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Dealing With Deflation Bias Through a Dismissal Contract

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Introduction

This paper studies the possibility of using dismissal contracts to counter deflation bias. Deflation bias can occur if the nominal short-term interest rate cannot turn negative even if the equilibrium value of the nominal short-run interest rate is negative. In this case the demand for investment funds and the supply of savings cannot be matched and output is below its 'natural' level. Persistent negative output gaps can lead to deflation (Summers 2015).

Nakata & Schmidt (2019) propose appointing a conservative central banker or assigning the central banker an inflation-based contract in order to incentivize the central banker to reduce deflation. These mechanisms have also been proposed to reduce inflation bias. However, from studies that have focused on inflation bias, it is known that appointing a conservative central banker can lead to a misalignment of preferences in case of a supply shock (Rogoff, 1985), while direct financial incentives provided by an inflation-based contract may not motivate the central banker enough (Walsh, 2002). Furthermore, it is argued that the New Zealand Reserve Bank Act of 1989 already represents some form of dismissal rule (Walsh, 1995B). Therefore Walsh (2002) proposed adopting a dismissal rule to incentivize the central banker to reduce inflation bias.

This paper aims to extend the literature on deflation bias by studying the possibility of mitigating deflation bias by adopting a central banker dismissal rule that would incentivize the central banker to keep inflation high in better states of the economy in order to mitigate deflation risk in worse states of the economy. Furthermore, several extensions are proposed to the Nakata & Schmidt (2019) model, such as adding a non-zero inflation target or considering the impact of a zero lower bound on inflation expectations on inflation and the output gap.

It is found that adding a dismissal rule to the central banker's contract incentivizes the central banker to increase inflation in the high natural rate state of the economy, which in turn also increases inflation in the low natural rate state of the economy. The effect of the dismissal contract on the output gap is ambiguous in both states. Furthermore, the existence of a zero-lower bound on inflation expectations nullifies the effect the existence of the zero-lower bound on the nominal interest rate has on output and the inflation rate in the non-ZLB state, but not in case the central bank has adopted a sufficiently high positive inflation target.

The literature review follows in section 2. In section 3, Nakata & Schmidt (2019) is discussed and several extensions of the model are proposed. In section 4 Walsh (2002) is discussed and one extension of the model is discussed. Section 5 discusses the addition of a dismissal rule to the Nakata & Schmidt (2019) model. Section 6 concludes the paper and section 7 is the bibliography.

Literature review

In the years prior to the COVID-19 crisis, economists often speculated about the notion of secular stagnation. Summers (2015) argued that the presence of a zero lower bound constraint on nominal interest rates could prevent the market from clearing, as the level of the nominal interest rate at which firm demand for loans would match the supply of loans cannot be reached if this level is below zero, and this could cause a prolonged period of output being below natural output (the output gap being negative) and the inflation rate being below the target level. The reasons the modern equilibrium interest rate might be lower than it used to be could be related to a decline in population growth, an increase in worldwide savings or a decline in investment demand due to lower marginal returns on investment (Summers 2015).

Eggertson (2006) stated that there are four requirements for sustained deflation to be a threat. The economy has to be subject to shocks, the nominal short-term interest rate has to be the central bank's only instrument, the central bank is unable to commit to future inflation rates and there has to be a zero lower bound on the short-term nominal interest rate. Under these circumstances shocks which drive the natural real interest rate to below zero can cause deflation, and the central bank is unable to overcome this by committing to future inflation. The inconsistency problem arguably is even worse than in the classical inflation bias case, since deflation usually is triggered by infrequent shocks and the central bank therefore does not have the means to build up a credible reputation in reducing deflation the same way it can build up a credible reputation in reducing inflation.

Nakata & Schmidt (2019) study a model in which the nominal interest rate is constrained by the zero-lower bound. The natural real interest rate has a positive non-crisis state and a negative crisis state. Since the frequency of shocks driving the natural rate to zero likely is different from the persistence of these shocks, the natural interest rate follows a two-state Markov process. Furthermore, expectations of future inflation play a role in determining current inflation, which creates an intertemporal link. The presence of the ZLB introduces a deflationary bias even in the high natural rate scenario, as the prospect of deflation in the ZLB scenario already reduces inflation and output in the high natural rate scenario.

Whether such effect holds in real life at least partially depends on the effectiveness of unconventional monetary policy. If policies such as forward guidance and quantitative easing are effective enough, the presence of the zero-lower bound potentially does not have a major impact on inflation expectations. Debortoli, Gáli & Gambetti (2019) found that several macroeconomic variables, such as inflation and the long-term interest rate, did not react differently to a demand shock when the short-term interest rate was closer to the zero-lower bound. This supports the idea that the presence of the zero-lower bound does *not* materially impact the economy, as unconventional monetary policy is an effective tool to get around the zero lower bound constraint. However, Gust, Herbst, López-Salido & Smith (2017) estimate a non-linear DSGE model and find that the presence of the ZLB accounted for a significant part of the post-2008 decrease in inflation.

Various solutions are proposed to this deflation bias problem. Eggertson (2006) suggested that fiscal policymakers could reduce deflation bias by committing to running persistently large budget deficits. This can impact inflation since persistently large budget deficits incentivize the central bank to keep interest rates low and inflation high, and high levels of expected inflation can lead to higher levels of inflation today. Therefore Eggertson (2006) proposes using the implications of the fiscal theory of the price level, which usually are used to warn against excessive government budget deficits, as a tool to reduce deflation bias by forcing the central bank to commit to higher future inflation levels.

Nakata & Schmidt (2019) also propose two solutions to the deflation bias problem. The first solution they propose is appointing a central banker who is more conservative than the population as a whole.

A conservative central banker can reduce the deflationary bias in the high natural rate state by prioritizing stabilizing inflation over stabilizing the output gap, which would also reduce deflation and increase output in the low natural rate state (as expected future inflation impacts current inflation). It is important to note that ‘conservative’ in this context means prioritizing inflation stability over prioritizing output stability. Therefore a conservative central banker would actually be willing to accept higher inflation (lower deflation) and a higher positive output gap in case of negative inflation, which might seem counterintuitive at first.

Nakata & Schmidt (2019) also studied a model in which the central bank puts the same weights on the inflation and output gap targets as society as a whole, and instead was assigned a linear inflation/output gap contract which was meant to incentivize the central bank to overcome deflationary bias. It was found that this approach had a more positive effect on welfare than appointing a conservative central banker, as it could prove an incentive for the central banker powerful enough to increase inflation to a positive level in the high rate state, which then also increased expected inflation and inflation in the low natural rate state.

However, both the conservative central banker and assigning an inflation contract could pose other problems. In order to better understand this, it is important to take a look on the literature on inflation bias, as the literature on institutional mechanisms to deter inflation bias is better developed than the literature on institutional mechanisms to deter deflation bias.

The incentives faced by central bankers have been thoroughly studied in the past, beginning with Barro & Gordon (1983). In this model the central bank sets the inflation rate, while output is dependent on the difference between inflation and expected inflation, since increasing surprise inflation would boost output by reducing labour costs, as wages were set based on expected inflation. While the central banker would ideally set the inflation rate at zero, the central banker faces the incentive to stimulate output by increasing inflation above the expected inflation rate. However, rational wage setters anticipate this and set their expected inflation rate at the exact rate at which the central banker would not be able to further increase his utility by trading off a higher level of output for a higher level of inflation. At this point inflation is higher than zero, but output is unchanged since the inflation was expected. Thus this equilibrium, which is called the time-consistent equilibrium, is less ideal than the zero-inflation equilibrium, which is called the commitment equilibrium. Overcoming this problem, where the central bank has the incentive to break its promise to keep the inflation rate low, has been the subject of many studies. Barro & Gordon (1983) already suggested that the time inconsistency problem could be averted if the game is played repeatedly and a reputational mechanism is present.

Another solution proposed by Rogoff (1985) would be to appoint a central banker who is significantly more conservative than the population as a whole. This means appointing a central banker who puts little weight on output stabilization relative to inflation stabilization. This solution does not directly deal with the time inconsistency problem, as the basic incentive to surprise wage-setters remains, but by appointing a more conservative central banker it is possible to greatly reduce the inflation rate even in the time-consistent equilibrium, as the amount of surprise inflation the conservative central banker is incentivized to introduce is lower than the amount of surprise inflation a less conservative central banker would be incentivized to introduce. However, Rogoff's model also includes supply shocks that move the Phillips Curve by pushing inflation up or down irrespective of the output gap. A central bank that is significantly more conservative than society as a whole would not let inflation increase by enough during a supply shock (and would reduce output in order to achieve that), therefore there also is an incentive to not pick a *too* conservative central banker.

Since picking a central banker who is more conservative than the population as a whole can lead to a suboptimal outcome if supply shocks are considered, another solution has to be considered. Walsh (1995A) proposed the use of performance contracts. The basic set-up is that the central banker determines the money growth rate after observing a signal about the supply shock, inflation is determined by money growth and a money velocity shock and output is produced by firms which set wages without observing a signal about the supply shock. Making the payment the central banker receives dependent on both the information about the supply shock and the inflation rate is found to reduce inflation bias without sacrificing the benefits of discretion in case of a supply shock, since the contract is adjusted for the supply shock signal.

Another solution that can be considered is a dismissal contract. If re-evaluation and dismissal occur on fixed time intervals, it is possible to incorporate a rule which states the central banker will be fired if inflation in the current period is higher than a certain target inflation rate. In reality central bankers might be more motivated by holding a prestigious office than by pure financial gain, this is an advantage of the dismissal contract (Walsh, 2002). The dismissal contract approach was studied in Walsh (2002). The utility function of the central banker includes both the social loss function and the utility the central banker derives from simply being in office. The presence of a dismissal rule is found to have a similar effect as the inflation contract proposed by Walsh (1995A), it induces the central banker to reduce inflation bias (in order to reduce the probability that inflation exceeds the critical value) while still maintaining some benefits of discretion during shocks, provided that the dismissal rule is adjusted for the output gap level.

