The Redistributive Effects of Quantitative Easing

Developing an illustrative model of the key mechanisms of Quantitative Easing

By Mark van der Plaat[*]

Abstract:

Since the Financial crisis of 2007-8, multiple central banks in the developed countries engaged in some kind of Quantitative Easing (QE), either large-scale asset purchases, improved central bank lending facilities or 'Operation Twist', to stimulate their economies. The current study develops an illustrative model that investigates the effect of these different types of QE-programmes on the distribution of income. All QE-policy discussed in the current study affect the income distribution by influencing the long-term and short-term interest rates. The initial effects of the different types of QE are ambiguous, however, when the economic agents react on the QE-policies income inequality increases unanimously. These results show that unconventional monetary policies, such as QE, have re-distributional side effects.

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"It would seem … that for every kind of capital-asset *there must be an analogue of the rate of interest on money" – (Keynes, 1936/2006)*

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1 *Introduction*

On July 26 2012 Mario Draghi, the president of the European Central Bank (ECB), announced that 'the ECB is ready to do whatever it takes to preserve the euro… and believe me, it will be enough' (ECB, 2012). In January 2015 the ECB put words into action and introduced the eurozone to the unconventional monetary policy tool of Quantitative Easing (QE), which aimed to achieve the price stability within its mandate (ECB, 2015a). QE is often regarded as a large-scale purchasing programme in which the central bank purchases long-term assets on the secondary market to cutback the long-term interest rates and increase inflation (Joyce, Miles, Scott, & Vayanos, 2012). The ECB was not the first central bank in the world to introduce such a programme. As early as 2001, the Bank of Japan (BoJ) introduced QE to stem the continuous price decline in the country and create a basis for sustainable economic growth (Ugai, 2007). Since the financial crisis of 2007-8, the Federal reserve of the US (the Fed) and the Bank of England (BoE) introduced QE as well. Because the national economies are different, central banks used different types of unconventional policies besides QE. The current study identifies two other unconventional policies that are also a type of QE, namely improved central bank lending facilities and 'operation twist'. Improved central bank lending facilities tend to decrease the long-term interest rate by improving the lending possibilities of banks at the central bank (Fawley & Neely, 2013; Joyce et al., 2012). Operation twist is very similar to the large-scale long-term asset purchasing programmes, but differs in the fact that the central bank sells short-term securities to fund the long-term securities purchases, hereby increasing the short-term interest rates as well as decreasing the long-term interest rates (Fawley & Neely, 2013; Joyce et al., 2012).

The current study investigates the effects of three QE-programmes on income inequality rather than focussing on the interest rates. Since the publication of Piketty's bestseller *Capital in the Twenty-First Century* in 2014 the discussion about income inequality in the developed world has been re-invigorated. He showed that since the 1970s income inequality is rising in the US and in many European countries. Income inequality arises when one has more resources earned from current and historic income than the other has. Normally, the government can redistribute income by means of taxes, subsidies and other social welfare measures. However, central banks redistribute income through changes in asset prices and income flows (Brunnermeier & Sannikov, 2012). QE could hypothetically also have an effect on the distribution of income.

The research question of the current study is as follows: *How does Quantitative Easing affect the distribution of income of private households?* To study these effects of QE, the current study introduces an illustrative model. The basic illustrative model is based on the large-scale asset purchasing programmes, because these are commonly used to achieve a QE-programme. The illustrative model is further extended to fit the other two QE-policies. In the general setup of the illustrative model, the actors are households, banks and the central bank; hereby illustrating the direct effects and the indirect effects of QE on households. In this way, households can directly invest in securities on the secondary market or can indirectly invest via banks. The variations in asset valuations lead to mutations in the wealth of households and subsequent changes in income inequality.

In summary, illustrative model suggests that QE as a large-scale long-term asset purchasing programme has few effects on the interest rates and income inequality. Firstly, when the central bank is credible, QE is able to decrease the short-term interest rate. Secondly, when the central bank purchases long-term securities, institutional investors are triggered to rebalance their portfolios, which increases the demand for long-term securities and decreases the long-term interest rates. The decline in short-term and long-term interest rates lead to an increase in security prices. If, and only if, the investing households, the bankers, are able to capitalise on the increased price of the long-term securities, income inequality increases. QE-policy 2, the improved central bank lending facilities, have similar effects on the interest rates and income inequality, only not affecting the short-term interest rates. QE-policy 3, operation twist, is somewhat different, here the long-term interest rates decrease, yet, the short-term interest rates increase, leading to a decrease in income inequality at first but to an increase in income inequality when households and banks react on the policy.

The following sections will discuss the results of the current study in more detail, starting with a review of conventional and unconventional monetary policy, of which QE is a part. Section 3 contains an in depth discussion of QE, which elaborates on the three different types of QE, why QE is distinctly different from conventional monetary policy and introducing the two most important channels of QE. Hereafter, the portfolio-balancing channel, which influences the long-term interest rates and is proposed by Kimura and Small (2006), is reviewed in section 4. Section 5 reviews and discusses the model of Gertler and Karadi (2013), which will serve as the basis of the illustrative model. Based on the information gained in sections 3 through 5, section 6 introduces the basic illustrative model. This model is based on QE as large-scale long-term asset purchasing programme. After the introduction of the basic illustrative model, section 6 further discusses the results and the propositions based on the presented model. The modified model including the two other forms of QE, which allows households and banks to invest in a greater variety of assets, is included in section 7. Finally, section 8 discusses and concludes the results of the current study.

2 *Conventional and Unconventional Monetary Policy*

This section discusses the differences between conventional and unconventional monetary policy. Firstly, this section discusses conventional monetary policy. What are the aims of this policy, how do central banks use conventional monetary policy and in what is situation this policy normally used. Secondly, unconventional monetary policy is discussed, including the difference in situation between conventional and unconventional monetary policy, the difference in aims and the difference in instruments.

Table 1: Open market operations

Central bank balance sheet			
Assets		**Liabilities**	
Domestic securities		Currency in circulation	(+)
Short-term	(+)	Deposits held by private banks	
Long-term			
Foreign securities			

Table 1 provides a schematic representation of a balance sheet of a central bank and the effect of open market operations. Adapted from: Krugman, Obstfeld, and Melitz (2015). When the central bank engages in expansionary open market operations the short-term securities on the asset-side of the balance sheet increase by purchasing short-term government securities. Simultaneously the currency in circulation on the liability-side of the balance sheet increases by using new money to finance the securities.

2.1 Conventional monetary policy

The current study defines conventional monetary policy as: the changes in the money supply of a specific country due to the creation or buying up of money by the central bank by means of open market transactions (Barro & Gordon, 1983; Blanchard et al., 2010). The two main aims of conventional monetary policy are to 'achieve low and stable inflation' (Joyce et al., 2012, p. F271), and 'to manage liquidity in the money market and steer short-term interest rates' (De Haan, Oosterloo, & Schoenmaker, 2015, p. 123). In practice, there are some differences among central banks. The mandate of three of the four biggest central banks, the ECB, the BoE and the BoJ only includes price stability. The mandate of price stability of the ECB and the BoE are conditional to the union's and government's economic policy respectively (BoE, 2016; EU, 2008, p. 101), whereas the mandate of the BoJ does not have such a conditionality (BoJ, 2016a). The fourth central bank, the Fed, goes a step further and uses its monetary policy for the adjustment of unemployment and stable long-term interest rates (Fed, 2016a).

The common instrument of conventional monetary policy are open market transactions. Open market transactions are operations in which central banks purchase – i.e. expansionary policy – or sell – i.e. contractionary policy – short-term securities on the open market[1] (Mundell, 1963). This definition includes two important factors: securities and the open market. Securities are 'fungible, negotiable instruments representing financial value' (De Haan et al., 2015, p. 156), these can be categorised in debt and equity securities and are also known as assets[2]. Open markets are markets in which resellers can purchase and sell excess securities (Lee & Whang, 2002), meaning that the original issuer of the securities is necessarily not involved.

Open market operations are executed by central banks as follows: central banks purchase short-term domestic securities on the open market in exchange for money, thereby increasing the money supply. The balance sheet of central bank increases when performing expansionary open market transactions (Table 1). On the assets side, the short-term domestic securities increase and on the liability

[1] Or secondary market

[2] Debt securities are specifically known as bonds

side, the currency in circulation increases, i.e. the money supply. The purchases have three effects. Firstly, an increased money supply stimulates inflation in the economy. Secondly, open market purchases increase the demand of specific securities, increasing their prices and, conversely, decreasing the respective yields. Thirdly, if investors are able to capitalise on the increased prices, e.g. sell the securities for higher prices than bought for, their wealth can increase, possibly increasing income inequality if not all economic agents are able to invest on the secondary market. As will become apparent section 3, QE as a long-term asset-purchasing programme has some similarities with open market transactions.

Conventional monetary policy is usually conducted in an environment with positive short-term interest rates. In such an environment, central banks are able to steer the interest rates and the corresponding inflation. Positive interest rate was previously assumed a necessary condition of monetary policy. The interest rates were assumed to be bounded by zero because, at a zero interest rate, securities and money become almost perfectly substitutable, meaning that money is more liquid that securities. As a result, investors strictly prefer money to securities and invest in money. Thus when the central bank engages in expansionary monetary policy, the increase in money supply has no effect on the interest rates, for investors would prefer money to securities, leaving the demand for securities and the interest rates unchanged. The next subsection shows that this assumption did not hold.

2.2 Unconventional monetary policy

After the financial crisis of 2007-8, conventional monetary policy proved to be inadequate. Conventional monetary policy could not deal with two issues.

Firstly, the Taylor-rule demanded negative or close to zero interest rates. The Taylor-rule was introduced by Taylor (1993) and is a rule how monetary policy can be applied in practice. In particular, the rule provides information on how the policy rate would have been set in the past in a particular country subjected to economic conditions (Gerlach-Kristen, 2003), which could be useful for setting the current policy rates. Generally, the original Taylor rule is as follows:

$$r_t = \rho + \pi^* + K_\pi(\pi_t - \pi^*) + K_y(y_t - y^*), \qquad (1)$$

where r_t is the real interest rate at time t, π^* is the inflation target, π_t is the actual inflation, y_t is the real output, y^* is the hypothetical potential output, and K_π and K_y are weight factors. It means that the actual interest rate is chosen based on the baseline path of the real interest $(\rho + \pi^*)$, the gap between the actual inflation and target inflation, and the output gap. Therefore, when the actual inflation decreases below the inflation target, the optimal interest rate will decrease as well; a similar argument holds for the output gap. After the financial crisis of 2007-8, for many developed countries the Taylor-rules indicated negative interest rates. This was mainly due to an increase in the output gap rather than in the inflation gap (Blanchard, Cerutti, & Summers, 2015; Nikolsko-Rzhevskyy & Papell, 2013). The negative Taylor-rule lead to the adoption of unconventional tools by central banks.

