Why did I write this thesis?
How goals explain actions

Bachelor Thesis Artificial Intelligence
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Abstract

Why did I write this thesis? There are several possible goals that one can come up with to explain my behavior, e.g. earning a bachelor degree or trying to publish. Explaining observed behavior with the goal causing that behavior is called goal inference. The best explanation will be inferred, from several possible goals, through a process of Inference to the Best Explanation (IBE). In the cognitive (neuro-)science literature goal inference is often modeled using probabilistic IBE. In this thesis two probabilistic approaches of IBE, Maximum Likelihood (ML) and Most Probable Explanation (MPE), are evaluated. Glass (2007) applied ML and MPE to medical diagnostic cases and demonstrated that these approaches have some problematic aspects. ML ignores prior probabilities and MPE overweighs them. The question I tried to answer was whether these problems generalize to the domain of goal inference. The approach taken was analogous to the approach of Glass. The predictions of the ML and MPE models in three scenarios of goal inference were compared to human intuitions. It turned out that the problem of the ML model was present in two of the three scenarios. This suggests that the problem of ML does generalize to the domain of goal inference. The predictions of the MPE model matched human intuition in all three scenarios. This result suggests that the problem of MPE does not generalize.

1 Introduction

Imagine you see a man in a restaurant, let’s call him Bob. It is 7 o’clock and every table is taken. Bob is sitting by himself. In front of him on the table is a rose. Bob checks his watch every couple of minutes and is looking around. After an hour, Bob stops looking at his watch and orders food instead.

How could the behavior of Bob be explained? A possible explanation is that Bob has a date with a woman who is late. When Bob orders food, an explanation could be that Bob has given up hope the woman will show up and has decided to eat by himself.

In this example we infer among other things that Bob has a date. However, there are other explanations of his behavior possible. Bob could instead just having
dinner on his own, the rose being left at the table by a couple that had dinner earlier. You do not know what the intentions of Bob are, you can only try to give a good estimated explanation of his behavior. We are unaware of the many inferences that we make in daily life to explain the behavior of others and we do not only make inferences to explain observed behavior, but also to explain e.g. our visual percepts (in visual perception), symptoms of diseases (in medical diagnosis) and communicative acts (in language processing). Imagine a friend set you up for a blind date. You want to know what the person looks like, and you ask: “Is he (or she) good-looking?” When your friend answers “He is very kind.”, you could infer that your friend says this because he does not explicitly want to say the person is not good-looking. This is an example of an inference in the language domain.

Explaining observed behavior by inferring a goal that explains the behavior is called goal inference. There are several ways to model goal inference. The purpose of this thesis is to assess the viability of two probabilistic approaches; Maximum Likelihood (ML) and Most Probable Explanation (MPE).

1.1 Inference to the Best Explanation

Goals or intentions are inferred through a process referred to as Inference to the Best Explanation (IBE) (Harman, 1965; Baker et al., 2008). The basic schema of IBE is that you start with a set of hypotheses and then infer the probable truth of a hypothesis, on the grounds that the hypothesis provides a better explanation of the data than competing hypotheses do (Okasha, 2000). IBE follows the pattern:

D is a collection of data (facts, observations, givens).
H explains D (would, if true, explain data).
No other hypothesis can explain D as well as H does.

Therefore, H is probably true. (Josephson and Josephson, 1996, p. 1)

For example, remember Bob sitting in the restaurant and ordering food. Possible explanations for the fact that Bob orders food could be that Bob is hungry, that Bob gave up hope his date will show up or that Bob orders for himself and his date who is using the bathroom. Bob giving up hope would the best explanation
if it explains the fact that Bob orders food and also explains this better than the other explanations do.

Inherent to IBE is one main form of doubt: the possibility of alternative and equally likely or even better explanations which are not considered (Josephson and Josephson, 1996). This doubt made Van Fraassen critique IBE with his “bad lot argument”. This argument comprises that our selection of a hypothesis as an explanation may well be the best of a bad lot (Okasha, 2000). Thus the set of hypotheses from which one hypothesis is chosen as “best” might be a set of only bad hypotheses. Consequently the hypothesis chosen as best explanation might be a bad explanation, because it is the best explanation of only bad explanations.

However there are certain arguments against the “bad lot argument”. For example two arguments given by Psillos (1996). First, Psillos emphasizes the role of background knowledge. He states that the choice of hypotheses for in the set is guided and constrained by the background knowledge one has. The background knowledge can hereby guide one to come up with good hypotheses, without missing better hypotheses. Second, Van Fraassen seems to conclude that one should first eliminate the possibility that the true hypothesis lies outside the set, before one can believe a hypothesis to be the best. If that is true, one would become a profound skeptic: one could hardly have any belief, since very few beliefs can be justified if justifying involves elimination of the possibility that the belief may be false.

What does best explanation actually mean in the Inference to the Best Explanation? The best explanation is not necessarily the same as the true explanation, and vice versa. For example, imagine you see someone sitting behind a computer and pressing buttons on the keyboard with his fingers. The best explanation probably is that the person is typing. However, the true explanation could be that the person had itchy fingertips and is trying to ease them by pushing them on the buttons of the keyboard. This example illustrates that best explanation does not necessarily means true explanation. There are different views on what “best” means and I will explain two of them below.

First, Psillos (2002) defines criteria for what the best explanation is (of which I will only explain some criteria). The first criterion is that the best explanation is the hypothesis that is favored by the background knowledge. If the background

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1Yet, one can argue background beliefs might also have been the best of a bad lot.
knowledge favors more than one hypothesis, the second criterion is that the best explanation is the hypothesis that explains all data. If none of the hypotheses explain all data, the third criterion is that the hypothesis that explains the most salient data is the best explanation. The second view on what “best” means, is formulated by Harman (1965). According to Harman “best explanation” means the hypothesis is simpler, more plausible, explains more and is less ad hoc than other hypotheses in the set.