Nakata & Schmidt (2019) never considered the dismissal rule (which can also be seen as a reappointment rule) as a method to mitigate deflation bias. Since a linear inflation contract might be hard to implement in practice, a more conservative central banker can lead to misalignment of preferences in case of supply shocks and the prospect of reappointment or dismissal is one major tool with which governments can punish or reward central bankers, I propose to extend the Nakata & Schmidt (2019) model with a dismissal rule. It is expected that a dismissal rule which specifies that the central banker will be dismissed if inflation is below a critical value will incentivize the central banker to focus at increasing inflation in the high natural rate state, which in turn also increases inflation in the low natural rate state through the expectations mechanism. This can be welfare-enhancing in the same way as appointing a more conservative central banker to deal with deflation was found to be welfare-enhancing by Nakata & Schmidt (2019), with the added benefit that the preferences of the central banker in case of a supply shock still align with the preferences of the government in case of a supply shock. Therefore the chosen research method of this thesis is a model-theoretical research method. This is in line with the previous research on this topic as well. A theoretical model can provide insights in the incentives faced by a central banker and clear the road for future empirical research to test possible hypotheses that are derived from the model. Finally, it would be hard to empirically test the effects of reappointment rules on inflation and output, since it is hard to estimate the appointment rules used by policymakers when appointing a central banker. These criteria also tend to change depending on the political party in power, as some political parties might prefer a more hawkish stance on monetary policy than other political parties. Therefore, for the purposes of this thesis, a model-theoretic research approach will be taken.

Nakata & Schmidt (2019)

General overview

Nakata & Schmidt (2019) study the effects of the zero lower bound on the nominal interest rate on inflation and the output gap. They do so in a setting where the central banker sets the output gap and inflation rate with respect to the Phillips Curve and an Euler equation determining the output gap. After establishing that the presence of the zero lower bound causes deflation even in periods in which the natural real interest rate is positive, they study various institutional frameworks to reduce this deflation bias, ranging from appointing a conservative central banker to assigning the central banker a performance contract with an inflation-based payment.

The equilibrium conditions in the Nakata Schmidt (2019) model are described by the following equations:

$$\pi_t = k y_t + \beta E_t \pi_{t+1}$$

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

Where π represents the inflation rate, y represents the output gap, i represents the nominal policy rate and r^n represents the real natural interest rate. It is important to note that $E_t \pi_{t+1}$ reflects expected inflation between period t and t+1 while i_t reflects the interest rate between period t and t+1. Therefore $i_t - E_t \pi_{t+1}$ reflects the expected real interest rate between period t and period t+1. The first equation states that the inflation rate depends positively on the output gap and expected future inflation, while the second equation states that the output gap in period t depends positively on the expected future output gap and the gap between the natural real interest rate and the expected real interest rate.

The first equation is the New Keynesian Phillips curve. The positive effect of future inflation on current inflation can be explained through pricing frictions. Suppose a company expects that at some point in the future it would be optimal for them to increase prices, but the company is not sure whether it will be able to increase prices at that date. In that case it would be optimal to move part of the price increase forward to make up for the possibility that prices cannot be increased in the future. Therefore expected future inflation can increase inflation today. Since this effect likely is smaller than one-for-one, β likely is smaller than 1.

The second equation can be seen as some sort of Euler equation. Rewriting the equation, we obtain:

$$E_t y_{t+1} - y_t = \sigma((i_t - E_t \pi_{t+1}) - r_t^n)$$

Thus, if the expected real interest rate exceeds the natural real interest rate, the future output gap will be larger than the current output gap. However, if the expected real interest rate is lower than the natural real interest rate, current output will be higher relative to future output (assuming natural output stays the same in all periods). This could be interpreted as a low interest rate leading people to shift consumption from t+1 to t, thus boosting output in period t at the expense of output in period t+1.

There are multiple different ways in which the real natural interest rate can be interpreted. It could also be interpreted as the real interest rate at which real money demand equals real money supply and demand for capital equals the supply of capital. However, in this context it is better to stick to the Euler equation comparison. The expected future output gap equals the current output gap if the expected real interest rate equals the nature real interest rate. This is reminiscent of the consumption Euler equation:

$$U'(C_t) = \beta(1 + r)U'(C_{t+1})$$

It can be found that $C_t = C_{t+1}$ if $r = \frac{1}{\beta} - 1$, as in that case the optimal consumption path would be stable over time, since the time preference effect is cancelled out by the reward for postponing consumption (the interest rate). It could be argued that the real natural interest rate in the Nakata-

Schmidt model represents the real interest rate at which consumption (and by extent, output) is stable over time.

The main implication of the zero-lower bound constraint is that i cannot drop below zero, but in the zero-lower bound state r^n drops below zero. This reduces output, which in turn also reduces inflation. Furthermore, since the output gap in period t is linked to the output gap in period t+1, the possibility of the natural rate dropping below 0 in period t+1 also influences output and inflation in period t. The states of the natural real interest rate are modelled as follows:

$$Prob(r_{t+1}^n = r_l^n | r_t^n = r_h^n) = p_h$$

$$Prob(r_{t+1}^n = r_l^n | r_t^n = r_l^n) = p_l$$

The real natural interest rate can take two values, the high value ($r_h^n > 0$) and the low value ($r_l^n < 0$). The probability that the real natural interest rate takes on the low value in t+1 if it took on the high value in t is p_h, the probability that the real natural interest rate takes on the low value in t+1 if it took on the low value in t is p_l. A two-stage Markov process is chosen since it can be expected that the persistence of the low state (the probability that the economy is in the low rate state again given that it already is in the low rate state today) is different from the probability of the low state hitting the next period given that we are in the high rate state today.

The utility functions of the government and central bank are modelled as follows:

$$u(\pi, y) = -\frac{1}{2}(\pi^2 + \lambda y^2)$$

Therefore the implicit targeted inflation rate and output gap both are zero. Any deviation from zero causes a decrease in utility. λ represents the weight being put on the output gap. As the core thesis of Nakata & Schmidt (2019) is that appointing a conservative central banker can reduce deflation bias, the central bank is allowed to have a different value of lambda. A lower value of λ indicates that a lower weight is put on the output gap target relative to the inflation target. For a λ of zero, the central banker puts no value on the output target and only focuses on reaching the inflation target (which is zero in this model).

In the model the central bank sets the output gap subject to the New Keynesian Phillips Curve, the Euler equation and the ZLB condition. First the Phillips Curve is substituted into the utility function in order to eliminate inflation from the equation.

$$u(\pi, y) = -\frac{1}{2}((ky_t + \beta E_t \pi_{t+1})^2 + \lambda y_t^2)$$

This utility function is maximized with respect to the output gap, after which the Phillips Curve is used to substitute the inflation rate back in to obtain the following first-order conditions:

$$0 = -k\pi_h - \lambda y_h$$

$$0 < -k\pi_l - \lambda y_l$$

One assumption that is being made by Nakata & Schmidt (2019) is that the central bank has no method of committing to future inflation rates, as the central banker does not take into account the impact of the inflation decision on expected inflation. This precludes the central banker from promising higher or lower future inflation rates in order to increase or decrease future inflation. The absence of commitment will play a major factor in deflation bias.

The reason the second first-order condition does not equal zero is that the central banker is not able to reach the optimal inflation rate in the low state, since that rate would require a negative value of i_t. Therefore the central banker set i_l at zero. The solution for π_l is found through making use of the Phillips Curve equations to eliminate y_l and y_h and solving the resulting system of equations for π_l and π_h. Since the resulting output gap in the low rate state is lower than the optimal output gap in the low rate state (which cannot be reached since i_l would have to drop below zero for that to happen), the second first order condition has a positive value, as the marginal welfare gain of increasing inflation and output in the low natural rate state is positive.

Furthermore, by making use of rational expectations it can be established that:

$$(E_t \pi_{t+1} | r_t^n = r_h^n) = (1 - p_h)\pi_h + p_h \pi_l$$

$$(E_t \pi_{t+1} | r_t^n = r_l^n) = (1 - p_l)\pi_h + p_l \pi_l$$

$$(E_t y_{t+1} | r_t^n = r_h^n) = (1 - p_h) y_h + p_h y_l$$

$$(E_t y_{t+1} | r_t^n = r_l^n) = (1 - p_l) y_h + p_l y_l$$

Therefore we obtain a system of equations which can be solved for the inflation rate, and the output gaps can be derived from those inflation rates. Nakata & Schmidt (2019) find the following equilibrium levels for inflation and the output gap:

$$\pi_h = \frac{A}{AD - BC} r_l^n, \qquad \pi_l = \frac{-B}{AD - BC} r_l^n,$$

$$y_h = \frac{\beta k p_h}{AD - BC} r_l^n, \qquad y_l = -\frac{(1 - \beta p_l) k^2 + (1 - \beta)(1 + \beta p_h - \beta p_l)\lambda}{k(AD - BC)} r_l^n$$

$$A = -\lambda \beta p_h < 0, \qquad B = \lambda + k^2 - \lambda \beta (1 - p_h) > 0, \qquad C = \frac{1 - p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - p_l > 0,$$

$$D = -\frac{1 - p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - (1 - p_l) < 0$$

The assumptions largely follow from each other and the first order conditions. Recall that the first order condition for the output gap in the low rate state had to be positive as the low state solution is a corner solution. Because of the zero lower bound constraint the central banker cannot increase the output gap up until the point that the indifference curve of the central banker is tangent to the constraint of the central banker (the Phillips Curve). This means that the marginal benefit of further increasing the output gap has to be positive. As A always is negative and B always is negative, the first order condition in the low rate state can only be positive if $AD - BC$ is negative.

It is found that the equilibrium inflation rate is negative in both the high and the low state while the equilibrium output gap is positive in the high rate state and negative in the low rate state. The intuition behind this is as follows. In the ideal scenario the inflation rate would be zero and the output gap would be zero. However, in the low-rate state the real natural interest rate is negative while the nominal interest rate cannot drop below zero, this causes a reduction in output and inflation in the low-rate state. Furthermore, since the inflation rate also depends on expected future inflation and there always is a risk of the real natural interest rate dropping below zero in the next period, inflation is depressed even in the high-rate scenario. In the high rate state the output gap is positive, since the central bank will boost output (through lowering the nominal interest rate) in order to reduce the deflation induced by future deflation expectations.

