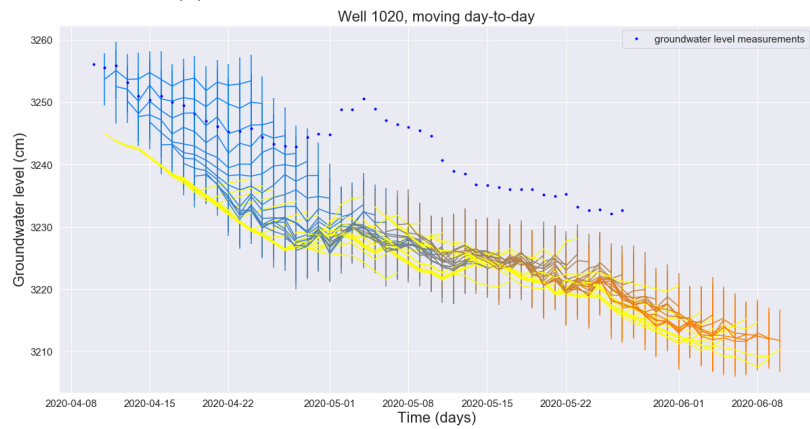




(a) Prediction using moving time steps



(b) Prediction using moving prediction steps



(c) Prediction using moving prediction day-to-day

Figure 20: Results of measuring well 1020 in Oldenzaal.