So far, I explained some of the background of IBE. In the next section I will go in some more detail and I will explain probabilistic models of goal inference.

1.2 Modeling goal inference

Inference to the Best Explanation is not only described in philosophy, it also attracted the attention of cognitive scientists. The cognitive science literature states, with the discovery of mirror neurons, that we understand others by simulating their behavior with our own mirror-neuron system (MNS). With this simulation we infer the intention(s) of others (Kilner et al., 2007a,b). The mirror-neuron system is believed to be involved in understanding actions and understanding intentions or goals of others (goal inference) (Iacoboni et al., 2005). Goal inference is in the cognitive (neuro-)science literature often modeled using probabilistic IBE (Rao et al., 2004; Baker et al., 2006; Verma and Rao, 2006; Grafton, 2009).

There are different approaches for modeling goal inference using probabilistic IBE. Two of them are Maximum Likelihood (ML) and Most Probable Explanation (MPE). According to the ML approach the best explanation of an observation is the hypothesis $H$, from the set of all possible hypotheses $\mathcal{H}$, with the maximum value of $P(E | H)$ (Glass, 2007). In other words, this notion of explanation defines a hypothesis the best explanation if it being true would make the evidence most probable. For example think of Bob who was sitting in the restaurant. I wanted among other things to explain the fact that Bob ordered food. The explanation I gave was that Bob had given up hope his date will show up and had decided to eat on his own. According to ML this is the best explanation if the evidence that Bob orders food is most probable given the hypothesis that Bob has given up hope. Bob being hungry would be a better explanation if it is more probable that
Bob will order food given the hypothesis that he is hungry.

MPE differs from ML in that it defines the best explanation as the hypothesis which is most probable given the evidence, i.e. a hypothesis H that maximizes $P(H \mid E)$. Again imagine Bob sitting in the restaurant and ordering food. I wanted to explain why Bob orders food. According to MPE, Bob giving up hope is the best explanation if, when Bob is ordering food, the most probable explanation is that he gave up hope his date will show up. Bob being hungry would be a better explanation if it is more probable that Bob is hungry given that he orders food.

Goal inference is often modeled using probabilistic IBE (like the ML and MPE approaches). However, there is a debate about whether people reason according to the rules of probability, because it seems to be in conflict with the finding that people are poor at explicit probabilistic reasoning. Namely, people make all sorts of errors in explicit probabilistic reasoning, for example overconfidence bias, conjunction fallacy, and base-rate neglect (Gigerenzer, 1991). According to Kahneman and Tversky (1973; 1974; 1981) intuitive predictions violate the statistical rules of prediction in systematic and fundamental ways.

The fact that people appear to be bad at explicit probabilistic, seems to imply people do not reason probabilistically. This seems to be incompatible with modeling goal inference using probabilistic IBE. However, this conflict is only apparent, because the fact that people are bad at explicit probabilistic reasoning does not mean that they do not implicitly reason in that way. An argument Chater (2006) gives is that people do not just struggle with probability, but with all branches of mathematics. Nevertheless, the fact that, for example, Fourier analysis is hard to understand does not imply it is not fundamental to audition and vision. Also, there are some researchers who suggest that people do reason according to the rules of probability, by showing that most so-called “errors” in probabilistic reasoning are in fact not violations of probability theory. For example the research of Gigerenzer (1991). Gigerenzer posed statistical problems in a different way, for example asking people to judge the frequency of events instead of asking people to make judgements about the probability of one event. He showed herewith that humans do not show errors in reasoning.

Oaksford and Chater (2009) make the case that cognition in general, and human everyday reasoning in particular, is best viewed as solving probabilistic infer-
ence problems. Finally, Chater et al. (2006) observe that probabilistic models are increasingly more used in cognitive and brain sciences. They suggest that probabilistic methods are likely to become important theoretical tools for understanding cognition.

In this section I tried to argue there is no reason to believe people do not reason probabilistically and that there is no reason to believe it is impossible to model (aspects of) human reasoning using probabilistic IBE (like the ML and MPE approaches do). However, both MPE and ML do have some problems with predicting human intuition when it comes to using prior probabilities. I consider these problems in the next section.

1.3 Problems of the ML and MPE approaches

The problem of the ML approach is that it does not take prior probability into account. Prior probability is the probability of a fact, observation or given before the evidence is known. Imagine that there is a patient with symptoms and that there are two diseases. Both diseases explain the symptoms observed in the patient. However, one disease is rare and the other one common. Intuitively, one would think the common disease would be the best explanation of the symptoms. Still, ML states that both diseases have the same probability to be the cause of the patients symptoms. Thus, in this example the prediction of ML does not correspond to human intuition.

The MPE approach does take prior probability into account. However, MPE overweighs this probability. Intuitively, a hypothesis could turn out to be the best explanation if it explains the evidence very well, even if the prior probability of the hypothesis is very small. However, MPE will not state that this hypothesis is the best explanation and is with this contrary to human intuition. For an illustration of this problem, imagine a man is ill and has certain symptoms. Also, there exists a very rare disease. Given his symptoms it would be very likely he has this rare disease. However, because the probability that one has the disease is so small, this hypothesis would, according to the MPE approach, not turn out to be the best explanation, even when people intuitively feel it is. Thus, MPE does take prior probability into account, unlike ML, but uses this probability too strong.
Glass (2007) showed that the problems of both models were present in the domain of medical diagnosis. The aim of this thesis is to find out whether these problems of ML and MPE generalize to the domain of goal inference. The approach taken here is analogous to the analysis of Glass. Glass made a comparison between several approaches by applying them to several simple medical diagnostic cases. My approach will be to apply the ML and MPE models to situations in which goal inference is needed. I will then analyze the outcomes and compare them to human intuition to see whether the problems, identified by Glass in the domain of medical diagnosis, generalize to the domain of goal inference.