The effect of increased central bank conservatism can be shown by adjusting λ. For a λ of 0, which means the central banker does not put any value on the output gap and instead only focuses on inflation, the following inflation rates and output gaps can be found:

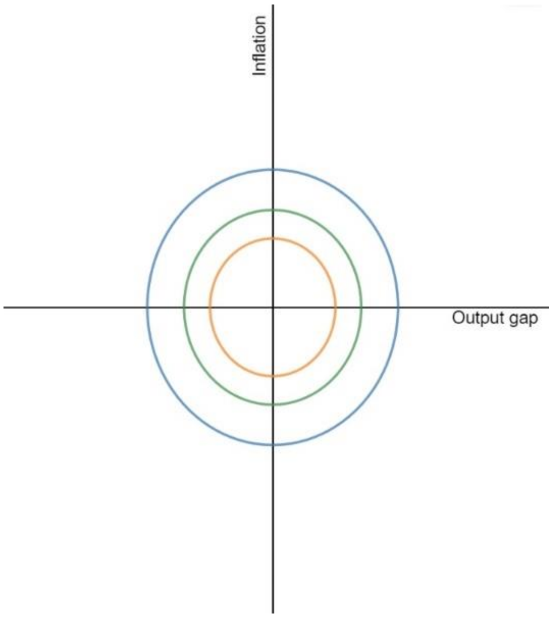
$$\pi_h = 0, \qquad \pi_l = \frac{1}{C} r_l^n, \qquad y_h = \frac{\beta k p_h}{-k^2 C} r_l^n, \qquad y_l = -\frac{1 - \beta p_l}{-kC} r_l^n$$

In this case the inflation rate collapses to zero in the high rate state and increases (while still remaining negative) in the low rate state. Therefore an ultraconservative central banker would lead to a higher inflation rate, as in this context central banker conservatism means putting a lower weight on the output gap target.

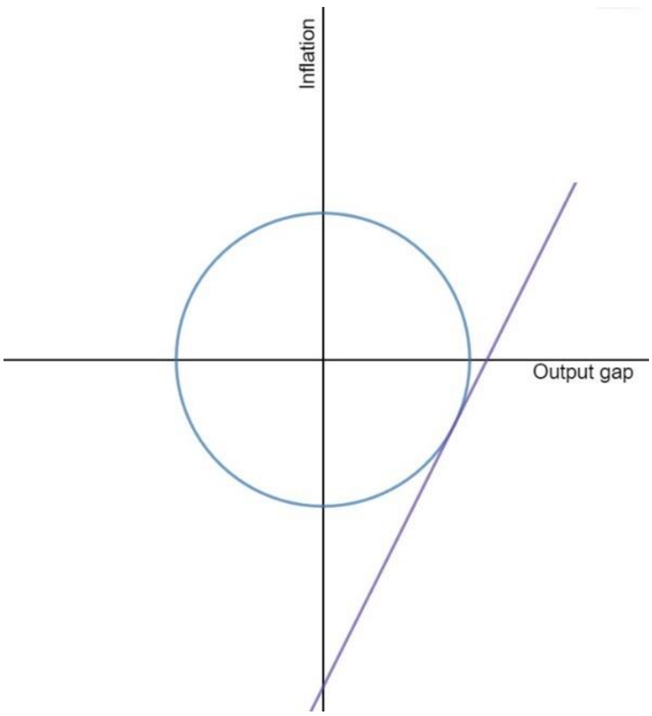
Graphical representation

The model can be illustrated in a graph that includes both the Phillips Curve and the utility function of the government in charge of appointing the central banker. The Phillips Curve captures the effect of the current output gap on the inflation rate. An increase in output causes a shift along the Phillips Curve, a change in expected inflation causes a shift of the Phillips Curve. If the expected level of inflation is zero, the Phillips Curve crosses the origin. A negative value of expected inflation means that the inflation rate is negative even when the output gap is zero, therefore the Phillips Curve crosses the vertical axis at a negative value of inflation.

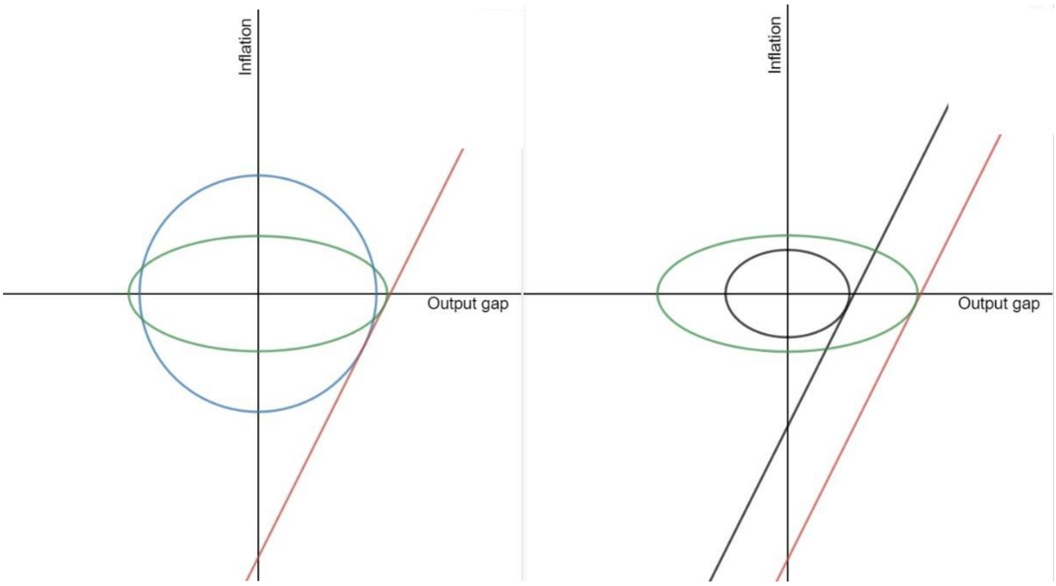
The utility function of the central bank is captured by the indifference curves. The indifference curves represent combinations of inflation and output gap between which the central banker is indifferent. As the targeted inflation rate and the targeted output gap both are zero, the highest possible level of utility is attained at the origin. Thus indifference curves closer to the origin reflect higher levels of utility.



Suppose the economy is in the high natural interest rate scenario (with a natural nominal interest rate higher than zero). The welfare optimum can be attained at the origin, and the origin can be attained if the expected level of future inflation is zero. However, the possibility that the natural real interest rate drops below zero in the next period depresses future inflation expectations. The Phillips Curve intersects with the vertical axis at a negative value of inflation. Therefore the Phillips Curve now is tangent to an inferior indifference curve, meaning that there is a negative inflation rate and a positive output gap in the new equilibrium.



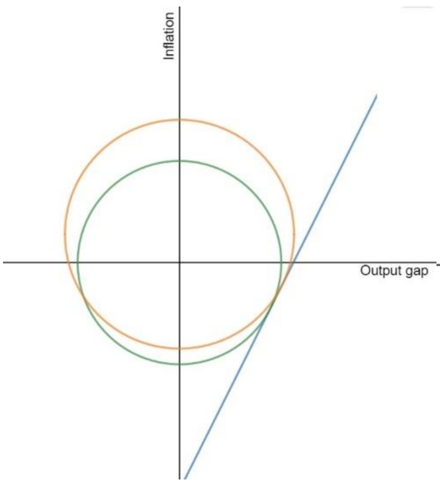
One possible solution to this deflation trap would be to appoint a more conservative central banker. This means appointing a central banker with a lower value of lambda, so that output gap deviations will have a weaker effect on overall utility. In the high natural interest rate scenario this has two implications. First of all, the indifference curves of the central banker become more clustered around low inflation levels. This means that the central banker will require a larger convergence towards zero of the output gap to compensate for a divergence from zero of the inflation rate. Furthermore, appointing a conservative central banker would also increase future expected inflation (assuming the conservative central banker remains in office for the next period). This causes an upwards shift *of* the Phillips Curve. The combination of the less steep indifference curve and a shifted Phillips Curve can reduce deflation without a major increase in the output gap.

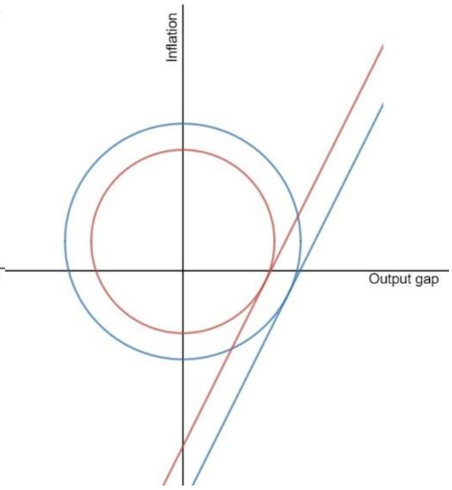


Another solution discussed in Nakata & Schmidt (2019) is to assign the central banker a contract that includes a payment which depends on inflation. The modified utility function would look like:

$$u_{cb}(\pi, y) = -\frac{1}{2}(\pi^2 + \lambda y^2) + f_{ic}\pi_t$$

Adding an inflation-based payment to the contract of the central banker would alter the indifference curve in two different ways. First of all, the indifference curve would shift up. The reason for this is as follows. For a negative level of inflation, the new inflation-based payment reduces utility and therefore staying at the same utility level as before the inflation-based payment would require an increase in inflation and an increase in utility. For a positive level of inflation, the new inflation-based payment increases utility. Since the disutility from inflation is quadratic, inflation has to be increased (and utility decreased) if you want to stay on the same utility level as before the inflation-based payment. Therefore the indifference curve shifts up. Furthermore, the indifference curve also broadens as changes in inflation require larger changes in the output gap in order to stay on the same utility level. Finally, the increase in the inflation rate caused by shifts in the indifference curve also increases expected inflation and therefore shifts the Phillips Curve up. This creates a similar effect as with the conservative central banker.