Secondly, markets, whose bubbles had burst during the financial crisis, became very illiquid and the solvency of the actors in these markets sharply decreased (De Haan et al., 2015). Markets that became illiquid during the financial crisis were, among others, housing markets and money markets (De Jong, 2011). Banks around the world had used the process of securitisation to pool, repack and resell housing loans to other banks and investors in order to diversify risk[3] (De Haan et al., 2015). When the housing bubble in the US burst, it became apparent that this risk was not diversified. Securitisation was supposed to sell the risky securities, but due to the bad credit rating and the complexity of the instrument, many risky securities remained on the balance sheet of the banks (De Haan et al., 2015). The result was that money market liquidity dried up. The money market provides short-term loans to and from financial institutions. The crash of the these markets was the incentive for the Fed and other central banks to provide liquidity to such markets (De Haan et al., 2015). Central banks around the world reverted to unconventional monetary policy tools to re-liquidise the markets with several measures, of which QE-policies are the best known.

2.3 Types of unconventional monetary policy

The previous subsection argued that central banks had to revert to unconventional tools to stimulate the economy. There are several forms of unconventional monetary policy tools, such as negative policy rates, forward guidance and balance sheet expansion. The negative policy rates are relevant to discuss, because it leads to the situation in which the central bank had to revert to QE. Forward guidance is relevant, because its mechanisms of influencing the interest rates are similar to these of QE. Balance sheet expansion also known as QE and is discussed in depth in the next section.

The first commonly used unconventional monetary policy tool is negative policy rates. This tool, among others, is used by the European Central Bank (ECB) and (Joyce et al., 2012) and by the Bank of Japan (BoJ). Negative policy interest rates by the central banks were designed to increase the incentive for banks to lend more money to other parties and disincentives banks to make use of overnight facilities of the central banks, which are used by banks to stall excess liquidity overnight. Furthermore, negative interest rates would also stimulate spending, because holding money would be less favourable. According to Bossone (2013/2016) critics argue, however, that negative interest rates would only lead to cash hoarding and safe asset accumulation instead of increased expenditures. QE in Japan, the eurozone, the United States and the United Kingdom was introduced in a situation of near-to-zero interest rates, making the situation in which QE was introduced unique.

Another unconventional monetary policy tool is forward guidance. Forward guidance is when a central bank communicates to hold the policy interest rate at its current level for some time in the future and thereby holding the interest rates low (Bossone, 2013/2016; De Haan et al., 2015). The mechanism

[3] Credit defaults swaps were used in combination with securitisation to further diversify risk. Credit default swaps are instruments that allows investors to take a position when the counterparty defaults.

Figure 1

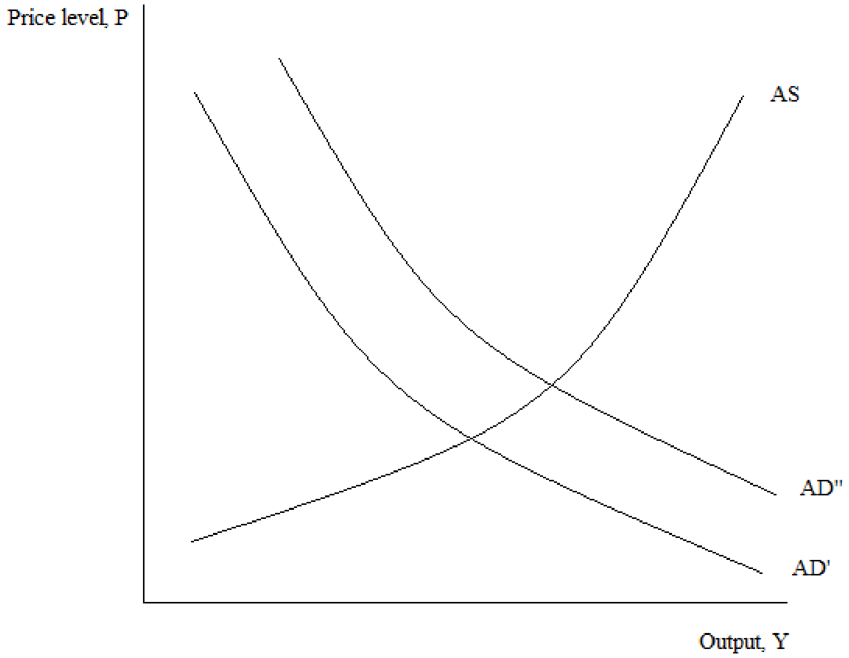

Figure 1 provides a schematic representation of how forward guidance affects the output and inflation in the AS-AD model, adapted from: (Blanchard, Amighini, & Giavazzi, 2010). When central banks' forward guidance is successful, it decreases the expectations of the interest rates, leading to a decrease in the interest rates. The lower interest rates lead to more investments by private companies, shifting the AD-curve upward from AD' to AD'', increasing the national income and the inflation rate.

behind forward guidance is that when the central bank communicates clearly to the market that it is willing to hold the interest rate on a certain level, market expectations will follow correspondingly. Especially at the zero-bound interest rates, a credible promise to hold the policy interest rate low can stimulate current aggregate demand and economic growth (Campbell, Evans, Fisher, Justiniano, Calomiris, & Woodford, 2012, p. 2). This relation can be captured in the AS-AD framework (Blanchard et al., 2010), which explains the relationship between output and inflation through the equilibrium between the aggregate demand and supply. It is assumed that forward guidance decreases the interest rates. Low interest rates make investments more attractive due to the lower interest costs. An increase in investment increases the aggregate demand or the AD-line[4] in figure 1 from AD' to AD''. To satisfy the increased demand, the suppliers of goods have to increase production. Increased production decreases unemployment, which increases the nominal wages. The increased nominal wages lead to an increase in the prices set by the firms, i.e. inflation rises. Figure 1 illustrates this as an increase along the AS-line[5]. This framework does not specify if the interest rates are short-term or long-term. A critical note though, only partial empirical evidence was found for the effects of forward guidance (Bossone, 2013/2016).

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[4] AD: $y = C(Y - T) + I(Y, i) + g; \frac{M}{P} = YL(i)$ (Blanchard et al., 2010)

[5] AS: $P = P^e(1 + \mu)F(1 - \frac{Y}{L}, z)$ (Blanchard et al., 2010)

3 *Quantitative Easing (QE)*

The term Quantitative Easing was created in Japan in the 2001 after a period of deflationary pressures and an upcoming recession (Joyce et al., 2012; Ugai, 2007). The policy interest rates were near to zero and pushed the BoJ to develop this new measure to stimulate the economy (Spiegel, 2006). This section discusses the differences between QE and conventional monetary policy, why central banks reverted to different QE-programmes, the three types of QE-policies and two important QE-channels.

3.1 QE and conventional monetary policy

QE is used by central banks in special circumstances. It has different aims from conventional monetary policy and makes QE strictly distinguishable to conventional monetary policy. Firstly, QE aims to decrease the long-term interest rates, providing stimulus to the economy (Christensen & Rudebusch, 2012). Secondly, QE aims to increase the amount of liquid assets in the economy (Den Haan, 2016), of which an increase in the money supply is most desirable. Thirdly, central banks seek to stimulate the entire economy of the country, in contrast to a specific market. When central banks only seek to stimulate one market, e.g. stimulate markets that are illiquid, they engage in credit easing (Fawley & Neely, 2013; Shleifer & Vishny, 2010).

Besides the differences in aims, QE-policies also differ from conventional monetary policies. The best-known form of QE are the large-scale long-term asset purchasing programmes (LSAPs). The current study identifies two other forms of unconventional policies used that could also be classified as QE. The first form are the improved lending facilities (ILFs) of the central bank, although this form does not involve purchases of assets on the secondary market, it has the same aims as LSAPs. The second form is ‘operation twist’, this form is very similar LSAPs, however, it does not increase the amount of liquid assets on the market by means of new money, but with short-term securities. The LSAPs return as QE in the illustrative model of section 6, section 7 includes the other types of QE in the illustrative model. The LSAPs are chosen as the basis of the illustrative model because the two channels of the policy are also applicable to other types of QE-policies.

3.2 The optimal choice of QE

The optimal form of a central bank’s QE-programme depends on the structure of the financial system. Before proceeding to the discussion of these three forms, two types of financial systems, a bank-based and a market-based financial system are briefly discussed.

In a bank-based system, financial intermediaries are important, whereas in a market-based system financial markets are important. The main difference between these two systems is that financial intermediaries have close business relationships with firms in which they invest, whereas investors in financial markets do not have any business relationships (Degryse & Van Cayseele, 2000). These business relationships give financial intermediaries an information advantage, which could constrain

Table 2

Type of QE	Operation of the central bank	Aims of QE		
		Decreasing long-term interest	**Increase liquid assets in the market**	**Macro-economic stimulus**
Policy 1: LSAP	Purchase long-term securities on the secondary market	Fulfilled	Fulfilled (money supply)	Fulfilled
Policy 2: ILF	New long-term loans to Financial intermediaries	Fulfilled	Fulfilled (money supply)	Fulfilled
Policy 3: Operation Twist	Purchase long-term securities and sell short-term securities on the secondary market	Fulfilled	Fulfilled (short-term securities)	Fulfilled

Table 2 provides a representation of the types of QE, how they operate and if the fulfil the aims of QE. Although different in operation the LSAP and ILF fulfil all the aims of QE, making them good instruments of QE. Operation twist is operational wise very similar to LSAP and only differs that operation twist finances the purchases with short-term securities. Consequently, the money supply is not increased though the short-term securities are.

investment projects of other firms (Levine, 2002). However, the lack of business relations could lead to less investments in projects due to the less available information (Levine, 2002).

The Fed, the BoE, the BoJ and the ECB based their use of different QE-programmes on respective the structure of their financial systems. These central banks were chosen to investigate in the current study, because their relative importance in the world economy. Based on GDP, the US and the eurozone have two of the biggest economies in the world (based on OECD figures). And based on the global foreign exchange market turnover, the four currencies of these central banks were the most traded in the world in 2013 (BIS, 2013).

Based on the amount of bank assets to GDP, Germany is considered to have the most bank-based financial system and the US the most market-based financial system in the world (Levine, 1997). Generally, the ideal types do not exist and financial systems are often a combination of bank-based and market-based. To study this, Kwok and Tadesse have developed a continuous measure[6] that shows if countries have more bank-based or market-based financial systems. Their results suggest that, overall; the UK and the US are predominantly market-based. Japan is relatively bank-based. The eurozone,

[6] Their measure is based on a variety of financial architecture indicators, which can be split into three variables (Kwok & Tadesse, 2006, pp. 232-233): i) the size of the stock markets relative to the size of the banking sector, ii) the activity of the stock markets relative to that of banks and (iii) the relative efficiency of stock markets *vis-à-vis* that of the banks.