If the predictions of the MPE approach do not fit human intuition and the predictions of the ML approach do fit, it suggests goal inference can be modeled using ML. If it is the other way around, it suggests goal inference can be modeled using MPE, like Baker (2007; 2008) does.

1.4 Overview

The remainder of this thesis is organized as follows. In Chapter 2 the analysis of Glass (2007) is described in detail. Glass made a comparison between several approaches by applying them to medical diagnostic cases. He concluded that the problems of ML (ignoring prior probabilities) and MPE (overweighing prior probabilities) were present in these medical diagnostic cases. In Chapter 3 I describe my approach. I applied the ML and MPE models to concrete scenarios, in which goal inference is needed. I chose the scenarios in such a way that they had the same structure as the situations Glass has shown to be a critical test of the ML and MPE approaches. In order to find out what the intuitions of people are according to the goal in the scenarios and whether they are converging, I conducted a small scale experiment. In Chapter 4 the experiment and the results are described. The outcome was that people did have converging intuitions on what the goal is in the three scenarios. These intuitions did not correspond to the predictions of the ML model. However, intuitions did correspond to the predictions of the MPE approach. I conclude with a discussion of the findings and their implications in Chapter 5 and 6.
2 Glass’ analysis

Glass (2007) made a comparison of several approaches by applying them to 4 medical diagnostic cases. These medical diagnostic cases constituted of two medical conditions which were believed to be causally related to brain tumors. The conditions were called $H_1$ and $H_2$. Glass tried to determine which of the conditions provided a better explanation of the occurrence of tumors. The conditions were mutually exclusive, but not exhaustive. That means that people who are susceptible to brain tumors, do not have to have one of these conditions. People cannot however have both conditions.

Glass gave four fictitious scenarios which specified prior probabilities of $H_1$ and $H_2$ within a group of susceptible people and the probabilities for developing a brain tumor given $H_1$ and given $H_2$. The prior probabilities and likelihoods of the four scenarios and the measures obtained (in Table 1) are given down here.

$P(E) =$ prior probability of having a tumor
$P(E \mid H_1) =$ probability that you, given that you have condition 1, have a tumor
$P(H_1 \mid E) =$ the probability of condition 1, given that you have a tumor

In all 4 scenarios: $P(E) = 1/10$

Scenario 1:
$P(H_1) = 1/25, P(H_2) = 3/50, P(E \mid H_1) = 1/2, P(E \mid H_2) = 1.$

Scenario 2:
$P(H_1) = 2/25, P(H_2) = 1/25, P(E \mid H_1) = 1/4, P(E \mid H_2) = 1.$

Scenario 3:
$P(H_1) = 9/25, P(H_2) = 1/50, P(E \mid H_1) = 1/6, P(E \mid H_2) = 1.$

Scenario 4:
$P(H_1) = 2/25, P(H_2) = 1/50, P(E \mid H_1) = 3/4, P(E \mid H_2) = 1.$
Scenario 1 was a very straightforward scenario. $H_2$ has a greater likelihood and a greater prior probability. In the other three scenarios $H_2$ had a greater likelihood and $H_1$ a greater prior probability. In scenario 1 and 2, ML and MPE agreed which hypothesis is the best, namely $H_2$. In scenario 3 and 4, ML and MPE did not prefer the same hypothesis.

As discussed in the Section 1.3, ML and MPE have some problematic aspects. The problem of the ML approach is that it does not take prior probability into account. MPE does take prior probability into account, but overweighs this probability. These problems arose in scenarios 3 and 4. In scenarios 3 and 4 the prior probabilities of $H_1$ were higher than the ones of $H_2$, still ML was in both scenarios in favor of $H_2$. And even though the prior probability of $H_1$ was larger in scenario 3 than in 4 (the prior probability of $H_2$ being the same in both scenarios), ML was in scenario 3 more in favor of $H_2$ than in scenario 4.

In both scenarios the likelihood of hypothesis $H_2$ was higher than the likelihood of $H_1$. The likelihood of $H_2$ was even 1 in both scenarios. Still, MPE was in both scenarios in favor of $H_1$. This is especially striking in scenario 3 where the likelihood of $H_1$ is 6 times as small as the likelihood of $H_2$ (1/6 and 1).

Glass stated human intuition favors $H_2$ in scenario 3 and $H_1$ in scenario 4. ML was thus in scenario 3 in correspondence with human intuition and in scenario 4 not. MPE corresponded in scenario 4 with human intuition. In scenario 3 MPE did not correspond to human intuition.
3 Extending Glass’ approach to the goal domain

The aim of this thesis was to determine whether problems known to plague probabilistic models of IBE generalize to the domain of goal inference. In order to do this, I constructed scenarios of goal inference that have the same structure as the medical diagnostic cases of Glass (2007) in which the problems of ML and MPE arose.

The scenarios of Glass in which the problems of ML and MPE arose, were scenarios 3 and 4. In these scenarios prior probability and likelihood pointed in different directions. In scenario 3 for example the prior probability of $H_1$ was the largest ($9/25 > 1/50$). However, $H_2$ had the largest likelihood ($1 > 1/6$). Thus in my scenarios the prior probability and likelihood of the hypotheses also pointed in different directions. This ratio between the hypotheses is of more importance than the exact probability assignments. The exact prior probability assignments are given for understanding and to be able to calculate what probabilities ML and MPE give to the hypotheses. Also, the exact probability ML and MPE give to the hypotheses is secondarily to the ordering they give to the hypotheses. For example, it does not matter whether MPE gives a probability of .01 to $H_1$ and probability .03 to $H_2$ or a probability of .1 to $H_1$ and probability of .3 to $H_2$. It is of more importance that $H_2$ has a (three times) higher probability than $H_1$ and that consequently $H_2$ is the best explanation.