The model with a non-zero inflation target

In the Nakata-Schmidt (2019) model it is implicitly assumed that the inflation target is zero, as the highest level of utility is obtained at an inflation level of zero and an output gap level of zero. However, in the real world it is more common for the central bank to target a low and stable inflation rate, for example an inflation rate of 2%. This can be included in the model by modifying the utility function:

$$u_{cb}(\pi, y) = -\frac{1}{2}((\pi - \pi^*)^2 + \bar{\lambda}y^2)$$

In this case the highest possible level of welfare is obtained at $\pi = \pi^*$and $y = 0$. To show how this affects monetary policy at the zero-lower bound we first assume that the zero-lower bound constraint does not hold. The following inflation rates and output gaps are derived:

$$\pi_l = \pi_h = \frac{k^2}{\lambda(1-\beta) + k^2}\pi^* > 0$$

$$y_l = y_h = \frac{(1-\beta)k}{\lambda(1-\beta) + k^2}\pi^* > 0$$

If we insert these values in the Euler equation we find that:

$$i_l - \frac{k^2}{\lambda(1-\beta) + k^2}\pi^* - r_l^n = 0$$

Therefore, in order for i_l to drop below zero (and the zero-lower bound constraint to kick in), $r_l^n < -\frac{k^2}{\lambda(1-\beta)+k^2}\pi^*$ has to hold. This means that in order to recreate the same zero-lower bound constraint as with a zero inflation target, the crisis state real natural interest rate in case of a positive inflation target has to be significantly lower than the crisis state real natural interest rate in case of a zero inflation target. This result is consistent with prior findings such as Williams (2009), who argued that circumventing the zero lower bound constraint was an argument for adopting higher inflation targets.

However, there may be a trade-off from adopting a non-zero inflation target. It should be noticed that in the situation with no zero lower bound constraint on nominal interest rates, adopting a positive inflation target leads to a positive output gap. In the absence of supply or demand shocks, it is not possible to have a positive inflation target while also balancing the output gap at zero. Therefore adopting a higher inflation target can lead to a suboptimal outcome if the risk of entering the zero lower bound state is very small. The reason for this can be found by rewriting the New Keynesian Phillips Curve:

$$y_t = \frac{1}{k}(\pi_t - \beta E_t \pi_{t+1})$$

The output gap is positive if current inflation is higher than $\beta E_t \pi_{t+1}$. An increase in the targeted inflation rate increases both current inflation and expected future inflation, but expected future inflation only impacts output by a factor of β. Therefore, for $\beta < 1$, a higher inflation target is associated with a higher level of output, since the inflation target would increase π_t by more than $\beta E_t \pi_{t+1}$ in that case.

β reflects the extent to which expected future inflation causes inflation today, since firms take into account the possibility that it will not be possible for them to adjust prices in the future because of pricing frictions. However, since there also is a possibility firms will be able to adjust future prices, expected future inflation does not result in a one-for-one increase in current inflation. A positive inflation target leads to positive expected future inflation. This in turn also increases current inflation, but not one-for-one. If inflation in the current period is lower than expected inflation in the future, costs in the current period will be lower as well, which means firms will find it attractive to boost output in the current period. This increases inflation in the current period some more, until it reaches the same level as expected future inflation.

A similar phenomenon was described by Alves (2014), who argued that the trade-off between stabilizing output and stabilizing inflation only disappears in the New Keynesian model if the inflation target is zero, while for any other inflation target the trade-off reappears.

Zero-lower bound on inflation expectations

The core mechanism behind deflation bias relies on expected inflation dropping below zero, but whether inflation expectations actually can turn negative is subject to debate. Gorodnichenko & Sergeyev (2021) study an American survey dataset of household inflation expectations. They find that the distribution of inflation expectations is very asymmetric, with inflation expectations never dropping below 0%. Since the United States rarely experienced deflation, they also studied a Japanese dataset and found that even Japanese consumers consistently underestimated the possibility of deflation. Furthermore, Gorodnichenko & Sergeyev (2021) find that reductions in the actual inflation rate cause substantially smaller changes in the expected inflation rate of households who already expected low inflation than in the expected inflation rate of households who expected higher levels of inflation, which is consistent with the idea of a zero-lower bound on inflation expectations.

What would this mean in the context of the Nakata-Schmidt model? The key mechanism in the model is that deflation expectations shift the Phillips Curve down and force the central bank into an equilibrium with deflation and a positive output gap in the high natural rate state. However, if inflation expectations are not able to drop below zero and the targeted inflation rate is zero, deflation in the low natural rate state will not impact inflation in the high natural rate state. However, if the targeted inflation rate of the central bank is higher than zero, it is possible that the same 'deflation bias' (this time meaning inflation below target level rather than deflation) takes hold, as there is nothing to prevent inflation expectations from dropping below target inflation (as long as target inflation is sufficiently above zero).

This can also be shown mathematically. Recall that in Nakata & Schmidt (2019) $\pi_h \leq 0$ and $\pi_l < 0$. Therefore the expected future inflation rate always is negative, both in the high-rate state and the low-rate state. However, if we use the first order conditions and constraints used by Nakata & Schmidt (2010) and impose $\pi_{t+1}^e = 0$, we obtain:

$$y_h = \big((1 - p_h)y_h + p_h y_l\big) + \sigma(r_h^n - i_h)$$

$$y_l = \big((1 - p_l)y_h + p_l y_l\big) + \sigma(r_l^n - 0)$$

$$\pi_h = k y_h$$

$$\pi_l = k y_l$$

$$0 = -k\pi_h - \lambda y_h$$

$$0 < -k\pi_l - \lambda y_l$$

From these equations we can deduce that π_h and y_h both equal zero. Furthermore, we can also establish that that π_l and y_l still are negative, as the zero-lower bound constraint prevents the central bank from attaining the optimal equilibrium. However, since the expected inflation rate is zero regardless of the actual equilibrium values for inflation, the link between both states is broken and the deflation in the low-rate state does not affect inflation in the high-rate state. The central banker also does not face any incentive to introduce surprise inflation as in a Barro-Gordon model. After all the central banker has reached its optimal point in the high-rate state and the central banker cannot further increase inflation in the low-rate state because of the zero lower bound. There also is no incentive to increase inflation in the high-rate state as there is no expectations mechanism that connects the high-rate inflation rate and the low-rate inflation rate.

It can also be shown that the zero-lower bound on inflation expectations does not affect equilibrium inflation rates and output gaps if the central bank works with a sufficiently high inflation target. Suppose the utility function of the central banker is rewritten to:

$$u_{cb}(\pi, y) = -\frac{1}{2}((\pi - \pi^*)^2 + \lambda y^2)$$

Where π^* reflects the target inflation rate. In this case, the first order conditions become:

$$0 = -k\pi_h + k\pi^* - \lambda y_h$$

$$0 < -k\pi_l + k\pi^* - \lambda y_l$$

The other conditions stay the same.

$$\pi_l = \frac{D}{AD - BC}k^2\pi^* - \frac{B}{AD - BC}r_l^n$$

$$\pi_h = -\frac{C}{AD - BC}k^2\pi^* + \frac{A}{AD - BC}r_l^n$$

Assuming the original conditions still hold, having a positive inflation target increases the inflation rate (since $-\frac{C}{AD-BC} > 0$ and $\frac{D}{AD-BC} > 0$). For a sufficiently high value of π^*, expected future inflation will be positive in both states. In that case the zero lower bound on inflation expectations simply does not influence the expected inflation rate. Finally, previously it also was established that a higher inflation target reduces the likelihood of the zero lower bound constraint being relevant, as the crisis state real natural interest rate has to become even more negative.

Central banker conservatism and asymmetric utility functions

A possible criticism of the definition of central banker conservatism used in Nakata & Schmidt (2019) is that it assumes conservative central bankers have a symmetric utility function, where overshooting the inflation target by a certain amount and undershooting the inflation target by the same amount both reduce utility by the same amount, but in reality it can be argued that many conservative central bankers have an asymmetric utility function and prefer undershooting their inflation target by a certain amount over overshooting their inflation target by the same amount. Whereas in Nakata & Schmidt (2019) a central banker is considered conservative if the central banker puts a low value on the output gap target, in reality so-called conservative central bankers might simply have a preference for low inflation. In order to reflect this, the utility function of the central banker can be changed to:

$$u(\pi, y) = -\frac{1}{2}((\pi_t - \pi^*)^2 + \lambda(\pi_t - \pi^*) + y_t^2)$$

With a λ of zero, the disutility of undershooting the inflation target by a certain amount equals the disutility of overshooting the inflation target by the same amount (assuming the output gap is the same in both cases). However, if lambda has a positive value, the central banker will prefer an inflation rate which is below the target inflation rate over an inflation rate that exceeds the target inflation rate by the same amount. Therefore, in this scenario a conservative central banker is defined as a central banker who prefers a lower rate of inflation rather than a central banker that prioritizes the inflation goal over the output gap goal. This could be a more accurate representation of the preferences of conservative central bankers.

$$\pi_l = \frac{D}{AD - BC}(\pi^* - \frac{1}{2}\lambda) - \frac{B}{AD - BC} r_l^n, \qquad \pi_h = -\frac{C}{AD - BC}(\pi^* - \frac{1}{2}\lambda) + \frac{A}{AD - BC} r_l^n$$

$$A = -\frac{\beta p_h}{k^2} < 0, \qquad B = 1 + \frac{1 + k - \beta(1 - p_h)}{k^2} > 0, \qquad C = \frac{1 - p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - p_l > 0,$$

$$D = -\frac{1 - p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - (1 - p_l) < 0$$

$$AD - BC < 0$$

As expected, a higher value of lambda (a central banker who prefers undershooting the inflation target over overshooting the inflation target) causes lower levels of inflation. Therefore, if central bank conservatism is defined as aversion to inflation (as it often is in the media), then a more conservative central banker obviously leads to lower inflation rates rather than higher inflation rates, even in a zero lower bound setting.