Table 3: Policy 1 – LSAPs

Central bank balance sheet

Assets		**Liabilities**	
Domestic securities		Currency in circulation	(+)
Short-term		Deposits held by private banks	
Long-term	(+)		
Foreign securities			

Bank balance sheet

Assets		**Liabilities**
Reserves	(+)	Deposits accounts
Loans		Central bank loans
Securities		
Short-term		
Long-term	(–)	

Table 3 provides a schematic representation of the balance sheet of a central bank and of a bank and the effects of large-scale long-term asset purchases on them, adapted from: (Blanchard et al., 2010). When a central bank engages in large-scale asset purchasing programmes, it augments the long-term domestic on the asset-side of the balance sheet; at the same time the currency in circulation on the liability-side increases for the purchases are financed by new money. On the bank's balance sheet something different happens: the purchases of the central bank lead to a decrease of the long-term securities on the asset-side of the balance sheet but to an increase in reserves the bank can use to finance new investments hereby increasina the national income.

however, is difficult to place in the scale according to this measure. But based on the findings of Levine (1997), Levine (2002) and Kwok and Tadesse (2006), eurozone is relatively more bank-based.

3.3 Three policies of QE

There are three different types of QE seen in the years after the financial crisis of 2007-8. Table 2 presents the three different types of QE, how they operate and if the three aims as described above are met. The three QE-policies are discussed in this subsection.

3.3.1 Policy 1: Large-scale long-term asset purchases

The first QE-policy, QE as an LSAP, was first used by the BoJ (Ugai, 2007) and later used by the Fed, the BoE and the ECB in the aftermath of the financial crisis of 2007-8 (Fawley & Neely, 2013). Effectively, this policy entails the purchase of safe long-term securities, including long-term government bonds and long-term mortgage-backed securities (MBS) (Krishnamurthy & Vissing-Jorgensen, 2011), inflating the balance sheet of the central bank and, as a result, increasing the money supply. Backed securities, including mortgage-backed securities, are distinctly different from other securities in two ways: i) if a backed security defaults, investors can fall back on the underlying collateral pool and ii) banks are obliged to include the underlying collateral on their balance sheets (Schwarcz, 2011). The central bank of the country has to pursue the goal of the programme, e.g. an inflation target, until the goal is completed for QE-policy 1 to effectively affect the expectations

Notice that in such a QE-programme, the central bank purchases long-term as opposed to short-term securities of the open market operations. A second distinction is the difference in scale between QE as described and open market operations. By the end of 2012 the Fed had accumulated 3,152 trillion dollars in government, agency, and mortgage-backed securities (Fawley & Neely, 2013), which corresponds approximately with 20% of the GDP of the US in 2012 (according to the World Bank). This is in contrast with the net open market purchases of the fed in 2011 and 2012, which were 638 billion dollars and 34 billion dollars respectively (Fed, 2016b).

If a central bank engages in QE-policy 1, the programme provides a stimulus for the economy in the following way: when committed to QE, the long-term domestic securities of the central bank increases. Since these securities are purchased with money, the currency in circulation on the central banks liability side increases. The long-term securities on the balance sheet of the central bank consist of long-term government and high-grade long-term private securities. Table 3 is a schematic representation of the balance sheets of a central bank and banks. Note the difference between QE and open market operations – tables 1 and 3.

On the banks' balance sheets the following events occur: purchases of the central bank lead to a decrease in long-term securities on the banks' balance sheets. The purchases of the central bank provide the banks with new reserves in terms of money. Banks can use these reserves to provide firms with fresh long-term loans or purchase long-term securities on the secondary market, effectively rebalancing their portfolios and stimulating the economy. Central banks, though, are not constraint by purchasing long-term securities from banks only. A QE-programme like this can also purchase long-term assets from individual investors. This is similar to purchases from banks, namely that individual investors sell part of their long-term securities for money, which they use to purchase new securities, funding firms' investments and therefore stimulating the economy. This technique is therefore suitable for every financial system.

Even though the financial systems of the US, the UK, Japan and the Eurozone are different, all central banks engaged in QE-policy 1. The Fed and the BoE very quick in engaging in long-term asset purchases. Their programmes started in 2008 and 2009 respectively. Both largely focus on the purchase of government bonds on the secondary market (Fawley & Neely, 2013; Krishnamurthy & Vissing-Jorgensen, 2011). In the beginning of the 2000s the BoJ combined improved central bank lending facilities and LSAP (Ugai, 2007). The BoJ abandoned the programme in 2006 (Ugai, 2007) only to reinstate it in 2010 (BoJ, 2010a). The ECB introduce LSAPs officially in 2015 (ECB, 2015a). During the course of all programme all central banks increased the amount with which they purchased long-term securities. For more in depth information see appendix A.

3.3.2 Policy 2: Improved central bank lending facilities

Improved central bank lending facilities (ILFs) are a second type of QE. This type of QE is essentially for bank-based financial systems, since the central bank provides long-term loans to banks in exchange

Table 4: Policy 2 – ILFs

Central bank balance sheet

Assets		**Liabilities**	
Domestic securities		Currency in circulation	(+)
Short-term		Deposits held by private banks	
Long-term	(+)		
Foreign securities			

Bank balance sheet

Assets		**Liabilities**	
Reserves	(+)	Deposits accounts	
Loans		Central bank loans	(+)
Securities			
Short-term			
Long-term			

Table 4 provides a schematic representation of the balance sheet of a central bank and of a bank with respect to the effects of Improved central bank lending facilities, adapted from: (Blanchard, 2010). When the central bank improves its lending facilities it effectively augments the long-term domestic assets on the asset-side of the balance sheet by providing more loans to banks, herewith increasing the currency in circulation on the liability-side of the balance sheet. On the banks' balance sheet on the asset-side the reserves increase, which can be used for new investments and the central bank loans on the liability-side of the balance sheet increase as well – keep in mind that these are long-term loans.

Table 5: Policy 3 – Operation twist

Central bank balance sheet

Assets		**Liabilities**
Domestic securities		Currency in circulation
Short-term	(–)	Deposits held by private banks
Long-term	(+)	
Foreign securities		

Bank balance sheet

Assets		**Liabilities**
Reserves		Deposits accounts
Loans		Central bank loans
Securities		
Short-term	(+)	
Long-term	(–)	

Table 5 provides a schematic representation of the balance sheet of a central bank and of a bank with respect to operation twist, adapted from: (Blanchard et al., 2010). The central bank purchases long-term securities financed by selling of short-term securities hereby increasing the long-term domestic securities and decreasing the short-term domestic securities on the asset-side of the balance sheet. On the banks' balance sheet the opposite happens, namely the short-term securities increase and the long-term securities decrease on the asset-side hereby twisting the yield-curve.

for a greater variety of eligible collateral (Fawley & Neely, 2013; Joyce et al., 2012). As a result, banks will be able to receive more long-term loans of the central banks on less collateral, increasing the lending facilities of banks (Fawley & Neely, 2013; Joyce et al., 2012). How this form of QE works, is explained in table 4. Here, the central bank does not purchase long-term securities, but obtains them by lending to banks; this increases the long-term domestic securities and the money supply. On the banks' balance sheets, the liabilities increase with the amount they borrow from central banks and the assets increases with the amount of money supply gained from the policy.

The idea is that the banks will now lend out their newly gained reserves to private firms or consumers, who, in turn, use this money to invest. The interest rates on the loans decrease, due to an increased supply of loans (Fawley & Neely, 2013). This means that the more fresh loans are created, the higher the market liquidity. The problem of QE-policy 2 is that it hinges on the willingness of banks to provide loans; in a crisis, banks may be less willing to lend money.

The ECB, before it committed to the in QE purchasing programmes, it engaged in ILFs. This measure is called the fixed-rate tender, full allotment (FRFA) programme. An FRFA involves repurchasing operations (repos) by the ECB for an increased amount of eligible collateral or assets (Joyce et al., 2012). A repo operation is when a central bank sells assets while obtaining the right and obligation to repurchase it at a specific time and at a specific price (De Haan et al., 2015, p. 162). This effectively means that banks could borrow money from the central bank more easily at a fixed rate. The BoJ has introduced this type of QE in 2001. The goal of this was 'to change the main operating target for money market operations from the uncollateralized overnight call rate to the outstanding current account balances (CABs) held by financial institutions at the BOJ, and provide ample liquidity to realize a CAB target substantially in excess of the required reserves' (Ugai, 2007, p. 2).

3.3.3 Policy 3: Operation twist

QE-policy 3 is a variation on the QE purchasing programme. The Fed used this strategy and is known as 'operation Twist' (Joyce et al., 2012), and as of yet no other central bank has used this technique (Fawley & Neely, 2013). The total operation had a size of $667 billion in total by the end of 2012 (Fed, 2013). Although operation twist does not increase the money in the market, it does increase the amount of liquid assets by selling short-term securities for long-term securities. The Fed designed it to have an effect on the long-term and short-term interest rates and was not aimed at a specific market (Joyce et al., 2012). Operation twist is therefore not a pure form of QE, but since the effects were meant to be economy-wide, this study regards it as a form of QE. When engaged in operation twist, central banks sell short-term bonds in order to purchase long-term bonds (Joyce et al., 2012) (Table 5). This technique will twist the yield-curve by decreasing the long-term interest rates relative to the short-term interest rates. Similar to the asset-purchasing QE, this form of QE is suitable for both bank-based and market-based financial systems.

3.4 Two transmission channels of QE

As discussed in section 3.3.1, from this subsection onward QE is considered as an LSAP, considering all big central banks have used such a QE-programme (Fawley & Neely, 2013; Joyce et al., 2012). This makes it the most widely used and most known QE-policy. The channels discussed in this subsection return in the illustrative model in section 6. In section 7, the other two policies are separately entered into the basic illustrative model. Hereafter, section 7 discusses the effects of each policy on the economy and income inequality. QE affects the long-term and short-term interest rates in several ways. The current section introduces the two most important channels making QE-policy 1 possible.

Generally the nominal yield of a n-year bond is as follows (Fawley & Neely, 2013):

$$r_{t+i}^{b} = r_{t+i} + \tau_{t+i,n}^{b}, \tag{2}$$

where r_{t+i}^{b} is the expected nominal interest rate at time $t + i$ on a long-term bond and r_{t+i} is the average expected overnight (short-term) nominal interest rate over the following n-years at time $t + i$, $\tau_{t+i,n}^{b}$ is the term-premium on a bond of n-years at time $t + i$. This equation illustrates that there are two possible channels for QE to affect real bond yields, namely: i) the term premium may fall (the portfolio-balancing channel) and ii) the expected path[7] of the policy interest rates may fall (the signalling channel).

3.4.1 Channel 1: The portfolio-balancing channel

The first channel, the portfolio-balancing channel, affects the term-premium on long-term assets, because the central bank purchases long-term securities on the market. As will be explained in more detail in section 4, these purchases lead to a lower supply of securities in the market, decreasing the term-premium on long-term assets, leading to a decrease in only the long term interest rate (Joyce et al., 2012; Kimura & Small, 2006). A term-premium is the mark-up on the interest on a specific medium- or long-term asset to compensate investors for the duration risk they are exposed to. Duration risk relates to the length of a loan (Boquist, Racette, & Schlarbaum, 1975): the longer the loan, the further away the final payment, the higher the risk of default. Investors will demand more return when the duration risk is higher, which is captured in the term-premium.