After I constructed the scenarios, I made predictions, using ML and MPE, about the goal inferred in the scenarios. The next step was to compare these predictions with human intuition of the goal in these situations. Since I wanted to know what human intuition was, I used stories and not just probabilities like Glass did. With the use of these stories, I performed a small scale experiment in which people were questioned about their intuitions of the goal in the scenarios.

I used one control scenario. In this scenario the prior probability and likelihood do not point in different directions. However, of the two goals given as an explanation of Bob’s behavior, one goal was “impossible”. I used this scenario to be able to ascertain whether people have read the scenarios properly. If for example 80% of the people chooses the “impossible” goal, it suggests that these people did not read this scenario properly. Then perhaps 80% of the people also did not read the
other scenarios properly. This means that these people may had chosen a different goal if they had read the scenarios properly. If this happens, the scenarios will have to be clarified. To see whether people have converging intuitions in these scenarios, these scenarios will also have to be tested.

The scenarios and the probabilities are given in the next sections.

3.1 Scenario A

Bob is having a picnic in the park with his girlfriend. They are sitting in the grass and are facing each other. In between them on the blanket lie 6 objects. These objects are: a loaf of bread, butter, cheese, a cup, a muffin and a basket.

Now, imagine Bob picks up the cup.

Why does Bob pick up the cup? Does Bob want to clean up or does Bob want to dig up dirt?

$H_A = \text{Bob wants to clean up.}$

$H'_A = \text{Bob wants to dig up dirt.}$

$E_A = \text{evidence: Bob picks up the cup.}$

| $P(H_A)$ | $P(H'_A)$ | $P(E_A|H_A)$ | $P(E_A|H'_A)$ |
|----------|-----------|--------------|--------------|
| .18      | .01       | 1/6          | 1            |

I assume the prior probability of Bob wanting to clean up to be lower bounded by .07 and upper bounded by 1: $0.07 \leq P(H_A) < 1$. The prior probability of Bob wanting to dig up dirt is obviously very small and therefore I assume it to be .01: $P(H'_A) = .01$. For numerical illustration, I set $P(H_A)$ at .18. It is defined this exact probability because then the hypotheses have the same ratio as used in Glass’ scenarios (the prior probability of one hypothesis being 18 times the prior probability of the other hypothesis).

When Bob wants to clean up, he can grab 6 objects. Therefore, I assume the probability of Bob picking up the cup, given that he wants to clean up ($P(E | H_A)$)
to be $1/6$. When Bob wants to dig up dirt, the only object he can use is the cup. It is unlikely that Bob wants to dig up dirt with one of the other objects, for example the cheese. Therefore I assume these probabilities to be 0. Consequently I assume the probability of Bob picking up the cup when he wants to dig up dirt ($P(E | H'_A)$) to be 1.

The probability of the evidence, seeing Bob picking up the cup, can be defined by equating the likelihoods with the priors and adding them up. However, not all possible hypotheses are given and thus it is impossible to calculate the probability of the evidence. Hence, I did not calculate $P(E_A)$, but set it, in imitation of Glass (2007), at $1/10$.

Scenario A corresponds to Glass’ scenario 3. The likelihood assignments ($P(E | H)$) are exactly the same. The probability assignments of the hypotheses in the two scenarios are different. However, they do have the same structure. In Glass’ scenario 3 the probability of $H_1$ is 18 times as large as the probability of $H_2$. In scenario B $P(H_A)$ (being .18) is also 18 times as large as $P(H'_A)$ (being .01).

3.2 Scenario B

Bob owns a car. Bob can unlock his car using either his key or his remote. The key can also be used, of course, to lock the car. But because the remote is broken it cannot be used to lock the car.

Now, imagine Bob drives with his car to the supermarket. He parks his car in the parking lot. Bob goes into the supermarket for an hour. After that, he walks to his car in the parking lot and puts his key in the lock.

Why does Bob put his key in the lock? Does Bob want to open his car or close it?

$H_B = \text{Bob wants to open the car.}$

$H'_B = \text{Bob wants to close the car.}$

$E_B = \text{evidence: Bob puts his key in the lock.}$

In the scenario it is said that Bob went to the supermarket for an hour. Therefore it is more probable that Bob wants to open his car than that he wants to close it. However, there is a possibility that Bob, while shopping, realized he forgot to
lock his car and came back to the parking lot to lock it. I therefore assume that this probability of Bob wanting to close the car is not 0, but $0 \leq P(H_B') < .35$.

Bob can only have two goals when he puts his key in the lock, lock or unlock the car. Therefore $P(H_B) + P(H_B')$ should be 1. The probability of Bob wanting to open his car is therefore $.65 < P(H_B) \leq 1$. For numerical illustration I use $P(H_B) = .8$ and $P(H_B') = .2$.

When Bob wants to open his car, he has two options: use the key or the remote. I therefore assume the probability that Bob uses the key when he wants to unlock the car ($P(E \mid H_B)$) to be $.5$. When Bob wants to close his car, he has only one option: use the key. Consequently, the probability that, when Bob wants to open the car, he uses the key ($P(E \mid H_B')$) should be 1. However, there is a possibility (though small) that Bob forgets the remote cannot be used to lock the car, I assume thus that the probability is not 1, but $.9$.