Walsh (2002)

General overview

Walsh (2002) studies the effectiveness of a dismissal rule in reducing inflation bias caused by the central bankers' inability to credibly commit to a low inflation rate. Therefore the main topic of interest of Walsh (2002) differs from the main topic of interest of Nakata & Schmidt (2019), however the mechanism studied in Walsh (2002) could also be relevant for the problem of deflation bias as studied by Nakata & Schmidt (2019).

The inflation problem studied in Walsh (2002) is very similar to the problem studied in Barro & Gordon (1983). There are 3 equilibria in the Walsh (2002) model. The commitment equilibrium in which the central bank promises a certain inflation rate and the private sector expects the central bank to honour its promise (in this case the inflation rate in the commitment equilibrium is positive since the government has a positive inflation target), the reneging equilibrium in which the private sector expects the central bank to honour its inflation promise but the government introduces surprise inflation in order to increase output (real wages are set based on expected inflation, so if inflation is higher than expected inflation, real wages decline and firms have an incentive to increase production), and the time consistent equilibrium in which the private sector anticipates the surprise inflation and sets the expected inflation rate at a rate at which the central bank does not have the incentive to introduce further inflation.

The key problem in Walsh (2002) is this inability of the central bank to commit to the commitment equilibrium. Because of the central banker's output target, the central banker will always have the incentive to introduce surprise inflation (to lower real wages and increase output) if the public expects the central banker will keep inflation equal to target inflation. Because the private sector is rational, the private sector incorporates the ‘surprise’ inflation in its wage demands and the extra inflation does not actually increase output. Walsh (2002) proposes to adopt a dismissal rule under which the central banker is fired if inflation exceeds a certain threshold value. By doing this, Walsh (2002) incentivizes the central bank to reduce the reneging and time consistent inflation rates, which improves welfare.

Just as the conservative central banker solution, the dismissal rule does not actually directly address the time inconsistency problem. Even with a dismissal rule (or conservative central banker), the central banker still is unable to reach the commitment equilibrium. These solutions therefore are designed to reduce the reneging & time-consistent inflation rates instead of making it more likely that the central banker is able to maintain the commitment equilibrium.

Walsh (2002) found that the dismissal contract could reduce inflation bias much like appointing a conservative central banker or assigning an inflation-based payment could, but unlike the conservative central banker, the dismissal contract allows for flexibility in case of output shocks, while potentially being more practically feasible than the inflation-based payment.

In the Walsh (2002) model inflation is determined by the following equation:

$$\pi_t = \pi_t^e + ay_t + e_t$$

Where π_t denotes the inflation rate between period t-1 and period t, y_t denotes the output gap and e_t denotes an supply shock. The difference between this equation and the Phillips Curve in Nakata & Schmidt (2019) is that in this equation the inflation rate is partially determined by expected inflation in the current period rather than expected future inflation. Therefore this Phillips Curve is more akin to the version of the Phillips Curve in Barro & Gordon (1983) than the New Keynesian Phillips Curve in Nakata & Schmidt (2019). A possible justification for including expected inflation in the current period in the model is that contracts often are set based on expected inflation. If inflation in the current period is higher than the level of inflation that was expected in the current period, this represents a decline in labour costs (as wages don’t immediately adjust) and an increase in output. In absence of a supply shock, the inflation rate equals the expected inflation rate if the output gap is zero.

The output gap is determined by the following equation:

$$y_t = -b(i_t - \pi^e_{t+1}) + u_t$$

Where y_t denotes the output gap, i_t denotes the nominal interest rate between period t and period t+1, $E_t\pi_{t+1}$ denotes the expected inflation rate between period t and period t+1 and u_t denotes the demand shock. Therefore $i_t - \pi^e_{t+1}$ represents the expected real interest rate between t and t+1. In this equation there is a negative relationship between the output gap and the real interest rate. In the absence of a demand shock, a positive expected real interest rate causes a negative output gap while a negative real interest rate causes a positive output gap. In the absence of a demand shock, the output gap will equal zero when the nominal interest rate equals the expected inflation rate in the future period (when the expected real interest rate equals zero). There are a few differences between the output function in the Walsh model and the output function in the Nakata-Schmidt model. First of all, there is no explicit natural interest rate in the Walsh model, though it could be argued that in the Walsh model the real natural interest rate is implicitly assumed to be zero as the output gap is zero for $i_t - \pi^e_{t+1}$ in the absence of any output shocks. Furthermore, there is no explicit intertemporal link between output gaps in the Walsh model. In the Nakata-Schmidt model the output gap is stable over time if the real interest rate equals the natural real interest rate, while in the Walsh model a real interest rate equal to zero (the implied natural real interest rate) is consistent with an output gap value of zero (in the absence of an output gap shock) rather than an output gap which is stable over time. Therefore the effect of interest rates on output gaps in the Walsh model is better interpreted as going through the effect of the interest rate on the equilibrium in the goods market and money market than through intertemporal consumption shifting.

Furthermore, both the output equation and the Phillips Curve include a variable denoting shocks (u_t for demand shocks and e_t for supply shocks). It is assumed that the central bank can only witness the true value of the shock at the end of the period and makes policy on the basis of a forecast (u^f_t and e^f_t).

The disutility function belonging to the central bank is defined as:

$$U = -\lambda_1 y_t + \frac{1}{2}\lambda_2 y_t^2 + \frac{1}{2}(\pi_t - \pi^*)^2$$

Deviations from the target inflation rate (π^*) cause reductions in welfare, as do deviations from a zero output gap. Since positive output gaps typically do not provide as much disutility as similarly-sized negative output gaps do, the first term was added to ensure the central banker would target a positive output gap. Another justification for this is that regulations and distortionary taxes typically drive the natural output level below the socially optimal output level, which incentivizes the central bank to target a positive output gap. An alternative way of modelling this would be to model the output gap similarly as inflation and have a positive output gap target.

One of the first things noted by Walsh (2002) is that a dismissal rule based solely on inflation cannot be optimal, since the optimal inflation rate depends on the size of the supply shock, and the presence of shocks are the reason a dismissal rule would be considered in the first place. As mentioned in the literature overview, one of the main problems posed by the conservative central banker solution to inflation bias is that appointing a conservative central banker means the central banker does not allow inflation to increase by enough during a supply shock and instead would sacrifice the output target for the sake of keeping inflation low. Therefore an optimal dismissal rule that avoids the pitfall of a conservative central banker would have to include other factors as well as the inflation rate. However, as both supply shocks and demand shocks are unobservable until the end of the period, u_t and e_t cannot be included in the equation. Furthermore, it is assumed that u_t^f and e_t^f are only observed by the central bank and not by the government. As an alternative, Walsh (2002) proposed to dismiss the central banker if:

$$\pi_t > \beta_0 - \beta_1 y_t$$

In absence of an output gap the central banker will be fired if the inflation rate exceeds threshold value β_0. However, in case of a negative (positive) output gap, the threshold inflation rate is higher (lower) than β_0. This ensures that the central banker will not automatically be fired in case of an unanticipated supply shock, as reducing inflation by reducing output will also increase the inflation rate at which the central banker is dismissed. Therefore the central banker does not have to counteract the full supply shock in order to be reappointed.

The main function of the demand shock in the Walsh (2002) model is to make it impossible for the government to infer the value of the supply shock and to force the government to find a different way to adjust the dismissal rule for shocks. The main topic of interest in the Walsh (2002) paper is the response of a central bank to anticipated *supply* shocks. The reason for this is that an anticipated demand shock can be 'sterilized' by the central bank since demand shocks move the output gap and inflation in the same direction. However, supply shocks force the central banker to pick between the inflation target and the output target, as supply shocks increase inflation without changing output. This can cause a divergence of preferences when the central banker is more conservative than the government, as a more conservative central banker will let inflation increase by less than the government would want (Rogoff, 1985). Therefore the supply shock poses a greater threat to the conservative central banker solution as proposed by Rogoff (1985).

By inserting the Phillips Curve into the dismissal rule and rearranging, we obtain:

$$0 > \beta_0 - \pi_t^e - \beta_1 y_t - a y_t - e_t$$

The probability of being reappointed equals the probability that the preceding equation holds. An increase in the output gap reduces the probability of reappointment in two ways, as it increases the actual inflation rate while reducing the threshold inflation rate at which the central banker will be fired.

Another challenge faced by Walsh (2002) was to determine how to deal with reappointment and the utility of reappointment. Since one of Walsh's assumptions was that the central banker will be replaced by an identical central banker, dismissal rules will only affect the decision of the central banker if the central banker directly attaches value to being in office, as the output gap and inflation levels would be the same under the replacement central banker. Since the central bank can only set inflation and the output gap on the current date, the central bank's decision problem can be broken down in a series of one-period problems in which the central bank minimizes:

$$U = -\lambda_1 y_t + \frac{1}{2}\lambda_2 y_t^2 + \frac{1}{2}(\pi_t - \pi^S)^2 - \theta_t V_t$$

$$\theta_t = F(\beta_0 - \pi_t^e - \beta_1 y_t - a y_t - e_t)$$

Furthermore, it should be noted that Walsh (2002) implicitly assumes that $e_{t+1}^f = 0$ and $E_t e_{t+1} = e_{t+1}^f$. The second assumption simply states that the expected future supply shock must equal the forecasted future supply shock. This assumption is not made in period t since the supply shock that matters is the supply shock that is revealed in period t and it is possible that the supply shock revealed at the end of period t differs from the forecasted supply shock, but the supply shock in period t+1 is not revealed in period t and therefore there is no reason why the expected value of this supply shock should differ from the forecasted supply shock. The first assumption simply states that the forecasted supply shock in period t+1 is zero. This is done to simplify the model. A possible justification for this is that there is not enough information in the present to already forecast supply shocks that are sufficiently far removed from the current period.