The key assumption of this channel lies in the idea of imperfect asset substitutability (Joyce, Tong, & Woods, 2011/2016). Under perfect asset substitutability, all assets can replace any other asset irrespectively of the characteristics of the assets such as duration or the organisation that issued it. Meaning that if long-term securities are substitutable the purchase of one type of long-term security decreases the interest rate of another type as well. A second important element is the assumption of a balanced portfolio of institutional investors[8]. A balanced portfolio means that institutional investors require a well-diversified portfolio of investments. This means that investors create portfolios with the highest possible return subjected to the lowest risk possible by investing in multiple assets (Brealey,

[7] An interest rate path is evolution of a certain interest rate through time. When the expected path fall interest rates will decrease.

[8] Institutional investors pool money of many individuals to invest for a specific goal or in a specific manner – e.g. mutual funds, insurance companies and pension funds. All over the developed world institutional investors are important in the financial markets and have a portfolio with a large amount of assets (Ferreira & Matos, 2008).

Myers, & Allen, 2011). This implies that institutional investors have a preferred habitat, meaning that investors have specific preferences for assets of specific maturities (Krishnamurthy & Vissing-Jorgensen, 2011). If the portfolio is unbalanced, institutional investors will try to rebalance their portfolios. Whether the preferred habitat demand is broad or narrow depends on the substitutability of the assets. When assets are highly substitutable, the preferred habitat is broad and *vice versa*.

QE transforms a part of the portfolio of institutional investors from long-term government bonds and long-term backed private securities into money. Institutional investors now have more liquidity, but on the other hand have an unbalanced portfolio because money is an imperfect substitute to long-term government bonds. They use the newly acquired liquidity to rebalance their portfolios, e.g. purchase long-term securities, until their portfolios are balanced again. The rebalancing of portfolios and QE itself lead to an increased demand in long-term assets (Joyce et al., 2011/2016), which will cause the investors to demand less compensation for the duration risk (Fawley & Neely, 2013; Kimura & Small, 2006). In the illustrative model in section 6, this channel is responsible for the decrease in the long-term interest rates.

There is some evidence for the portfolio-balancing channel. For example, Christensen and Rudebusch (2012) found that the portfolio-balancing channel dominated the signalling channel in the UK. They used the announcements of QE in the UK as events that triggered changes in the yields in gilts (government bonds from the UK). They found that, overall, the yields on the long-term gilts decreased more than the short-term gilts. This was confirmed when they decomposed the response of the term structure of the instantaneous forward rates into forecasted future spot rates and instantaneous forward term premiums. The effect shown by Christensen and Rudebusch (2012) was also found by Joyce et al. (2012) and Ugai (2007) for the US and UK, and Japan respectively. Based on the evidence, the portfolio-balancing channel returns in the basic illustrative model, because it is an important driver of the decrease in long-term interest rates. Because this channel is relatively comprehensive and important for QE as an LSAP, it is discussed in more detail in section 4 using the paper of Kimura and Small (2006).

The studies discussed here use the approach of decomposing the yield curves and then extract the effect of the signalling. In such an approach, the researchers split the total movements of the interest rates into several factors that influence the movement of the interest rate. The upside of such a method is that specific factors can be isolated so they can be studied separately. The downside is that splitting the interest rate movements into several factors is often by construct, in other words: the influencing factors are dependent. This means that splitting the yield-curve into specific factors is often difficult and ambiguous. The results of this techniques are therefore ambiguous, as they can be created by construct or be the real effect.

Figure 2

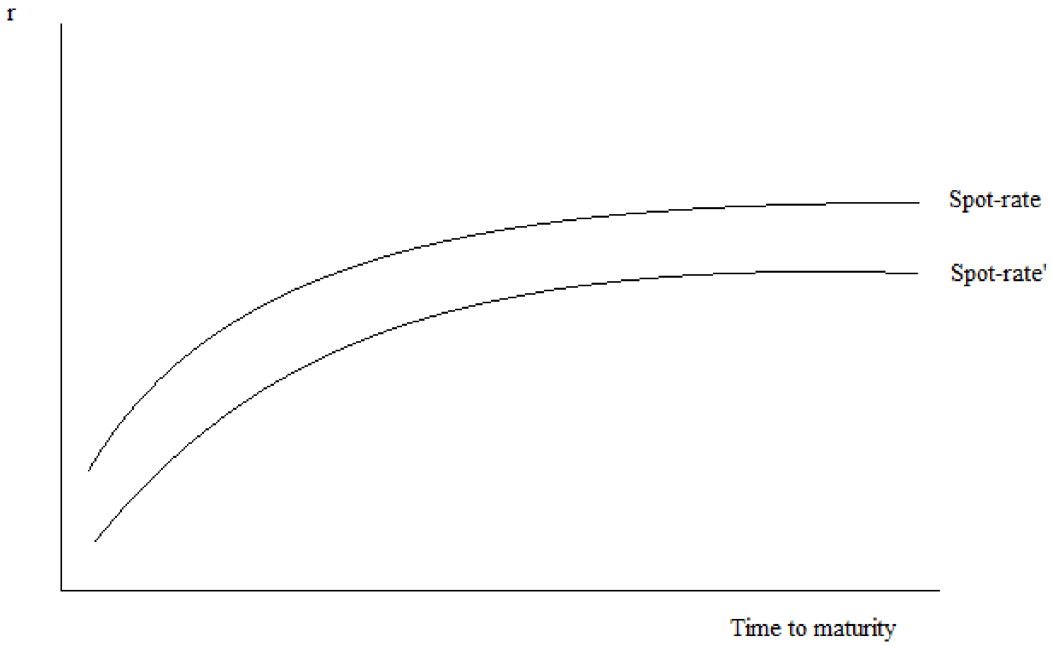

Figure 2 provides a schematic representation of the spot-rate curve. Here the spot-rate curve represents all interest rates on all assets of different maturities from short-term to long-term. When the signalling channel functions properly, the short-term interest rate decreases. Referring to equation (2) when the short-term interest decreases the long-term interest rate decreases similarly leading to an equal decrease over the entire spot-rate curve, decreasing the sport-rate curve to spot-rate'.

3.4.2 Channel 1: The signalling channel

The second channel through which QE affects long-term interest rates on securities is the signalling channel. As can be seen in equation (3), this channel works through the expectations of future interest rates of the market. Rewritten, the expectations about the interest rate are the following (Rudebusch, 1995):

$$r_t^d = \frac{1}{n}\left[r_t + E_t \sum_{j=1}^{n-1} r_{t+j}\right], \tag{3}$$

where r_t^d is the average expected overnight nominal interest rate at time t, r_t is the nominal interest rate at time t and $E_{t+i} \sum_{j=1}^{n-1} r_{t+i+j}^D$ are the expectations of all nominal interest rates in the future over n – 1 periods.

Equation (3) states that if economic agents expect lower interest rates in the future, the nominal short-term interest rates today will decrease accordingly. Whether the agents believe a low interest rate to persevere depends on the credibility of QE and the communication of the QE-programme by the central bank. Announcements of central banks give economic agents (or the market) information about the future state of the economy and policy path of the central bank (Christensen & Rudebusch, 2012). Using this information, they can infer whether the interest rates will decrease or increase. In many theoretical models, a central bank is credible and the announcements are credible when the central banks attains a higher expected utility by following the announcement compared to when the central bank reneges (Blinder, 1999). This channel resembles forward guidance in that it influences the interest rates through the expectations of economic agents.

Figure 2 shows the nominal spot-rate curve of securities with several maturities. The figure shows that when the time to maturity of a security increases the corresponding interest rates increases as well. In figure 2 a decrease in the short-term interest rate is a downward shift of the spot-rate curve – to spot-rate'. In other words, the nominal interest rate decreases for all maturities.

For Japan Ugai (2007) presented some confirming evidence for the signalling channel in multiple studies. All the papers discussed by Ugai have found a lowering effect on the yield curve by the signalling effect of the BoJ during the first period of QE (as described above). Furthermore, the majority of the papers discussed by Ugai found a larger decrease in the yield curve under QE than under the prior zero-interest rate policy. The results presented here suggest downward pressure on the yield curve of the Japanese government bonds in the period 2002-06 due to QE, even more so than the zero-interest rate policy could achieve.

A similar story emerges for the QE1 and QE2-programme of the Fed (see Appendix A). Krishnamurthy and Vissing-Jorgensen (2011) found that yields significantly declined several times during the QE1 programme for most maturities of the treasury bonds and MBSs, and all of the agency bonds. They use similar techniques as the papers discussed in Ugai (2007). When these declines were split into the several channels, they found that the signalling effect on treasury bonds with a maturity of five to ten years is associated with a 20 to 40 basis points decrease. The authors found similar evidence for the QE2 period, with a slightly less strong downward pressure on the yield curve than during QE1.

Based on the evidence of the signalling channel in Japan and the US, one can assume that this channel may be salient during times of QE and is an important factor in the decrease of long-term interest rates in multiple countries. This channel will therefore return in the basic illustrative model in section 6.

4 *Transmission channel 1: The portfolio-balancing channel (Kimura & Small, 2006)*

The portfolio-balancing channel is one of the two channels that is used in the basic illustrative model in section 6. The framework of the basic illustrative model is based on the paper of Gertler and Karadi (2013), in which this channel together with the signalling channel is introduced. This section discusses the portfolio-balancing channel based on Kimura and Small (2006). Firstly, the research topic of the paper is introduced. Secondly, a model of the portfolio-balancing channel in a CAPM-setting is discussed.

4.1 The setting of the paper

Kimura and Small (2006) study the effects of the portfolio-balancing channel during the period of QE in Japan that started in March 2001 with a formal model and empirically. According the authors, some believe the portfolio-balancing channel did not function properly during the QE-period, because the capital position of financial intermediaries had been impaired due accumulation of non-performing loans[9]. This lead to a fall in asset prices and a subsequent recession. As a result, financial intermediaries were less willing to take on portfolio risks by purchasing risky securities. Thus, the QE-programme of the BoJ was seen as ineffective, because the financial intermediaries and institutional investors were reluctant to purchase new assets with the increased reserves.

The authors argue otherwise. They argue that financial intermediaries and institutional investors in a crisis are willing to rebalance their portfolios, hereby decreasing the long-term interest rate, because they are averse to business cycle risks. Business cycle risks are the variations in return on securities due to the economic cycle (Perez-Quiros & Timmermann, 2000). This means that the higher the correlation of securities with the business cycle, the more the returns will decrease during an economic crisis. The underlying mechanism argued by the authors is as follows: when in an economic downturn, investors[10] invest more in safe assets, such as government bonds, which are negatively correlated with business cycles. The QE-policy 1 transforms government bond-holdings of all investors into money, which leads to a more pro-cyclical portfolio of the financial intermediaries and institutional investors. In turn, these investors will rebalance their portfolio in order to create a more counter-cyclical portfolio. This rebalancing behaviour increases the demand for counter-cyclical securities and decreases the demand for pro-cyclical securities. As a result, interest rates on cyclical securities – equities and low-grade private debt securities – will rise and interest rates on counter-cyclical securities – government bonds and high-grade private debt securities – will decrease.