The probability of the evidence, seeing Bob putting his key in the lock, can be defined by equating the likelihoods with the priors and then counting them up, see equation 1. Using this equation, the probability of the evidence $P(E_B)$ is .58.

$$P(E_B) = P(E_B \mid H_B)P(H_B) + P(E_B \mid H_B')P(H_B') \tag{1}$$

Scenario B corresponds to Glass’ scenario 4. The probability assignments of the hypotheses ($P(H)$) have the same ratio as in Glass’ scenario 4, $H_1 (.8)$ being four times as large as $H_B' (.2)$. The likelihood assignments ($P(E \mid H)$) do not correspond perfectly. However, the likelihood of $H_B$ ((in Glass’ scenario $P(E \mid H_2) .75$, in scenario B this is .5)) is also smaller than the likelihood of $H_B'$ (in Glass’ scenario $P(E \mid H_2) 1$, in scenario B this is .9).
3.3 Scenario C

Bob is at home. Bob has a television. To adjust the volume, Bob can do two things. On the television there is a button with which he can decrease or increase the volume. Bob also has a remote. However, the remote is old and some buttons on the remote are broken. Bob can increase the volume with the remote, but not decrease it.

Imagine, Bob is at home, watching a movie on his television. Suddenly outside there is a lot of noise, there are children playing. Bob grabs his remote.

Why does Bob grab his remote? Does Bob want to decrease the volume or does Bob want to increase the volume?

\[ H_C = \text{Bob wants to decrease the volume.} \]

\[ H'_C = \text{Bob wants to increase the volume.} \]

\[ E_C = \text{evidence: Bob grabs his remote.} \]

| \( P(H_C) \) | \( P(H'_C) \) | \( P(E_C|H_C) \) | \( P(E_C|H'_C) \) |
|-------------|--------------|----------------|----------------|
| .4          | .6           | .01            | .5             |

This scenario is a control scenario. It is used to be able to ascertain that people have read the scenarios properly and processed the information correctly. In this scenario only one goal is possible given the action performed. If people choose the “impossible” goal, it suggests they did not read the scenarios properly.

In the scenario, Bob has only two possible goals: decrease or increase the volume. \( P(H_C) \) and \( P(H'_C) \) thus need to count up to 1. In the scenario it is told that Bob is watching a movie, he probably wants to hear the movie. However, Bob could also be wanting to hear what is going on outside. Therefore I assume \( P(H_C) \) to be lower bounded by .2 and upper bounded by .8 and \( P(H'_C) \) to be lower bounded by .2 and upper bounded by .8: .2 \( \leq P(H_C) \leq .8 \) and .2 \( \leq P(H'_C) \leq .8 \). For numerical illustration I use \( P(H_C) = .4 \) and \( P(H'_C) = .6 \).

When Bob wants to decrease the volume, he has only one option: use the button.
on the television. However, because Bob could have been forgotten the remote cannot decrease the volume this probability is not assumed to be 0. However, it is very unlikely Bob forgets this, because it is a lot of trouble to get up to decrease the volume, therefore I assume the probability of Bob grabbing the remote given that he wants to decrease the volume \( P(E_C | H_C) \) to be .01. When Bob wants to increase the volume, he has two options: use the remote or use the button on the television. Therefore I assume the probability of Bob grabbing the remote given that he wants to decrease the volume \( P(E_C | H'_C) \) to be .5.

The probability of the evidence, seeing Bob grabbing the remote, can be defined by equating the likelihoods of the hypotheses with their priors and then counting them up, see equation 2. Using this equation, the probability of the evidence \( P(E_C) \) is .304.

\[
P(E_C) = P(E_C | H_C)P(H_C) + P(E_C | H'_C)P(H'_C)
\]

This scenario does not correspond with any of Glass’ scenarios.

### 3.4 Model predictions

In this section I will apply ML and MPE to the scenarios to make predictions about the goal inferred in the scenarios. Hereby, I will make frequent use of conditional probability distributions, where a conditional probability of an event \( A \) given \( B \) is defined as:

\[
P(A | B) = \frac{P(A, B)}{P(B)} \text{ for } P(B) > 0
\]

Bayes’ theorem offers an alternative way to compute conditional probabilities:

\[
P(A | B) = \frac{P(B | A)P(A)}{P(B)}
\]

since \( P(A, B) = P(B, A) = P(B | A)P(A) \), according to the definition (3).

As explained in Section 1.2, ML defines the best explanation as being the hypothesis \( H \) with the maximum value of \( P(E | H) \). Using definition 4, the best explanation according to ML can be defined using equation 5.
\[
P(E \mid H) = \frac{P(H \mid E)P(E)}{P(H)}
\]  

(5)

However, I already defined \( P(E \mid H) \) in the scenarios. For example, I assumed the probability that Bob wanted to clean up given that he grabs the cup (\( P(E_A \mid H_A) \)) to be 1/6. Therefore, I will not use equation 5. Instead, I will use the likelihoods given in the scenarios to identify the best explanation according to ML.