Walsh (2002) finds that the time-consistent inflation rate and output gap *without* dismissal contract are:

$$\pi_t = \pi^S + \frac{\lambda_1}{a} + \frac{\lambda_2}{\lambda_2 + a^2} e_t^f + a\left(u_t - u_t^f\right) + (e_t - e_t^f)$$

$$y_t = -\frac{a}{\lambda_2 + a^2} e_t^f + (u_t - u_t^f)$$

As π^S is the target inflation rate, $\frac{\lambda_2}{\lambda_2 + a^2} e_t^f$ shows the response to the forecasted supply shock and $a\left(u_t - u_t^f\right) + (e_t - e_t^f)$ show the effects of the unforecasted supply & demand shocks on inflation, $\frac{\lambda_1}{a}$ is the inflation bias. Therefore, eliminating $\frac{\lambda_1}{a}$ is the objective. A first thing to note is that the inflation bias in Walsh (2002) isn't affected by λ_2, the variable which measures conventional central bank conservatism (the weight the central banker puts on the output gap target). Instead, inflation bias is the result of λ_1, which measures the central banker's bias towards a positive output gap. In theory inflation bias could be eliminated by setting λ_1 to zero, and this would not impact the way the central bank responds to forecasted supply shocks (which is influenced by λ_2). Therefore, by picking a central banker with $\lambda_1 = 0$ and λ_2 at the same value as the government, it is possible to eliminate inflation bias while still maintaining a central banker who responds to unanticipated supply shocks the way the government wants the central banker to respond. Therefore, the trade-off between minimizing inflation bias and aligning the central banker's response to a forecasted supply shock with the government's preferred response to a forecasted supply shock disappears. However, this likely is the result of a (small) flaw in the model. If the disutility function were rewritten to:

$$U = \frac{1}{2}\lambda_1(y_t - y^*)^2 + \frac{1}{2}(\pi_t - \pi^S)^2$$

It is possible to capture the fact that a central banker will prefer a small positive output gap over a small negative output gap while also maintaining the usual trade-off between minimizing inflation bias and having a central banker who aligns with the government when it comes to the optimal way to respond to supply shocks. Effectively, in the Walsh model it is possible to have a central banker who does not care about the output gap but still is biased towards a positive output gap ($\lambda_1 > 0, \lambda_2 = 0$). This does not make a lot of sense, but a small change in the utility function could fix this.

Finally, when including the dismissal contract in the central banker's utility function, Walsh (2002) finds that:

$$\pi_t = \pi^S + \frac{\lambda_1}{a} + \frac{\lambda_2}{\lambda_2 + a^2} e_t^f + \frac{1}{a} E_t \frac{\delta\theta_t}{\delta y_t} V_t + a\left(u_t - u_t^f\right) + \left(e_t - e_t^f\right)$$

$$y_t = \frac{a\left(\frac{\delta\theta_t}{\delta y_t} V_t - E_t \frac{\delta\theta_t}{\delta y_t} V_t\right) - a e_t^f}{\lambda_2 + a^2}$$

Therefore, the same inflation rate as in the commitment equilibrium can be attained if:

$$\frac{1}{a} E_t \frac{\delta\theta_t}{\delta y_t} V_t = -\frac{\lambda_1}{a}$$

First of all, it should be noted that $\frac{\delta\theta_t}{\delta y_t}$ always is negative, since an increase in output reduces the probability of being reappointed. The extent to which an increase in output reduces the probability of being reappointed depends on β_1, a higher value of β_1 increases the extent to which an increase in output reduces the probability of reappointment. Therefore, a higher bias towards a positive output gap (λ_1) requires the government to set a higher value of β_1 in return.

The zero-lower bound in the Walsh (2002) model

The zero-lower bound is not present in the Walsh (2002) model. This means that the nominal interest rate can be adjusted to accommodate any combination of inflation and the output gap that is consistent with the Phillips Curve. However, it is possible to determine the parameter values at which the inflation rate chosen by the central banker would require a negative nominal interest rate. First of all, we adjust the output function in order to include the natural real interest rate. This is so we can study the effects of a changing natural real interest rate, as Summers (2015) hypothesized the declining equilibrium real interest rate was a major factor in the rising relevance of the zero lower bound. Furthermore, including the natural real interest rate could reflect the fact that investment demand and the supply of savings are matched at a positive real interest rate, because the time value of money is positive. This leads to the following output equation:

$$y_t = -b(i_t - r_t^n - \pi_{t+1}^e) + u_t$$

Following the steps of Walsh (2002) leads to the following equation for the nominal interest rate:

$$i_t = \frac{1}{b}\left(\frac{a}{\lambda_2 - a^2}\right)e_t^f + r_t^n + \pi^S + \frac{\lambda_1}{a}$$

Furthermore, it can be found that the nominal interest rate is negative when:

$$-\frac{1}{b}\left(\frac{a}{\lambda_2 - a^2}\right)e_t^f - \frac{\lambda_1}{a} - \pi^S > r_t^n$$

Since the inflation bias $\frac{\lambda_1}{a}$ is positive and the socially optimal inflation rate π^S likely is positive, a negative real natural interest rate or a sizable negative forecasted supply shock is necessary to drive the nominal interest rate negative. In the Walsh model there is no zero lower bound constraint, however if such a constraint were present we could model it in the same way as Nakata & Schmidt (2019) do. First of all, as in Nakata & Schmidt (2019), we assume that there are two states of the economy: one state with a positive real natural interest rate and one state with a negative real natural interest rate which is sufficiently low to trigger the zero lower bound constraint on the nominal interest rate. The probabilities of being in each state in the future period depend on the current state:

$$Prob(r_{t+1}^n = r_l^n | r_t^n = r_h^n) = p_h$$

$$Prob(r_{t+1}^n = r_l^n | r_t^n = r_l^n) = p_l$$

For the low rate state, the solution for inflation is found by setting i_t at zero, inserting $\pi_{t+1}^e = p_l\pi_l + (1 - p_l)\pi_h$, inserting the solution for π_h found in the high rate state and making use of the Phillips Curve equation to eliminate the output gap from the equation. For the high rate state the solution for inflation is found by minimizing the utility function with respect to inflation and the output gap subject the Phillips Curve as budget constraint. Rational expectations are assumed to hold. For the sake of simplicity, forecasted shocks are ignored.

$$\pi_l = -\frac{(1 - p_l)}{p_l}\left(\frac{1}{a}\lambda_1 + \pi^S\right) - \frac{1}{p_l}r_l^n + au_t + e_t$$

$$\pi_h = \frac{1}{a}\lambda_1 + \pi^S + au_t + e_t$$

$$y_h = u_t,\;\; y_l = u_t$$

First of all, in the Walsh (2002) model with zero lower bound constraint, the presence of the zero lower bound does not affect output or inflation in the high-rate state at all. This is because expected future inflation (and thus inflation in the low rate state) does not feature in the version of the Phillips Curve used in Walsh (2002). As long as expectations are rational, the (forecasted) output gap will be zero because the eventual inflation rate will be anticipated by the private sector and wages will be set accordingly.

In the low rate state, a decrease in the real natural interest rate actually *increases* inflation. This is in sharp contrast to the Nakata & Schmidt (2019) model where a decrease in the real natural interest rate decreases inflation in both the high rate state and the low rate state. This also harkens back to the differences between the Phillips Curves used in both papers. In the Walsh (2002) paper, the expectations-augmented Phillips Curve is used. In this variant of the Phillips Curve, in absence of a supply or demand shock, the output gap can only be positive if inflation is higher than expected inflation. However, this will not occur if expectations are rational and the time consistent equilibrium is reached. Therefore the time consistent output gap cannot deviate from zero unless there is a demand shock. Thus, if we ignore demand shocks, insert $i_t = 0$ (since the ZLB constraint holds) and insert $y_l = 0$, we obtain:

$$\pi_{t+1}^e = -r_t^n$$

This equation essentially is the so-called Friedman Rule, which states that the central bank should pursue a deflation rate which balances out the real natural interest rate such that the nominal interest rate is zero. The reason for this is that the social cost of creating extra money is virtually zero, so therefore the opportunity cost of holding that money (the nominal interest rate) should be zero as well (Friedman, 1969).

The economic intuition behind this effect can be found in the dynamics of money demand. Suppose the real natural interest rate is negative while the expected inflation rate is zero (the future price level is expected to be the same as the current price level). In this case, holding money becomes attractive relative to investing in real assets. Money demand will increase, which pushes down the current price level relative to the future expected price level. This process continues until the expected inflation rate (the change in the price level between periods) cancels out the negative real interest rate and investing in real assets is as attractive as holding money. If the real natural interest rate is assumed to be negative forever, the inflation rate will also be positive forever as the price level in a period will always be lower than the expected price level in the next period.

This exercise clearly shows that expected future inflation is the mechanism through which the zero lower bound constraint can affect output and inflation even in states of the economy in which the zero-lower bound is not binding.

Glasner (2018) studies a similar equation:

$$i_t - \pi_{t+1}^e = r_t$$

Assuming the nominal interest rate drops to zero and the zero lower bound constraint holds, we arrive at a very similar equation. However, Glasner (2018) assumes that inflation expectations are exogenous while the real interest rate can adjust. Therefore, a decrease in inflation expectations has to trigger an increase in the real interest rate. This is achieved through a decrease in the value of real assets in the current period relative to the value of real assets in the future. Therefore, Glasner (2018) argues that this dynamic of a binding zero-lower bound constraint on nominal interest rate, declining inflation expectations and a sharp decline in asset values after the financial crisis can be explained through this effect.

The model

Set-up

In many respects, the model outlined in this section of the paper follows the template of Nakata & Schmidt (2019). The equilibrium values of inflation and the output gap are determined by the New Keynesian Phillips Curve and the Euler equation:

$$\pi_t = ky_t + \beta E_t \pi_{t+1} + e_t$$

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) + u_t$$

While the expected future inflation rate features in the model, it is later shown that this rate equals the weighted average of high-state inflation and low-state inflation, which are variables determined within the model. The values for the supply shock and demand shock are revealed at the end of the period, thus the central banker cannot take these values into account when making a decision on inflation and the output gap. Making the Phillips Curve stochastic is important as an important part of the central banker utility function is the utility of reappointment. If inflation were fully deterministic, the probability of reappointment would be either 0 or 1, which would complicate the optimization problem. To simplify the initial analysis, it is assumed that the forecasted supply shock and forecasted demand shock both are zero.