Beside the model, the authors empirically test their hypothesis that QE leads to more portfolio rebalancing and thus a decrease in long-term interest rates. The authors find two effects of the portfolio-balancing channel. Firstly, the QE-policy increased the demand for counter-cyclical assets that are substitutes for the long-term Japanese government bonds, decreasing their interest rates. Conversely, this lead to a decline in demand for pro-cyclical assets, increasing their risk-premiums. The effect on pro-cyclical assets is small but significant and is conditional on close to zero policy interest rates. Namely, in above-zero interest rates situations the central bank can alter the policy interest rate downward to create a similar effect. Secondly, the QE-policy of the BoJ leads to a decrease in volatility and in corresponding returns in some asset markets, decreasing the overall market risk. In addition to the evidence provided in section 3.4.1, the evidence provided by Kimura and Small (2006) indicate that the portfolio-balancing channel is important for the functioning of QE.

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[9] Are loans where the borrower defaults on the payments.

[10] Either institutional investors or financial intermediaries.

4.2 Portfolio-balancing channel in a CAPM-setting

Kimura and Small (2006) propose a portfolio balancing model based on the CAPM[11]. The model is built around the premise that some financial asset prices rose while others fell. If the signalling channel were the driver of the changes in assets prices, one would expect every asset price to decrease. The authors use the CAPM to define the asset pricing and enter a measure of the market return as a function of business cycles into the model to illustrate the effects of the portfolio-balancing channel. Kimura and Small (2006) define the asset pricing according to the CAPM as follows:

$$E\left[r_t^j - r_t^f\right] = Cov\left[r_t^m, r_t^j\right] \frac{E\left[r_t^m - r_t^f\right]}{Var\left[r_t^m\right]}, \tag{4}$$

where r_t^j is the return on asset j, r_t^m is the return on the market portfolio, r_t^f is the risk free rate of return, $E\left[r_t^j - r_t^f\right]$ is the risk-premium on asset j, $E\left[r_t^m - r_t^f\right]$ is the risk-premium on the market portfolio, $Cov\left[r_t^m, r_t^f\right]$ is the covariance of the return on the market portfolio and the risk free rate of return, $Var\left[r_t^m\right]$ is the variance of the return on the market portfolio and $\frac{E\left[r_t^m - r_t^f\right]}{Var\left[r_t^m\right]}$ is the market price of risk. Assumed is that the market portfolio comprises of several types of assets, j, such as money, equities, government bonds and different types of corporate bonds. In a few steps, the authors rewrite equation (5) as (see appendix B for more details):

$$E\left[r_t^j - r_t^f\right] = k\rho\left[r_t^m, r_t^j\right]\sqrt{Var\left[r_t^j\right]}\sqrt{Var\left[r_t^m\right]}, \tag{5}$$

where k is constant and equal to the market price of risk, $\frac{E\left[r_t^m - r_t^f\right]}{Var\left[r_t^m\right]}$ and $Cov\left[r_t^m, r_t^f\right]$ is split into the correlation coefficient and the variances. Equation (5) expresses that the excess return on asset j, defined as the risk-premium on top of the risk free rate of return, depends on the volatility of the market and the correlation between asset j and the market, which means that if either change, the risk-premium on asset j changes as well.

To define the asset returns and to model the economic situation, i.e. an economic recession and low policy rates, in which the BoJ engaged in long-term asset purchases, the authors introduce a measure of business cycle downturns. For an expositional case, the authors assume that the *ex post* returns are governed by:

$$r_t^j = \lambda_0^j + \lambda_1^j Z_t + \varepsilon_t^j, \tag{6}$$

where Z_t is the variable for business cycles and ε_t^j captures the specific return of asset j. The authors assume that $Z_t < 0$, meaning that there is a business cycle downturn. Equation (6) shows that the economic situation of a country affects the return on asset j. The authors further assume that

[11] Original notation is used in this section, see appendix G for all symbols

$Cov[Z, \varepsilon_t^j] = 0, \forall j$. With the specification of the asset returns (equation (7)), the authors define the return on the market portfolio:

$$r_t^m = \lambda_{0,m} + \lambda_{1,m} Z_t + \varepsilon_{m,t} = \sum_{j=1}^{N} j_j \lambda_0^j + \sum_{j=1}^{N} w_j \lambda_1^j Z_t + \sum_{j=1}^{N} w_j \varepsilon_t^j, \tag{7}$$

where w_j is the share of asset j in the market portfolio and $\lambda_{1,m}$ is a measure of cyclicality of the portfolio. Assumed is that the market portfolio is pro-cyclical $\lambda_{1,m} > 0$.

As argued before, interest rates of pro-cyclical and counter-cyclical securities could be different. In the model, this means that λ_1^j differs between asset classes. For government bonds $\lambda_1^N < 0$ (for which j = N), meaning that these are counter-cyclical securities. Investors are more inclined to invest in these safe government bonds in an economic crisis. Investment behaviour like this is associated with a lower interest rate and capital gains in economic downturns due to the increased demand. Equities are more pro-cyclical than government bonds, implying that $\lambda_1^j > \lambda_1^N$ and $\lambda_1^j > 0$. The authors suggest that high-grade private debt securities behave like similar to government bonds and low-grade government bonds like equities. The following analysis assumes that government bonds and high-grade private debt securities are strictly negative, and equities and low-grade private debt securities are strictly positive.

The authors substitute equation (7) in equation (5) and hereby illustrate the effect of QE – a decrease in w_N – on the variance of the market portfolio and the covariance of asset j and the market, thereby illustrating the effect the workings of the portfolio-balancing channel.

4.2.1 Transmission channel 1.1: Changes in variance

The variance of the market portfolio is (see appendix C for the derivation):

$$var[r_t^m] = \left(\sum_{j=1}^{N} w_j \lambda_1^j\right)^2 var[Z_t] + \sum_{j=1}^{N} w_j^2 var[\varepsilon_t^j], \tag{8}$$

where both variances of the business cycle, $var[Z_t]$, and the asset specific return, $var[\varepsilon_t^j]$, are positive. Differentiating $var[r_t^m]$ with respect to w_N yields:

$$\frac{\partial var[r_t^m]}{\partial w_N} = 2\left(\sum_{j=1}^{N} w_j \lambda_1^j\right) \lambda_1^N var[Z_t] + 2 w_N var[\varepsilon_t^N]. \tag{9}$$

It means that the first term on the right-hand side is negative due to the assumption of $\lambda_1^N < 0$ and the second term is positive. There could be situations in which $2\left(\sum_{j=1}^{N} w_j \lambda_1^j\right) \lambda_1^N var[Z_t] > 2 w_N var[\varepsilon_t^N]$, which means that the variance of the market portfolio of financial intermediaries and institutional investors increase if the central bank engages in QE. Returning to equation (5). An increase in the market variance due to QE leads to a lower risk-premium in the market portfolio of financial intermediaries and institutional investors, creating an incentive to invest in new long-term or higher risk assets because the

Table 6

Affected part of the CAPM	Effect of LSAPs	
	Counter-cyclical securities	Pro-cyclical securities
Variance of the market portfolio	If $2\left(\sum_{j=1}^{N} w_j\lambda_1^j\right)\lambda_1^N var[Z_t] > 2w_N var[\varepsilon_t^N]$, than variance of the market portfolio increases for both counter-cyclical and pro-cyclical securities, leading to a decrease in the term-premium.	
Covariance of the market portfolio	Decreases for counter-cyclical securities, leading to a decrease in the term-premium.	Increases for pro-cyclical securities, leading to an increases in the term-premium.

Table 6 provides a summary of the findings of the paper of Kimura and Small (2006). LSAPs have an effect on the variance and the covariance of a portfolio of an investor. When the variance decrease due to LSAPs, the term-premium on counter-cyclical and pro-cyclical securities decreases. The covariance of the market portfolio decreases for counter-cyclical and increases for pro-cyclical securities leading to a respective decrease and increase of the term-premium.

financial intermediaries and institutional investors are business cycle risk averse, decreasing the term-premium long-term assets as defined in equation (2).

4.2.2 Transmission channel 1.2: Changes in covariance

The second result of the model are the changes in covariance. The authors give the covariance as follows:

$$Cov\left[r_t^m, r_t^j\right] = \lambda_1^j \lambda_{1,m} Var[Z_t] + w_j Var\left[\varepsilon_t^j\right]. \tag{10}$$

When differentiating $Cov\left[r_t^m, r_t^j\right]$ with respect to w_N the first-order conditions are:

$$\frac{Cov\left[r_t^m, r_t^j\right]}{\partial w_N} = \left(\lambda_1^N \lambda_1^N + \lambda_{1,m}\right) Var[Z_t] + w_j Var[\varepsilon_t^N] > 0, \qquad \forall j = N \tag{11}$$

$$\frac{Cov\left[r_t^m, r_t^j\right]}{\partial w_N} = \lambda_1^j \lambda_1^N Var[Z_t] \underset{<}{\overset{>}{=}} 0 \; as \; \lambda_1^j \underset{<}{\overset{>}{=}} 0, \qquad \forall j \neq N \tag{12}$$

Equation (11) states that outright purchases of long-term government bonds of the central bank will decrease the covariance of the market portfolio and government bonds, because $\lambda_1^N < 0$. This decreases the risk-premium in equation (5). The covariance of the market portfolio and high-grade private debt securities show a similar reaction ($\lambda_1^j < 0$), however the effect of the covariance of the market portfolio and purchases of high-grade private debt is less strong. The outright purchases of long-term government bonds will increase the covariance between the cyclical assets and the market portfolio ($\lambda_1^j > 0$), see equation (12). This will increase the risk premium in equation (5).

In conclusion, the model of Kimura and Small (2006) suggests two effects of QE as outright purchases of government bonds in a situation when there is a business cycle downturn and near-to-zero interest rates. Firstly, the variance of the portfolio of financial institutions or investors could increase, suggesting an increase in cyclicality of the portfolio. This will lead to portfolio rebalancing behaviour of these private agents and a subsequent decrease of the risk-premium on government bonds. Secondly, QE decreases the covariance between the market return and return on government bonds, meaning that the risk premium on government bonds will decrease. High-grade private debt securities show similar behaviour, albeit less strong. Equities and low-grade private debt securities, however, show opposite behaviour: purchases of government bonds lead to an increase in the risk-premium because investors seek to rebalance their portfolio with counter-cyclical assets. Table 6 presents a summary of the findings of the model of Kimura and Small (2006).

5 *Policy 1: Large-scale asset purchases (Gertler & Karadi, 2013)*

This section discusses the paper of Gertler and Karadi (2013), which serves as the basis for the basic illustrative model in the next section. The authors setup a DSGE-model, in which banks are able to invest in private securities and government bonds in order to model an LSAP as a monetary policy tool within a macroeconomic environment. This section only discusses the relevant mathematical setup of the model.