MPE defines the best explanation as the hypothesis \( H \) that maximizes \( P(H \mid E) \). Using definition 4, the best explanation according to MPE can be defined using equation 6. I will identify the best explanation according to MPE using this equation.

\[
P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}
\]  

(6)

In scenario A and B the hypotheses that ML and MPE identify as best explanation differ (see Table 5 and 6). In scenario A, ML identifies as best explanation that Bob wants to dig up dirt (\( H'_A \)). MPE in contrast favors the explanation that Bob wants to clean up (\( H_A \)). In scenario B, ML identifies that Bob wants to lock the car as being the best explanation (\( H'_B \)). MPE identifies as best explanation that Bob wants to open his car (\( H_B \)). In scenario C both ML and MPE are both in favor of the explanation that Bob wants to increase the volume (\( H'_C \)).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>ML</th>
<th>MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario A</td>
<td>( H'_A )</td>
<td>( H_A )</td>
</tr>
<tr>
<td>Scenario B</td>
<td>( H'_B )</td>
<td>( H_B )</td>
</tr>
<tr>
<td>Scenario C</td>
<td>( H'_C )</td>
<td>( H'_C )</td>
</tr>
</tbody>
</table>

Table 5: Hypothesis that best explains evidence according to ML and MPE

The choice of ML and MPE for the hypothesis they favor does not depend on the exact prior probability assignments given in Chapters 3.1, 3.2 and 3.3. As explained I gave lower and upper bounds to the prior probabilities, but set them at a specific value for numerical illustration. Within the ranges of the lower and upper bounds, ML and MPE favor the same hypotheses they favor with the exact probability assignments.
Table 6: Rankings of the hypotheses according to ML and MPE

<table>
<thead>
<tr>
<th></th>
<th>ML</th>
<th>MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H'$</td>
<td>$H$</td>
</tr>
<tr>
<td>Scenario A</td>
<td>1/6</td>
<td>.3</td>
</tr>
<tr>
<td>Scenario B</td>
<td>.5</td>
<td>.9</td>
</tr>
<tr>
<td>Scenario C</td>
<td>.01</td>
<td>.5</td>
</tr>
</tbody>
</table>

The problem of ML is that it does not take prior probability into account. The choice of ML in scenario A and B shows this problem. In both scenarios the prior of $H$ is much higher than the prior of $H'$. Still, ML is in both scenarios in favor of $H'$. Especially in scenario A this is striking. The prior of $H$ is 18 times larger than the prior of $H'$. Even so, ML ignores this. It ranks the two hypotheses far from each other, with a clear advantage for $H'$, having a six times greater probability than $H$.

The problem of MPE is that it puts too much weight on the prior probabilities. However it seems to be that MPE does not have that problem here. In scenario A where the prior probability of $H_A$ is 18 times larger than the one of $H'_A$, it does rank $H'_A$ as best explanation.

4 Experiment

In order to test whether there is consensus about the goal of Bob in the scenarios, I conducted a small scale experiment. In this experiment I asked people about their intuitions of the goal of Bob in the 3 scenarios. Scenario C was a control scenario, it was used to be able to ascertain that people have read the scenarios properly.

4.1 Method

Participants Twelve people participated in the experiment, ranging in the age from 19 to 32 (average age being 22). Nine people were AI students, the other three people were a student of political science, a psychology student and a philosophy/cognitive science PhD student. Five participants were female and seven
were male.

**Procedure** People were asked whether they wanted to participate in a small scale experiment. They were told the experiment would not take much of their time. If they wanted to cooperate, the experiment form was given to them. Before the participants began reading, they were told that they had to read the front page thoroughly and only flip the page when they were ready reading (and filling in) the front page. Also it was told that if they had questions, they should ask these questions before they flipped the page. Participants were not allowed to ask questions or discuss with other people during the experiment. When participants were ready with the experiment, they gave the experiment form back.

**Design** To prevent order effects to arise, there were 8 versions of the experiment form. These 8 forms differed in the order of scenarios and answers. In the beginning of the form it is stated that there are no trick questions and no right or wrong answers. Also it is stated that people should choose an answer based on their first intuition. This in order to prevent people from not choosing the best explanation, but perhaps adjusting their answers to expected experiment goals.

In the following pages the three scenarios were explained, each on a different page. Each scenario was followed by a question about Bob’s intention and the two possible answers (\(H\) and \(H'\)). The scenarios were explained as clearly as possible. This in order to prevent people misunderstanding the situations and having a wrong interpretation of the scenario in mind when answering the questions. (See Appendix A for the form used in the experiment.)

### 4.2 Results

The results of the experiment are summarized in Table 7. In scenario A and B all people had the same intuition. In scenario A the intuition was that Bob wanted to clean up. In scenario B the intuition was that Bob wanted to open his car. However, in scenario C the intuitions were not as converging as in the first two scenarios. Two people thought that Bob wanted to decrease the volume of his television. The other ten people thought Bob wanted to increase the volume.

Scenario C was a control condition. The evidence was that Bob grabbed the remote. The two possible explanations were that Bob wanted to decrease (\(H_C\)
or increase ($H'_C$) the volume. However, in the scenario was told that the remote could only increase the volume. The fact that in scenario C two out of twelve people had the intuition that Bob wanted to decrease the volume, suggests that these people did not read the scenario properly. If we assume the same proportion of people did also not read scenario A and B properly, then perhaps these people would have had a different intuition if they did had read the scenarios properly. However, 83.33% (ten out of twelve people) having the same intuition still is a converging intuition. Therefore, it can be concluded that the intuitions about the goal in the scenarios are converging.

Table 7: Human intuition about the goal

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$H'$</th>
<th>$H''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

5 Discussion

Glass (2007) showed that ML and MPE have some problematic aspects in the domain of medical diagnosis. ML ignores prior probabilities and MPE overweighs them. The problem posed was whether the problems of ML and MPE generalize to the domain of goal inference. In order to test this, I applied both approaches to three scenarios of goal inference. Then I tested what human intuition about the goal was in the scenarios. Finally, I compared the predictions of the approaches with human intuition.