Again, the natural real interest rate is determined by a two-state Markov process:

$$Prob(r_{t+1}^n = r_l^n | r_t^n = r_h^n) = p_h$$

$$Prob(r_{t+1}^n = r_l^n | r_t^n = r_l^n) = p_l$$

Therefore, expected future inflation is:

$$E_h \pi_{t+1} = (1 - p_h)\pi_h + p_h \pi_l$$

$$E_l \pi_{t+1} = (1 - p_l)\pi_h + p_l \pi_l$$

The main deviation from the Nakata-Schmidt model comes in the form of the central banker utility function, as the central banker has to consider reappointment and the utility of being in office:

$$u(\pi, y) = -\frac{1}{2}\left(\pi_h^2 + \lambda y_h^2\right) + \theta_h V_h$$

$$u(\pi, y) = -\frac{1}{2}\left(\pi_l^2 + \lambda y_l^2\right) + \theta_l V_l$$

Where θ represents the probability of being reappointed and V represents the utility value of being in office the next period.

It is assumed that the central banker is altruistic and even derives disutility from inflation and the output gap while not in office, and that the central banker would be replaced by another central banker with identical preferences. Therefore the only thing that the central banker has to consider for reappointment is the utility of being in office the next period. Finally, it is assumed that there is no limit on the amount of times the central banker can be reappointed. Since the central banker cannot commit to future inflation rates and will always select the same inflation rate in the high-rate scenario, this problem can be modelled as a series of one-period decisions.

The central banker will be reappointed if:

$$\pi_t > \beta_0 - \beta_1 y_t$$

This can be rewritten as:

$$(k + \beta_1)y_t + \beta E_t \pi_{t+1} + e_t - \beta_0 > 0$$

$$\theta_t = F((k + \beta_1)y_t + \beta E_t \pi_{t+1} + e_t - \beta_0)$$

Inflation has to exceed a target value, as we wish to incentivize central bankers to increase inflation rates in the high rate period. Since it is assumed the government cannot observe the true value of the supply shock or demand shock, the target inflation rate cannot be adjusted for these shocks. The output gap is used instead to adjust the required inflation rate. A higher (positive) output gap increases the inflation rate and reduces the minimum inflation rate for reappointment, therefore a higher output gap increases the likelihood of being reappointed. The goal of this is to incentivize central bankers to adopt a more expansionary stance in the high rate state. It should be noted that $\frac{\delta\theta_t}{\delta y_t} > 0$ and $\frac{\frac{\delta\theta_t}{\delta y_t}}{\delta\beta_1} > 0$.

Since this model is a discrete time model and the central banker is either dismissed or reappointed at fixed time intervals, the dismissal rule can be seen as a reappointment rule and the length of any given period can be equated to the length of the standard central banker term. However, in that case the term length of a central banker would coincide with the frequency of the inflation decision. This does not make sense as in reality the term of a central banker lasts for several years while the central bank makes a decision on interest rates every few months.

Another possibility is to interpret it as a dismissal rule rather than a reappointment rule. However, this raises some accountability issues. While it might be possible to specify a clear rule based on easily observable variables, it still hands the government a powerful tool to influence monetary policy. If it is too simple to change the dismissal rule, the government could manipulate this rule to potentially dismiss the central banker at will. This could harm central bank independence if this rule is not properly laid out.

Results

The following first-order conditions were found:

$$0 = -k\pi_h - \lambda y_h + E_t \frac{\delta\theta_h}{\delta y_h} V_h$$

$$0 > -k\pi_l - \lambda y_l + E_t \frac{\delta\theta_l}{\delta y_l} V_l$$

As before, the fact that the first order condition for inflation and the output gap in the low natural rate state is positive reflects that output is suboptimally low and welfare could be increased by further increasing the output gap.

By making use of the first order conditions, the output function and the Phillips Curve, the following inflation rates and output gaps are found:

$$\pi_l = \frac{D}{AD - BC}\left(kE_t \frac{\delta\theta_h}{\delta y_h} V_h\right) - \frac{B}{AD - BC} r_l^n + e_t + ku_t$$

$$\pi_h = -\frac{C}{AD - BC}\left(kE_t \frac{\delta\theta_h}{\delta y_h} V_h\right) + \frac{A}{AD - BC} r_l^n + e_t + ku_t$$

$$y_l = \frac{\beta p_l - 1 + (\beta - 1)C}{AD - BC} E_t \frac{\delta\theta_h}{\delta y_h} V_h - \frac{1}{k} \frac{(1 - \beta p_l)k^2 + (1 - \beta)(1 + \beta p_h - \beta p_l)\lambda}{AD - BC} r_l^n + u_t$$

$$y_h = -\left(\frac{(1 - \beta)C - \beta p_h}{AD - BC}\right)\left(E_t \frac{\delta\theta_h}{\delta y_h} V_h\right) + \left(\frac{\beta p_h k}{AD - BC}\right) r_l^n + u_t$$

$$A = -\lambda\beta p_h < 0, \qquad B = \lambda + k^2 - \lambda\beta(1 - p_h) > 0, \qquad C = \frac{1 - p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - p_l > 0,$$

$$D = -\frac{1 - p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - (1 - p_l) < 0, \qquad AD - BC < 0$$

First, we assume that $AD - BC < 0$, as this originally was a necessary condition for equilibrium. As D is negative and C is positive, an increase in the value or the likelihood of reappointment leads to an increase in inflation in both the high natural interest rate state and the low natural interest rate state. This result was expected, the presence of the dismissal rule in the high natural rate state incentivizes the central bank to adopt a more inflationary policy in the high state. Through the expectations channel this increases inflation in the low state as well. Therefore $\frac{\partial \pi_h}{\partial (E_t \frac{\partial \theta_h}{\partial y_h} V_h)} > 0$ and $\frac{\partial \pi_h}{\partial (E_t \frac{\partial \theta_h}{\partial y_h} V_h)} > 0$.

Furthermore, since A is negative while B is positive, an increase in the low rate state natural real interest rate would cause an increase in both high rate state and low rate state inflation. Again, this result was expected since it reduces the gap between the actual nominal interest rate and the ‘equilibrium’ nominal interest rate.

The effects of the dismissal rule on the output gap are more ambiguous. We obtain the following expressions:

$$\frac{\partial y_h}{\partial(E_t \frac{\partial \theta_h}{\partial y_h} V_h)} = -\left(\frac{(1-\beta)C - \beta p_h}{AD - BC}\right)$$

$$\frac{\partial y_l}{\partial(E_t \frac{\partial \theta_h}{\partial y_h} V_h)} = \frac{\beta p_l - 1 + (\beta - 1)C}{AD - BC}$$

$\beta p_l - 1 + (\beta - 1)C$ and $(1-\beta)C - \beta p_h$ may be either positive or negative depending on the values of the parameters. On the one hand, the dismissal rule incentivizes the central banker to increase inflation by overshooting the output gap target. On the other hand, higher inflation increases expected future inflation levels, which increases the inflation level for any given output gap, and therefore reduces the necessity of an increase in the output gap to increase inflation. Graphically speaking there is both a shift along the Phillips Curve and a shift of the Phillips Curve. Which of these two effects dominates depends on the parameter values. It can be shown that the presence of the dismissal contract increases output in the high rate state if:

$$\frac{1-p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - p_l - \frac{\beta}{1-\beta} p_h > 0$$

$$\frac{\delta\left(\frac{1-p_l}{k\sigma}(1 - \beta p_l + \beta p_h) - p_l - \frac{\beta}{1-\beta} p_h\right)}{\delta\beta} = \frac{(1-p_l)}{k\sigma}(p_h - p_l) - \frac{p_h}{1-\beta} - \frac{p_h \beta}{(1-\beta)^2}$$

If we assume that $p_l > p_h$, as the likelihood of being in the negative real natural interest state the next period likely is higher if the economy already is in the negative real natural interest state in the current period, an increase in β reduces the likelihood that the presence of the dismissal contract increases output in the high rate state. β reflects the extent to which future inflation expectations increase inflation today. A higher value of β means that an increase in the expected future inflation rate leads to a bigger increase in the inflation rate today. Since the dismissal contract shifts the Phillips Curve through increasing inflation expectations, a higher value of β means that the dismissal contract leads to a bigger shift *of* the Phillips Curve.

In the low-rate state, the presence of the dismissal contract increases output ($\frac{\partial y_l}{\partial(E_t \frac{\partial \theta_h}{\partial y_h} V_h)} > 0$) if:

$$\beta p_l - 1 + (\beta - 1)\frac{1 - p_l}{k\sigma}((1 - \beta p_l + \beta p_h) - p_l) < 0$$

Welfare effects

Recall the utility function of the government:

$$u(\pi_h, y_h) = -\frac{1}{2}(\pi_h^2 + \lambda y_h^2)$$

$$u(\pi_l, y_l) = -\frac{1}{2}(\pi_l^2 + \lambda y_l^2)$$

In the original Nakata & Schmidt (2019) paper, it was found that (without any institutional arrangements to limit deflation bias) $\pi_l < 0$, $y_l < 0$, $\pi_h \leq 0$ and $y_h > 0$. Therefore, in the low natural rate state an increase in inflation and output could lead to an unambiguous increase in welfare, as both inflation and the output gap are brought closer to the target values of zero. In the high natural rate state an increase in inflation would lead to an increase in welfare, but an increase in the output gap (meaning a higher positive level of the output gap) would lead to a reduction in welfare. However, as previously shown, the sign of the effect of the dismissal contract on the output gap is ambiguous in both states and depends on parameter values. The dismissal contract does increase inflation in both states.

Thus, the dismissal contract would unambiguously increase welfare if $\frac{\partial y_h}{\partial(E_t \frac{\partial \theta_h}{\partial y_h} V_h)} < 0$ and $\frac{\partial y_l}{\partial(E_t \frac{\partial \theta_h}{\partial y_h} V_h)} > 0$. In this case the dismissal contract would increase inflation in both the high natural rate state and low natural rate state, increase output in the low natural rate state and reduce output in the high natural rate state.