5.1 Setting of the paper

Gertler and Karadi (2013) propose QE as an LSAP and their model is based on the three periods of QE the Fed has initiated – see Appendix A. The purpose of their paper is to 'develop a macroeconomic model that presents a unified approach to analysing [QE] as a monetary policy tool' (Gertler & Karadi, 2013, p. 7). Their DSGE model presents a comprehensive economy with many actors. The most important actors are banks, the central bank and households. The illustrative model proposed in the current study only considers the important actors, meaning that the other actors are not presented. As a result only the important actors are discussed in more detail. There are two reasons for omitting these actors. Firstly, including these actors does not provide more information as to how the central bank affects the excess returns, and consequently the banks and households in this model. Secondly, these actors are included by Gertler and Karadi (2013) to create a general equilibrium. However, the current study is not concerned with investigating the general equilibrium, but with the basic mechanisms of QE.

The first actor is the central bank. In contrast to common believe, the authors argue that in a situation of economic crisis the central bank replaces part of the banks' intermediation to decrease the excess return that have occurred due to the crisis. The excess returns are defined as the difference between the rate of return in market with arbitrage friction and markets with frictionless arbitrage. When there is frictionless arbitrage, banks and households are able to fully capture the excess returns on the market, meaning that there are no such excess returns occur on the market. The intermediation of the central bank leads to a decline in excess returns. The central bank intermediation is as follows: the central bank sells short-term government debt and with the proceeds purchases private securities and government bonds in the secondary market. Central bank intermediation is less efficient than bank intermediation, the authors posit. The short-term debt the central bank issues is very safe and creates a way for the central bank to elastically obtain funds[12].

Banks are intermediaries in the model and are, in contrast to the central bank, unable to elastically obtain funds from the market due to an agency problem. Banks divert a portion of their net-equity to the owners of the banks, the bankers. However, the owners of the deposits, the workers, do not want these practices, because it increases the chance of the bank going bankrupt. If the bank goes bankrupt, the depositors lose their money. That is why the workers withdraw part of their deposits if the banks divert money to the bankers. This means that the bankers are only able to divert a specific amount of money from the bank before the workers punish the banks by withdrawing a part of the deposits.

Households have multiple functions in the model. Firstly, households supply money in the form of deposits to banks and in the form of short-term government debt to the central bank. This ensures that the central bank and the banks are able to function as intermediates. Secondly, households are able to directly invest into private securities and government bonds albeit not as efficiently as banks. Thirdly, households are owner of the banks and are willing to divert funds of the bank to the households. The authors make a distinction between households that are owners of the banks, bankers, and those who do not, workers.

In general, the authors argue that government bond purchases are less efficient than purchases of private debt securities because the limits to arbitrage are weaker in the markets for government bonds than in markets for private debt securities. Reason being that the liquidity in the market for government bonds is greater than that of the private securities markets. When a market is liquid, transactions are abundant and do not affect the prices of the assets (Brunnermeier & Pedersen, 2009). Notice that here the purchases of government bonds are less efficient due to the greater market liquidity, whereas Kimura and Small (2006) argue that these purchases are more efficient due to the counter-cyclicality of the long-term government bonds.

In conclusion, the authors find that the transmission to real output and inflation of QE is very similar to conventional monetary policy. Unlike conventional monetary policy, QE is useful at the zero-bound interest rates. In addition, the authors find that QE is effective in decreasing the long-term interest rates. Besides the reduction in long-term interest rates, short-term interest rates are less like to rise over a specific time-period due to the expectations of low interest rates. The remainder of this section reviews each of the three actors in more detail.

5.2 The model

Before reverting to the players in the model, the private securities and the government bonds in the market are discussed, in which households, banks and the central bank invest in. Private securities are

[12] This means that the supply of short-term government debt is always met by the demand for it.

loans to non-financial private companies that use it for investing in their own production process. The government issues government bonds in order to finance its deficit. Both are longer than one period and are therefore defined long-term securities. Note that all the signs used here are those used in the paper of Gertler and Karadi (2013). The total supply of both private securities and government bonds is as follows:

$$S_t = S_{pt} + S_{ht} + S_{gt} \tag{13}$$

$$B_t = B_{pt} + S_{ht} + S_{gt} \tag{14}$$

where S_{pt} and B_{pt} are the total amount of private securities and government bonds that are intermediated by banks, S_{ht} and B_{ht} are the total amount of private securities and government bonds held by households and S_{gt} and B_{gt} are the total amount of private securities and government bonds held by the central bank.

5.2.1 Households

All households consume, save and supply labour. The households save by lending funds to banks or to the central bank. More importantly, a household can be two types, either worker or banker. Workers earn labour income, whereas bankers manage banks and transfer any earnings to the households. At any time, there are $1 - f$ members of the household workers and f bankers; both types of members can switch occupation through time. There is a probability of σ that the banker will stay banker next period with an average survival time of $\frac{1}{1-\sigma}$. The model has a finite horizon to prevent bankers from cumulating enough resources to fund all investments with their own capital. The household provides new bankers with a start-up fund equal to $\frac{X}{(1-\sigma)f}$. The authors use this as a schematic representation of how households evolve throughout time; since in reality less-endowed households gain wealth and well-endowed households lose wealth.

The utility of the households is a follows:

$$u_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right], \tag{15}$$

where the utility of the household, u_t, consists of the current consumption, C_{t+i}, minus the discounted consumption of the previous period, hC_{t+i-1}, and the labour time weighted by a factor of relative preference to work, $\frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi}$. Here is χ the relative risk aversion of households and φ the inverse elasticity of work effort which is the sum of the substitution elasticity and a measure of people's willingness to trade work for consumption over time (Reichling & Whalen, 2012). Furthermore, E_t is the expectance operator and β^i is the subjective discount rate. The household utility function states that the utility of the households increase when present consumption relative to past consumption increase, but decreases when the households provide more labour.

The household budget constraint is as follows:

$$C_t = W_t L_t + \Pi_t - X + T_t + R_t D_{ht-1} - D_{ht}, \tag{16}$$

where the consumption, C_t, consist of the labour income, $W_t L_t$, the transfers from banks to households, Π_t, the total transfer for household members to become banker, X, the lump-sum tax from the government to the household, T_t, the gross real return on deposits from the period $t-1$, $R_t D_{ht-1}$, and the total quantity of short-term debt the households acquire, D_{ht}. The short-term debt of the households consists of deposits held at the bank and short-term government debt. Both the deposits and short-term government bonds are one period assets. Because the budget constraint in equations (16) does not allow for direct investment into securities, households have to use intermediaries to invest their money into securities. In the next step, the authors allow households to purchase securities directly without intermediation of banks. Adding the possibility to invest directly into securities gives:

$$\begin{aligned} C_t + D_{ht} + Q_t\left[S_{ht} + \frac{1}{2}k(S_{ht} - \overline{S_h})^2\right] + q_t\left[B_{ht} + \frac{1}{2}k(B_{ht} - \overline{B_h})^2\right] \\ = W_t L_t + \Pi_t - X + T_t + R_t D_{ht-1} + R_{kt} S_{ht-1} + R_{bt} B_{ht-1}, \end{aligned} \tag{17}$$

where the holding costs for private securities and government bonds are $\frac{1}{2}k(S_{ht} - \overline{S_h})^2$ and $\frac{1}{2}k(B_{ht} - \overline{B_h})^2$ respectively, the return from period $t-1$ for private securities and government bonds are $R_{kt} S_{ht-1}$ and $R_{bt} B_{ht-1}$ respectively, and the investments in new securities of both types are $Q_t S_{ht}$ and $q_t B_{ht}$ containing the respective market price and the amount invested at time t. The costs are introduced to prevent the households to engage in frictionless arbitrage, however, there are certain amounts of assets the households can hold costless. The authors argue that this cost structure is a simple representation of the limited participation in asset markets that lead to incomplete arbitrage. Finally, the discount rate of the budget constraint of the households is equal to the marginal rate of substitution of the households.

The first-order conditions are as follows – maximised to C_t, L_t, D_{ht}, S_{ht} and B_{ht}:

$$uC_t W_t = \chi L_t^{\varphi}; \tag{18}$$

$$E_t \Lambda_{t,t+1} R_{t+1} = 1, \qquad \Lambda_{t,t+1} = \beta \frac{uC_{t+1}}{uC_t}; \tag{19}$$

$$S_{ht} = \overline{S_h} + \frac{E_t \Lambda_{t,t+1}(R_{kt+1} - R_{t+1});}{k} \tag{20}$$

$$B_{ht} = \overline{B_h} + \frac{E_t \Lambda_{t,t+1}(R_{bt+1} - R_{t+1})}{k}. \tag{21}$$

Equation (18) states the choice of the households to supply labour, equation (19) specifies the preferences of the households for deposits. In equation (18) when the wage rate, W_t, increases, households are more inclined to provide more labour. If the expected safe short-term interest rate in equation (19), R_{t+1}, increases, households are more inclined to hold deposits. Equations (20) and (21) state that the demand for each asset above its frictionless capacity level is increasing when the excess returns are high conditional to the marginal costs. Important to note is that $\Lambda_{t,t+1}$, is the marginal rate of substitution of households, this is used as a discount rate for the households as well as for banks.

Especially equation (20) and (21) are interesting for the workings of QE. The equations state that households will invest an amount into securities that is at least equal to the frictionless amount. Investment above this amount depends on two aspects, the first is the excess returns on the specific securities and the second are the marginal costs. If the excess returns on securities will increase, households are more inclined to invest in the specific securities. The households are less willing to invest in securities if the marginal costs of investment increase.

5.2.2 Banks

Banks are able to fully participate in financial markets and have an intermediary function. The banks are constraint in how much money they can obtain from households due to the agency problem as aforementioned. The authors set up the optimisation problem for a single bank before aggregating banks. This section does not mention the aggregation of the banks, because it does not change the argument of the paper.

First, the single bank's utility function is a function of the net income of the bank and of an agency problem. The utility function is given by

$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}, \tag{22}$$

where σ is the probability that the banker stays a banker in the next period and n_{t+i} is the total amount of equity of the bank at time t+i. The discount rate of the bank, $\Lambda_{t,t+i}$, is equal to the marginal rate of substitution of the households. The bank's utility function says that if the chances of remaining a banker increase or the expected discounted wealth increases, the utility increases.