It turned out that ML was not in correspondence with human intuition in scenario A and B. In scenario A, ML favored the hypothesis that Bob wanted dig up dirt ($H'_A$). People however chose the hypothesis that Bob wanted to clean up ($H_A$) as being the best explanation. In scenario B, ML favored the hypothesis that Bob wanted to close his car ($H'_B$). People favored the hypothesis that Bob wanted to open the car ($H_B$). In scenario C, ML was in correspondence with human intuition (both favored the hypothesis that Bob wanted to increase the volume.
However, this scenario did not function as a critical test of the models. In sum, ML failed to imitate human intuition (see Table 8) and ML did have the problem of ignoring prior probabilities in these scenarios of goal inference.

The predictions of MPE matched with human intuition in all three scenarios. In scenario A both human intuition and MPE favored the hypothesis that Bob wanted to clean up ($H_A$), in scenario B that Bob wanted to open his car ($H_B$) and in scenario C that Bob wanted to increase the volume ($H'_C$) (see Table 8). These results suggest that MPE does not have the problem of overweighing prior probabilities in the domain of goal inference.

The results of this analysis differ from the results of Glass. Glass applied ML and MPE to medical diagnostic cases. Hereby he found that both ML and MPE failed to imitate human intuition. In contrast, in my scenarios of goal inference MPE did match human intuition. It could be interesting to do more research with MPE to see whether it still matches human intuition in more scenarios of goal inference. Also, MPE could be applied to more domains in which inference is being used to analyze whether the predictions of MPE match human intuition and whether it has the problem of overweighing prior probabilities in these domains.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>ML</th>
<th>MPE</th>
<th>Intuition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$H'_A$</td>
<td>$H_A$</td>
<td>$H_A$</td>
</tr>
<tr>
<td>B</td>
<td>$H'_B$</td>
<td>$H_B$</td>
<td>$H_B$</td>
</tr>
<tr>
<td>C</td>
<td>$H'_C$</td>
<td>$H'_C$</td>
<td>$H'_C$</td>
</tr>
</tbody>
</table>

5.1 Methodological Considerations

Two methodological considerations to the analysis can be given. First, I do not have exact knowledge of the probabilities in the scenarios. For example, the prior probability of Bob wanting to close his car. I assume this probability to be .2. I cannot know for sure whether this probability is exactly correct, I can only try to give a good estimation. The results of my analysis would perhaps be different if I had used different probabilities. However, I defined the prior probabilities ranging from a lower bound to some upper bound. This in order to prevent the results being
to dependent of the exact probability assignments. In the case of the probability of Bob wanting to close his car this range was 0 till .35. Within these ranges ML and MPE favored the same hypothesis as they did with the exact probability I gave them. I did not do this with the likelihood assignments. These probabilities follow causality and can therefore be estimated following this causality. For example, think of the probability of Bob grabbing the remote when he wants to increase the volume. Bob only has two options to increase the volume, I thus assume this probability to be .5.

Second, only three scenarios were used of which one was a control scenario and did not function as a critical test. One could raise the question whether the analysis with the use of two scenarios is enough to be able to conclude whether ML and MPE are good models of human intuition in the domain of goal inference. However, the predictions of ML did not correspond with human intuition in the two scenarios. The predictions of ML did fit human intuition in one scenario. Yet, this scenario was not a critical test of the models, but a control scenario. It was a straightforward scenario, the prior probabilities and likelihoods did not point in different directions. Therefore it is very unlikely that ML will perform better in other scenarios that are not straightforward scenarios, like the control scenario is. Further research needs be done with the MPE approach, to be able to conclude it correctly predicts human intuition in scenarios of goal inference. The results of my analysis do however imply that it is possible for MPE to predict human intuition in the goal domain correctly.

5.2 The probabilistic mind debate

As discussed in the Introduction, there is a debate about whether people reason according to the rules of probability. The argument for the view that people do not reason according to the rules of probability was that it seems to be in conflict with the finding that people are poor at explicit probabilistic reasoning. People show all sorts of errors in explicit probabilistic reasoning; base rate fallacy being the most used as a demonstration of errors in probabilistic reasoning.

Base rate fallacy means that people ignore prior probabilities. The most
dominant view is that base rate fallacy is “a matter of established fact” (Koehler, 1996). Often the base rate fallacy (and other errors) is being used to demonstrate people do not reason according to the rules of probability.

I found that in the three scenarios of goal inference, people did not neglect prior probabilities. This result adds to the growing number of literature that questions whether there really is a base rate fallacy. Base rate neglect seems to come and go. In some tasks people ignore priors, and in other tasks they do use priors in their predictions (Gigerenzer and Hoffrage, 1995; Evans et al., 2000; Goodie and Fantino, 1996). Koehler (1996) concludes that base rates are almost always used. However the degree to which base rates are used depends on task representation and structure. When tasks are represented in a way that encourages people to use base rates (e.g. defining tasks in frequentist terms), people are more likely to solve the task using base rates (Barbey and Sloman, 2007; Gigerenzer and Hoffrage, 1995; Laming, 2007; Betsch et al., 1998; Stolarz-Fantino et al., 2006; Cosmides and Tooby, 1996; Evans et al., 2007). There are even researches that find a overweighing of base rates (Teigen and Keren, 2007; Medin and Bettger, 1991). Thus, the view that base rate neglect does not exist, only in certain tasks which are mostly completely different than those existing in real life (Koehler, 1996; Laming, 2007) becomes much more widespread. My results add to this view.

The possibility that people do not show a base rate fallacy, suggests people do integrate prior probabilities in their predictions. This takes away the foundation for the view that people do not reason according to the rules of probability and it supports the view that the human mind reasons according to rules of probabilities, which is supported by e.g. Oaksford and Chater (2009).