In addition to the signs of the effect of the dismissal contract on inflation and the output gap, the magnitudes of the effects of the dismissal contract on inflation and the output gap also matter. However, since the government can directly influence $\frac{\delta \theta_t}{\delta y_t}$ (through adopting a different value of β_1)

and indirectly influence V_t, it can be expected that the government maintains some form of control over the magnitude of those effects.

Increasing β_1 increases the positive effect of increasing output on the likelihood of being reappointed. Therefore $E_t \frac{\delta\theta_h}{\delta y_h}$ will be higher for a higher value of β_1. Thus a higher value of β_1 likely is optimal if the equilibrium inflation rates without dismissal contract are lower, for example because r_l^n is lower.

Credibility of the dismissal rule

One issue raised in Walsh (2002) was whether a dismissal rule could be credible. The idea was that by promising to adhere to the dismissal rule, the government would lower inflation expectations (in the time-consistent equilibrium) as the public expects the central banker to reduce inflation relative to the scenario without dismissal contract. However, in this case the government could indicate to the central banker that the government would not dismiss the central banker in case inflation exceeds the amount of inflation permitted by the dismissal contract. Therefore the central banker would be free to introduce surprise inflation and boost output. Effectively, the government would be reneging on the dismissal contract.

In the context of this model, the government could use the dismissal rule to increase inflation expectations, which increases inflation in both the low rate state and the high rate state. In the high rate state the government could then 'convert' this extra inflation in a lower output gap (recall that y_h likely is positive while the optimal value of y_h is zero). This can be shown by solving the optimization problem in the high natural rate state (minimizing the utility function without dismissal rule component with respect to the Phillips Curve) and taking expected future inflation as a given:

$$\pi_h = \frac{\beta\lambda}{\lambda + k^2} E_t\pi_{t+1} + e_t + ku_t$$

$$y_h = -\frac{k\beta}{\lambda + k^2} E_t\pi_{t+1} + u_t$$

$$E_t\pi_{t+1} = p_h\left(\frac{D}{AD-BC}kE_t\frac{\delta\theta_h}{\delta y_h}V_h - \frac{B}{AD-BC}r_l^n\right)$$
$$+ (1-p_h)\left(-\frac{C}{AD-BC}kE_t\frac{\delta\theta_h}{\delta y_h}V_h + \frac{A}{AD-BC}r_l^n\right)$$

However, the same comments made in Walsh (2002) still apply here. In an infinite horizon setting, reneging on the dismissal contract would cost the government its credibility in exchange for a small one-period gain. If the government puts sufficient weight on future welfare, it is very unlikely that reneging on the dismissal contract would be beneficial for the government. One could argue that governments are extra myopic since they face elections and a high probability of not being in office the next period. However, reneging on the dismissal rule would permanently damage the credibility of the dismissal rule. Even if the government loses power in the next period, it might be in power again in one of the periods thereafter and therefore will have a vested interest maintaining the dismissal rule. Furthermore, reneging on the dismissal rule becomes less attractive if the central banker is more conservative. This is logical, since a conservative central banker will make less use of ‘surprise deflation’ in order to cool down output, as the conservative central banker does not value the output target. A final argument that could be made against the credibility of the dismissal rule is that in a game with two governments, the incumbent government might not value the future utility loss of reneging on the dismissal since it is possible the *other* party reneges at some point in the future. This could lead to a situation in which both competing parties are incentivized to renege before the other does. However, this would need to be studied further and is beyond the scope of this paper.

Forecasted supply shocks

As stated before, one of the main weaknesses of the conservative central banker solution is that appointing a conservative central banker will lead to diverging preferences between the central banker and the government in case of a supply shock (Rogoff, 1985). A highly conservative central banker will allow a lower share of the forecasted supply shock to pass through to inflation than the government would like.

Forecasted supply shocks could be incorporated in the model by changing the Phillips Curve and output function the central banker operates with to:

$$\pi_t = ky_t + \beta E_t \pi_{t+1} + e_t^f$$

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) + u_t^f$$

Therefore, the central banker sets policy assuming u_t^f and e_t^f are the values for the demand shock and supply shock. At the end of the period, the true values of u_t and e_t are revealed.

However, including forecasted supply and demand shocks provides a difficulty for the model. Since expected future inflation affects current inflation in the model, forecasts of future supply and demand shocks could already affect inflation today. Thus an assumption about future supply and demand shock forecasts has to be made as well. It would be realistic to assume that future supply and demand shock forecasts have different values than supply and demand shock forecasts in the current period, but this would complicate the model by allowing the expected values of π_h and π_l to differ over time. Therefore, it is assumed that both the demand shock and the supply shock will have the same value over time. This is an unrealistic assumption, but it simplifies the model and does not change the main logic of the model. Furthermore, this assumption could be defended on the grounds of shocks often being at least somewhat persistent. After adding forecasted shocks, the following inflation rates and output gaps are obtained:

$$\pi_l = \frac{D}{AD-BC}\left(\lambda e^f + kE_t \frac{\delta\theta_h}{\delta y_h} V_h\right) - \frac{B}{AD-BC}\left(\frac{1-p_l}{k\sigma} e^f + r_l^n + \frac{1}{\sigma} u^f\right) + (e_t - e^f) + k(u_t - u^f)$$

$$\pi_h = -\frac{C}{AD-BC}\left(\lambda e^f + kE_t \frac{\delta\theta_h}{\delta y_h} V_h\right) + \frac{A}{AD-BC}\left(\frac{1-p_l}{k\sigma} e^f + r_l^n + \frac{1}{\sigma} u^f\right) + (e_t - e^f) + k(u_t - u^f)$$

$$y_l = \frac{1}{k}\frac{\beta p_l - 1 + (\beta - 1)C}{AD-BC}\left(\lambda e^f + kE_t \frac{\delta\theta_h}{\delta y_h} V_h\right) - \frac{1}{k}\frac{(1-\beta p_l)k^2 + (1-\beta)(1+\beta p_h - \beta p_l)\lambda}{AD-BC}\left(\frac{1-p_l}{k\sigma} e^f + r_l^n + \frac{1}{\sigma} u^f\right) + (u_t - u^f)$$

$$y_h = -\frac{1}{k}\left(\frac{(1-\beta)C - \beta p_h}{AD-BC}\right)\left(\lambda e^f + kE_t \frac{\delta\theta_h}{\delta y_h} V_h\right) + \left(\frac{\beta p_h k}{AD-BC}\right)\left(\frac{1-p_l}{k\sigma} e^f + r_l^n + \frac{1}{\sigma} u^f\right) + u_t$$

A number of observations can be made. First of all, a positive forecasted demand shock increases the inflation rate in both the high rate state (unless lambda is zero) and the low rate state. The reason it increases inflation in the low rate state is that the central bank cannot further reduce the nominal interest rate to 'sterilize' the forecasted demand shock. The reason it increases inflation in the high rate state is that a positive future forecasted demand shock increases inflation in the low rate state, which in turn increases inflation today (recall that the forecasted demand shock has the same size in both period t and period t+1). Furthermore, a positive supply shock increases inflation in both the high rate state (unless lambda is zero) and the low rate state.

It would seem that the presence of the dismissal contract does not affect the extent to which supply shocks are passed on to inflation. However, this is not correct. The likelihood of not being dismissed partially depends on the expected future inflation rate, and a positive forecasted supply shock increases both the expected future inflation rate (assuming the supply shock is persistent) and the inflation rate which the central bank expects to set in the current period. Therefore, the effect of an increase in output on the likelihood of not being dismissed also depends on the forecasted supply shock, with a higher value of expected inflation likely meaning that the marginal effect of increasing output on the likelihood of being reappointed reduces (as the central banker likely will not be dismissed anyway). Therefore, a high value of e^f likely reduces the value of $E_t \frac{\delta\theta_h}{\delta y_h} V_h$. Thus, the presence of the dismissal contract reduces the extent to which a positive supply shock is passed on to inflation. This means that a dismissal contract may also share the same drawback as the conservative central banker, namely that the central banker response to an increase in e^f is different from what the government would like.

For a lambda of zero (a central banker who only puts value on the output gap target), high rate state inflation collapses to:

$$\pi_h = -\frac{1}{-k^2}\left(E_t \frac{\delta\theta_h}{\delta y_h} V_h\right) + (e_t - e^f) + k(u_t - u^f)$$

As expected (for $e_t = e^f$) the forecasted supply shock does not influence inflation in the high rate state. This shows the drawback of the conservative central banker approach.

Conclusion

The main topic of this paper was the study of deflation bias caused by the zero lower bound constraint. The Nakata & Schmidt (2019) model states that the presence of the zero lower bound on nominal interest rates can cause deflation both when the zero lower bound is binding (the real natural interest rate is negative and the nominal interest rate cannot drop below zero to adjust) and when the zero lower bound is not binding (when the real natural interest rate is positive and the equilibrium nominal interest rate is positive). Furthermore, the presence of the zero lower bound causes output to increase in periods with a high real natural interest rate while output decreases in periods with a negative real natural interest rate.

Furthermore, it is confirmed that future inflation expectations are the main mechanism through which the zero lower bound affects inflation and output even in a period with a high real natural interest rate and positive nominal interest rate. By setting a zero lower bound on inflation expectations, the link between the high rate state and the low rate state is broken.

Finally, it is shown that adding a dismissal contract which states that the central banker is fired if a certain minimum inflation rate (which decreases if output increases) is not reached increases inflation in both the high natural rate state and low natural rate state. The effect of the dismissal rule on output is ambiguous in both states. Assuming the dismissal rule cannot factor for supply shocks, the dismissal rule likely shares the same drawback as the conservative central banker with regard to the central bank response to supply shocks. This is because a positive forecasted supply shock reduces the effect an increase in output has on the likelihood of being reappointed, therefore even a central banker with the same preferences as the government will react more conservatively in response to a large positive supply shock if a dismissal contract is present.

Future research could focus on the credibility of the dismissal rule, particularly in a political system with two competing parties. It is possible that each party faces the incentive to breach the dismissal contract in the high rate state (and thus allow the central bank to reduce the suboptimally high output level) before the other party does. Similar logic could also be applied to study the effect of party competition on the credibility of appointing a more conservative central banker.

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