There are two constraints to which the utility function of the bank is constraint, the balance sheet constraint and the incentive constraint. These are:

$$Q_t s_t + q_t b_t = n_t + d_t, \tag{23}$$

$$where\ \ n_t = R_{kt} Q_{t-1} s_{t-1} + R_{bt} q_{t-1} b_{t-1} - R_t d_{t-1}, and \tag{24}$$

$$V_t \geq \theta Q_t s_t + \Delta\theta q_t b_t. \tag{25}$$

Where s_t and b_t are the amounts the bank invest in private securities and government bonds respectively, n_t is the net amount of equity of a bank at time t, d_t are the amount of deposits obtained from households, θ is the amount of funds the bank can divert of the private securities and $\Delta\theta$ is the amount of funds the bank can divert from the government bonds; $0 \leq \Delta < 1$. The authors define the diversion of funds like this, because the authors argue banks can more easily divert funds from their private loan portfolio because households have more difficulties in monitoring them than government bonds. Equations (23) and (24) are the bank's balance sheet constraint, stating that the bank's investments are constraint by the balance sheet of the bank and cannot invest unlimitedly. The left side of equation (25) depicts the total assets of a bank consisting of total investment in private securities and government bonds and the right side are the liabilities of a bank consisting of the net worth in equity of the bank and the total amount of received deposits. The equity of the bank evolves as equation (24) suggests the sum of the total return on private securities and government bonds of the previous period minus the cost of the deposits of the previous period. Equation (25) is the incentive constraint and effectively says that that banker can gain a specific sum equal to the right-hand side but when doing so loses in franchise value (the left-hand side).

The effect of the incentive constraint can be seen in the following equations – maximized with respect to R_{kt+1}, R_{bt+1} and R_{t+1}:

$$E_t\widetilde{\Lambda}_{t,t+1}(R_{kt+1} - R_{t+1}) = \frac{\lambda_t}{1+\lambda_t}\theta, \qquad \widetilde{\Lambda}_{t,t+1} \equiv \Lambda_{t,t+1}\Omega_{t+1}; \tag{26}$$

$$E_t\widetilde{\Lambda}_{t,t+1}(R_{bt+1} - R_{t+1}) = \Delta\frac{\lambda_t}{1+\lambda_t}\theta; \tag{27}$$

where the expected excess returns on private securities and government bonds are $E_t\widetilde{\Lambda}_{t,t+1}(R_{kt+1} - R_{t+1})$ and $E_t\widetilde{\Lambda}_{t,t+1}(R_{bt+1} - R_{t+1})$ respectively, λ_t is the lagrangian multiplier of the incentive constraint (equation (26)) and the $\widetilde{\Lambda}_{t,t+1}$ is the augmented stochastic discount factor of the bank – Ω_{t+1} is the shadow value of a unit of net worth to the bank at time t+1. If the incentive constraint is binding or not, has two interpretations. First, when the incentive constraint is not binding, i.e. $\lambda_t = 0, \forall\, t$, banks acquire assets to the point where the discounted return on each asset equals the discounted costs of these assets. A QE-programme of the central bank would in this situation only lead to a displacement of intermediation from banks and households to the central bank without any effect on the asset prices. Second, when the incentive constraint is binding, limits to arbitrage will emerge that could lead to positive excess returns in equilibrium, meaning that a QE programme in such a situation increases the demand for securities and thereby decrease the excess returns. Moreover, notice that the effect of QE on the excess returns of government bonds are smaller than the effect on the excess returns of private securities due to the definition of Δ.

5.2.3 Central Bank

The last agent, the central bank can purchase either safe private securities, S_{gt}, or long-term government bonds, B_{gt} – each for its respective market price. To finance these purchases the central bank issues riskless short-term debt securities. Contrary to banks, the central bank does not have an agency problem for it honours its debt. The balance sheet constraint of the central bank is:

$$Q_tS_{gt} + q_tB_{gt} = D_{gt}. \tag{28}$$

Equation (28) states that the central bank can expand its investments in securities equal to the expansion of short-term government debt. Here the authors assume that if the central bank makes any profits it is transferred to the treasury and money transfers from the treasury cover any losses.

Lastly, the authors note that arbitrage is weaker for government bonds and that QE should move the yield on government bonds less than the yields on private securities, this can be depicted as:

$$E_t\widetilde{\Lambda}_{t,t+1}(R_{kt+1}-R_{t+1}) = \Delta E_t\widetilde{\Lambda}_{t,t+1}(R_{bt+1}-R_{t+1}). \tag{29}$$

Equation (29) means that when the expected discounted excess returns on private securities decrease, the excess returns on government bonds only decrease by a fraction of it. This is contrary to the idea of Kimura and Small (2006), who argue that the effect of purchasing of government bonds is more effect due to the high counter-cyclicality of these securities.

In conclusion, Gertler and Karadi (2013) present a model in which actors interact with each other. The most important actors for the current study are households, banks and the central bank. The most important notions of the model are: i) in a financial crisis the excess returns on assets increases, ii) the central bank replaces part of the intermediation of banks in a crisis situation, decreasing the excess returns, iii) the households are able to participate in the financial market but not efficiently, iv) households are either workers or banks, meaning that different effects on both types of household members could occur and v) the central bank is able to elastically obtain funds from households, who supply funds to the financial intermediaries and the central bank. The general mechanism of the model is when excess returns of assets in the economy are high due to a financial crisis and banks are unable to fully capture these excess returns the central bank can decrease the excess returns in the economy by pursuing QE – effectively increasing the demand on these assets. Consequently, a decrease in the excess returns should lead to a decrease of investments in securities by the households and the banks. The authors, however, do not elaborate on the mechanism of how asset-purchases affect the excess returns, this is important for the basic illustrative model and will therefore be added.

6 *Policy 1: The basic illustrative model*

This section introduces the illustrative model. The model of Gertler and Karadi (2013) is taken as a departure. The illustrative model proposed in the current study adds two transmission channels, the portfolio-balancing channel, see section 4, and the signalling channel, see section 3. The portfolio-balancing channel is especially interesting, because it clearly illustrates the effects of QE-policy 1 and QE-policy 3 on the long-term interest rates. The signalling channel is interesting in the illustrative model, for it demonstrates the effect of QE-policy 1 on the short-term interest rates, an aspect that is missing in the model of Gertler and Karadi (2013). The model of Gertler and Karadi (2013) is simplified for a three reasons. Firstly, the current study is not interested in investigating a general equilibrium, but solely to the workings of QE. A simplification of their model therefore suffices to illustrate the effects of QE. This means that only the households, banks and the central bank are used in the current study. As a result, some variables presented in the previous section either are simplified or drop out entirely. Secondly, including more actors than the three aforementioned to the model probably does not lead to different results. The current study is interested in the transmission of QE to households. However, banks are added because they directly affect the households and are directly affected by QE. Thirdly, the additions of the two transmission channels require different mechanics than present in the model of Gertler and Karadi (2013). As a result, some mechanics unique to their model are omitted or changed to serve the two transmission channels. The notation is, where possible, equal to that in section 4 and 5, see appendix G for a description of the notation used.

Besides simplification, there are a few changes to the basic framework of Gertler and Karadi (2013). Firstly, the definition of the excess returns of Gertler and Karadi (2013) differs. As argued in section 3.4, QE should have an effect on the term-premium (Fawley & Neely, 2013) and not on the difference between the frictionless interest rate and the interest rate on a market with market frictions. The results may be very similar, though in the remainder of the study the excess returns are defined as the difference between the long-term interest rate and the short-term interest rate. This has the additional effect that the agency problem of banks is ineffective in affecting the excess returns and this therefore omitted from the model. Secondly, the framework of Gertler and Karadi (2013) does not elaborate on the mechanisms of QE and only argues that the central bank increases the total market demand in assets. To illustrate the effects of QE in more depth, the balance portfolio channel and the signalling channel are added to the model separately, meaning that the central bank is not defined as a player with an optimisation problem but enters exogenously as two separate channels. Consequently, an additional assumption is made, namely the central bank in unconstraint and the funding of the central bank does not return in this model; due to the new assumption, households only hold bank deposits. Thirdly, the long-term government bonds and the long-term high-grade private securities of the model of Gertler and Karadi (2013) are combined for reasons of parsimony and are henceforth called counter-cyclical securities. Notice that households and banks can only invest in these securities. Section 7.3 allows households and banks to investment in pro-cyclical securities. Fourthly, there are two types of interest rates, a short-term interest rate on deposits and long-term interest rate on counter-cyclical securities.

The remainder of the section is set up as follows. Firstly, the model setup is presented. Thereafter, the general results are presented. Lastly, the perturbation results are presented, here the results of the QE-policy are perturbed to check the effect of QE on the interest rates. These changes are then linked to the results of the households and banks leading to four propositions.

6.1 The model set-up

The model setup is divided in to the timing of the events, households, banks and the central bank.

Figure 3

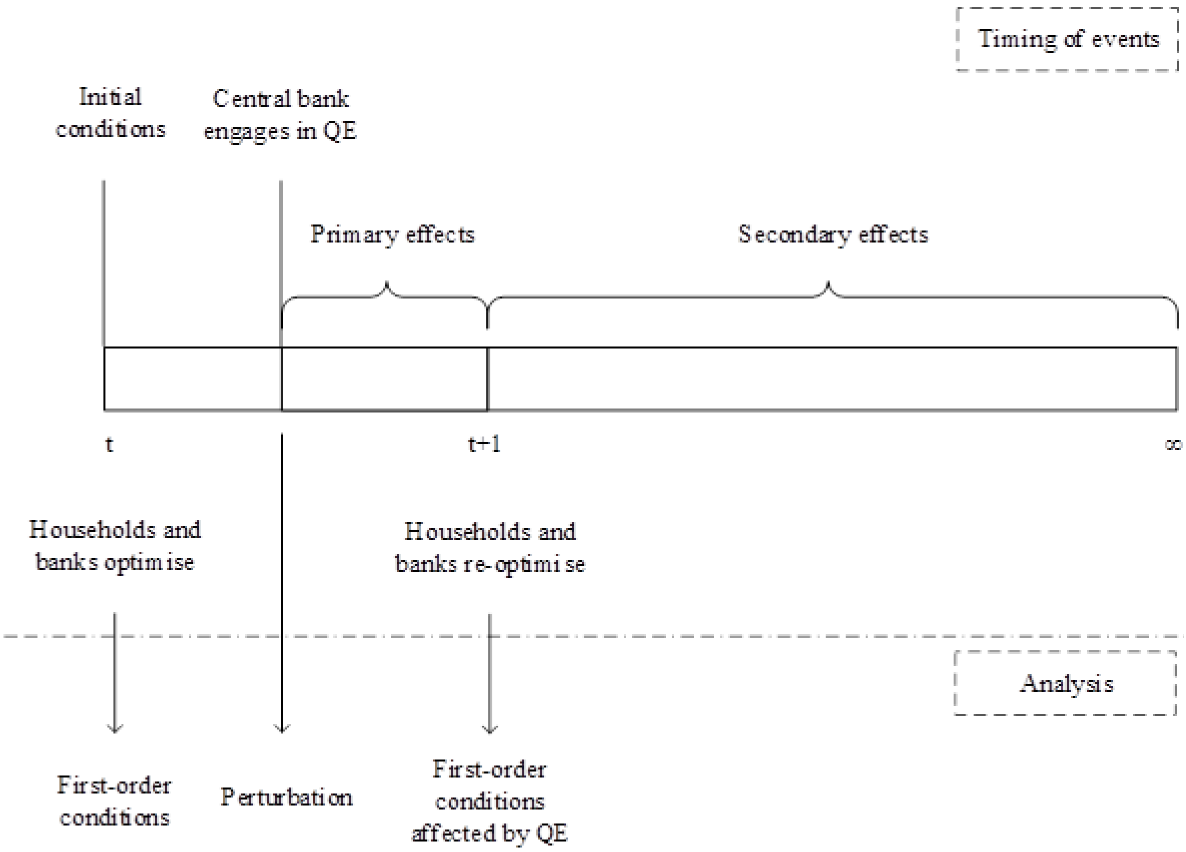

Figure 3 provides a timing of events of the model. The time span of the model is to infinity. For the analysis three moments are of special interest. At time is t households and banks optimise their relative optimisation problems, after this the central bank engages in QE, leading to the primary effects – i.e. before the households are able to react on the policy. At time $t + 1$ households and banks are able to react on the policy by re-optimising their respective optimisation problems. The corresponding analysis is as follows: firstly the first-order conditions are calculated, secondly, the QE-variables are perturbed and then the first-order conditions that are affect by the perturbation is discussed.