5.3 Probabilistic models of cognition

Using probabilistic models to model (parts of) human reasoning, means describing human reasoning according to probability theory. Humans however do not only take into account probabilities, they also have regard for utilities. People, for example, take into account how big the consequences of a certain decision are. By not taking into account utilities, these models do not fully reflect the way human
beings reason. However, it is proven to be possible to explain human cognition in probabilistic terms (Chater et al., 2006; Oaksford and Chater, 2009) and more often probabilistic models are used to model aspects of human cognition, for example language processing (Jurafsky, 1996) and musical cognition (Temperley, 2004).

How can goal inference best be modeled using probabilistic models? The results in this thesis suggest that MPE could be a good candidate. The predictions of the MPE approach corresponded with human intuition in the three scenarios I used. However, as discussed in Chapter 5.2, only three scenarios are used. Thus in order to be able to conclude that MPE predicts human intuition in goal inference correctly, further research needs to be done by applying MPE to more scenarios of goal inference. However, Glass (2007) showed that MPE does at least in the domain of medical diagnosis not predict human intuitions correctly. Therefore MPE as a model of not only goal inference but of explanatory inference in general does not seem to be very strong.

Glass also showed that ML has problems with predicting human intuition. In his paper Glass tried to overcome the problems of the ML and MPE in the domain of medical diagnosis and defined the Coherence approach. This approach defines the best explanation as the hypothesis which is the most coherent with the evidence, and has the maximum value for $C(H, E)$, see equation 7.

$$C(H, E) = \left( \frac{1}{P(H | E)} + \frac{1}{P(E | H)} - 1 \right)^{-1} \quad (7)$$

According to Glass the Coherence approach captures our intuitions better than the ML and MPE approaches do. Interestingly, I found that Coherence fitted human intuitions in all three scenarios (see Table 9). I suggest further research with the Coherence approach by applying it to more scenarios of goal inference and by applying it to scenarios of other domains of human inference.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Intuition</th>
<th>Coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario A</td>
<td>$H_A$</td>
<td>$H_A$</td>
</tr>
<tr>
<td>Scenario B</td>
<td>$H_B$</td>
<td>$H_B$</td>
</tr>
<tr>
<td>Scenario C</td>
<td>$H'_C$</td>
<td>$H'_C$</td>
</tr>
</tbody>
</table>
6 Conclusion

Probabilistic models of IBE are often used in the cognitive (neuro-)science literature to model goal inference. Two of these models are ML and MPE. It is known that ML and MPE have problems with predicting human intuition (Glass, 2007). The analysis in this thesis showed that ML did not predict human intuition correctly in the used scenarios of goal inference. This result suggests that ML has problems with predicting human intuition in the domain of goal inference. More research needs to be done to ascertain this. However, it is not very likely ML will predict human intuition correctly in other scenarios of goal inference. MPE, in contrast to ML, predicted human intuition correctly in the scenarios of goal inference. This result suggests that MPE does not have the problem of overweighing prior probabilities in the domain of goal inference. However, in order to be certain that the problem of MPE is not present in the domain of goal inference, more research with MPE needs to be done. It would also be interesting to apply MPE to scenarios of other inference domains. Will the predictions of MPE still correspond to human intuition in these domains? MPE could for example be applied to scenarios of language perception.

I also found that the Coherence approach predicted human intuition correctly in the three scenarios of goal inference. Glass showed that ML and MPE have problems predicting human intuition in medical diagnostic cases and he defined the Coherence approach. The Coherence approach predicted human intuition in medical diagnostic cases correctly. Coherence thus predicts human intuition correctly in scenarios of medical diagnosis and in scenarios of goal inference. However, Glass defined this model as a compromise between ML and MPE to correspond to human intuition in the medical diagnostic scenarios he used. Coherence thus seems to be a ad hoc model, it is made to fit the data in the medical diagnostic cases and lacks a theoretical basis.

In sum, the results of my analysis suggest that the problem of ML does generalize to the domain of goal inference. The problem of MPE does not seem to generalize.
Acknowledgements

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References


A Experiment Form

Instructions
On the next pages you will find three scenarios about a fictive character named Bob. For each scenario there is one question.

Read the scenarios in the given order. When you have read one scenario and answered the question, you can turn the page. Do not return to a previous scenario.

There are no trick questions and no right or wrong answers. Choose your first intuition.
If you have any questions, you can ask them now.

Please fill in the following information before you turn the page.

Age:

Gender: Female/Male

Profession or Study:

Scenario 1

Bob owns a car. Bob can unlock his car using either his key or his remote. The key can also be used, of course, to lock the car. But because the remote is broken it cannot be used to lock the car.

Now, imagine Bob drives with his car to the supermarket. He parks his car in the parking lot. Bob goes into the supermarket for an hour. After that, he walks to his car in the parking lot and puts his key in the lock.

Why does Bob put his key in the lock? (pick the most plausible explanation)

(a) Bob wants to unlock his car.
(b) Bob wants to lock his car.

Scenario 2
Bob is having a picnic in the park with his girlfriend. They are sitting in the grass and are facing each other. In between them on the blanket lie 6 objects. These objects are: a loaf of bread, butter, cheese, a cup, a muffin and a basket.

Now, imagine Bob picks up the cup.

Why does Bob pick up the cup? (pick the most plausible explanation)

(a) Bob wants to clean up.
(b) Bob wants to dig up dirt.

Scenario 3

Bob is at home. Bob has a television. To adjust the volume, Bob can do two things. On the television there is a button with which he can decrease or increase the volume. Bob also has a remote. However, the remote is old and some buttons on the remote are broken. Bob can increase the volume with the remote, but not decrease it.

Imagine, Bob is at home, watching a movie on his television. Suddenly outside there is a lot of noise, children began playing on the street. Bob grabs his remote.

Why does Bob grab his remote? (pick the most plausible explanation)

(a) Bob wants to decrease the volume.
(b) Bob wants to increase the volume.
Thank you for your time and cooperation!