6.1.1 Timing of events

Unique to this illustrative model is the timing of events. The definition of the timing of the events is important for understanding the four propositions that follow in section 6.3. The model has an infinite time horizon of which three events are specifically important for the analysis. Figure 3 provides a schematic illustration of the timing of the events. At time is t, households and banks optimise their respective optimisation problems. Households choose their optimum level of labour, deposits and investments in counter-cyclical securities, whereas banks only choose their optimal level of counter-cyclical securities. After households and banks have optimised, the central bank commences the QE-policy. Important to notice is that at this point in time households and banks cannot react on the policy, meaning that the perturbation results of the QE-policy on the original equilibrium at time is t lead to the primary effects. After the central bank engages in QE, households and banks are able to react on the policy at time is $t + 1$. At this time households and banks re-optimise their respective portfolios based on the perturbation due to QE, leading to secondary effects of QE.

Notice that the time in the model is discrete, meaning that the time discontinuously jumps from one time-period to another. For illustrative purposes, this has the advantage that different effects can be studied without these effects happening almost simultaneously. The timing of events now has the advantage that it can show the primary and secondary effects in the market. The primary effects are purely the result of the QE-programme, leading to a change of the interest rates. The secondary effects are the sole effects of the reaction of households and banks on the policy, leading, possibly, to a different effect on the interest rates. Because there is a response time between the engagement in QE and the reaction of households and banks, the primary and secondary effects do not coincide. The length of the response time, however, is unknown. Such a setup illustrates what happened in practice when central banks engaged in QE. There was a primary shock when central banks started their programmes and through time, but when agents reacted on the policy, the shock was diluted (Christensen & Rudebusch, 2012; Schenkelberg & Watzka, 2013). This means that there first is a large decrease in the long-term interest rate before settling in a lower level than *ex ante* QE. This setup also helps to disentangle the effects of QE on income inequality.

6.1.2 Households

Households in the model are very similar to the households in Gertler and Karadi (2013), in which households consume, provide labour, save and invest on the secondary market. There are two types of households, workers (w) and bankers $(1-w)$. Workers receive labour income, $W_t L_t$ in equation (32). Bankers receive additional income from the banks where they work, Π_t^{1-w}. Furthermore, it is assumed that workers cannot invest directly invest in counter-cyclical securities, B_t^h, whereas bankers can. This assumption is made to illustrate the different endowments of households in the economy that could lead to an alteration in the distribution of income. The simplified household utility function is as follows:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i [uC_{t+i} - \alpha L_{t+i}], \tag{30}$$

where the utility of the household, U_t, depends on the utility of consumption of the households, $\mathrm{uC_{t+i}}$, minus the relative utility lost by providing labour, αL_{t+i}. Here both uC_{t+i} and αL_{t+i} are convex functions, where their respective first-order and second-order conditions are greater than zero and smaller than zero. The E_t is the expectance operator at time t and β^i is the subjective discount factor. Comparing the utility function to that of Gertler and Karadi (2013), equation (16), notice that here no utility is gained from past consumption, because the current study argues that past consumption has no effect on current utility. Including such a system is useful when modelling rational addiction, see Becker and Murphy (1988). Rational addiction is not present in this model and is therefore excluded. Additionally, the structure of adding past consumption to a utility function is not generally used. Furthermore, the utility lost due to labour is simplified, because QE does not affect the labour income directly and it suffices to simplify it for the analysis. Labour income is thus included in the model to illustrate the difference in income between workers and bankers. QE can affect labour income by affecting the productivity of non-financial firms (Gertler & Karadi, 2013). In short, QE decreases the costs of capital for non-financial firms that increase investments in capital goods. These investments increase the demand in labour, increasing the wage of labour. As a results labour income increases. However, Gertler and Karadi (2013) illustrate that the increase in capital investments by non-financial firms is not instantaneous and can take some time periods. This means that this mechanism is not active at $t+1$ in this model and is therefore not a necessity.

The budget constraint of the households is left relatively unchanged compared to the model of Gertler and Karadi (2013) (equation (17)). In the model of Gertler and Karadi (2013) the taxes are added to fund the government, but because the government is not present in this model, the taxes are omitted. Because the government does not invest in counter-cyclical securities, it is not added to the model. In addition, it is assumed for illustrative reasons that bankers and workers are not able to change status. Changing of function between bankers and workers does not affect the workings of QE in this model. This means that the transfers from a worker to become a banker, X, drops out. It is therefore assumed that the amount of workers w and bankers $1-w$ is fixed. Furthermore, as argued earlier, the counter-cyclical securities are taken as one. The one-period new budget constraint is as follows:

$$\begin{aligned} C_t = W_t L_t + \Pi_t^{1-w} + r_t^d D_{t-1}^h + r_t^b B_{t-1}^{1-w} - D_t^h \\ - p_t \left[B_t^{1-w} + \frac{1}{2} k \left(B_t^{1-w} - \overline{B^{1-w}} \right)^2 \right], \end{aligned} \tag{31}$$

$$\Lambda^i = \beta \frac{u' C_{t+1}}{u' C_t}. \tag{32}$$

Where D_t^h is the total amount of deposits and short-term government debt held by all types of households in which $h = w + (1-w)$, r_t^d is the nominal risk free interest rate on deposits, B_t^{1-w} are the counter-cyclical securities in which only bankers can invest in and r_t^b is the nominal return on counter-cyclical securities. In addition, p_t is the respective market prices of counter-cyclical securities. Notice here that the household budget constraint is one period, when made intertemporal all future values of the budget constraint are discounted by the households' marginal rate of substitution, equation (32), which is assumed constant over time – see Appendix D for the intertemporal household budget constraint.

An addition to the framework of Gertler and Karadi (2013), a transversality condition imposed. Acting as a boundary condition, the transversality condition provides a limit to the optimisation problem and is needed when working with Hamiltonians (Chiang & Wainwright, 2005). It prevents stock from being overused or used less when time runs to infinity. The imposed transversality condition is as follows:

$$\lim_{I \to \infty} \Lambda^I D_{t+I+1}^h = 0, \tag{33}$$

where the discounted investment in deposits, $\Lambda^i D_{t+I+1}$, is zero when time reaches to infinity. This condition is necessary, otherwise households would not consume their entire wealth when reaching infinity if $\lim_{I \to \infty} \Lambda^I D_{t+I+1} > 0$ or consume more than their wealth when reaching infinity if $\lim_{I \to \infty} \Lambda^I D_{t+I+1} < 0$. Both scenarios are improbable and thus the above transversality condition is defined. A similar argument can be made for investments in counter-cyclical securities, therefore a second transversality condition is imposed:

$$\lim_{I \to \infty} \Lambda^I B_{t+I+1}^{1-w} = 0. \tag{34}$$

6.1.3 Banks

The banks in the model are also similar to the banks in the model of Gertler and Karadi (2013). The utility function is simplified compared to what Gertler and Karadi (2013) present. It is still a function of the maximisation of the net-equity of the bank, but the agency problem is omitted because the non-frictionless arbitrage of banks does not lead to excess returns as argued earlier. The new utility function of the aggregate bank is as follows:

$$V_t = E_t \sum_{i=0}^{\infty} \Lambda^i N_{t+i}, \tag{35}$$

where Λ^i is the discount rate of the bank and is equal to the marginal rate of substitution of households, equation (32) and N_{t+i} is the total net-equity of the bank at time $t+i$. Notice that, compared to the banks' utility function of Gertler and Karadi (2013), equation (22) the condition $\sum_{i=1}^{\infty}(1-\sigma)\sigma^{i-1}$ drops out, because of the previously made assumption that bankers and workers cannot change positions.

The banks' balance sheet constraint has slightly changed from the balance sheet constraint of Gertler and Karadi (2013), because the counter-cyclical securities are taken as one. Firstly, the banks' balance sheet here is taken as an aggregate of all banks in the model's economy. Secondly, the way the equity of the bank builds up has changed – equation (37). Here the definition of Gertler and Karadi (2010) is used, because it more clearly depicts the effect of excess returns on the net equity of the bank. The one-period banks' balance sheet constraint is as follows:

$$p_t B_t^p = N_t + D_t^h; \tag{36}$$

$$N_t = \left(r_t^b - r_t^d\right) p_{t-1} B_{t-1}^p + r_t^d N_{t-1}. \tag{37}$$

Here $p_t B_t^p$ is the total value of banks' investments in counter-cyclical securities. Because of omitting the agency problem in the banks' maximisation problem, the incentive constraint as defined by Gertler and Karadi (2013) drops out. Similar to the households, the banks' balance sheet constraint is one period; appendix D introduces the intertemporal balance sheet constraint.

Important to note is that the income of the banker, Π_t^{1-w}, depends on the amount of profit – or the increase in net-equity, $\left(r_t^b - r_t^d\right) p_{t-1} B_{t-1}^p$ – of the bank itself. When the profit is high, the banker will receive a relatively high amount of income. This is a representation of the bonus-culture within banks. This relationship is not modelled explicitly for reasons of parsimony.

As with households, banks have an imposed transversality conditions as a boundary condition:

$$\lim_{I\to\infty} \Lambda^{I} B^{p}_{t+I+1} = 0, \tag{38}$$

meaning that when time approaches infinity the banks' investments in counter-cyclical securities is equal to zero. Were this either greater than or smaller than zero than the banks would either increase their balance sheets with more than infinity or with less than infinity.

6.1.4 *Central bank*

Before the QE channels are discussed, the market is discussed. Similar to Kimura and Small (2006) the market consists multiple assets, namely:

$$Y^{T}_{t} = B_{t} + A_{t}. \tag{39}$$

Where Y^{T}_{t} is the total assets in the market, B_t are the total counter-cyclical securities consisting of long-term government bonds and long-term high-grade private securities, and A_t are the total pro-cyclical securities consisting of long-term low-grade private and equities. The qualification of the securities are very similar to Kimura and Small (2006), only now the economy is moneyless.

When the central bank purchases counter-cyclical securities, they do not return on the market at a later point. The amounts of counter-cyclical securities in the market are defined as follows:

$$B_{t} = B^{p}_{t} + B^{1-w}_{t} - B^{g}_{t}, \tag{40}$$

Where B^{p}_{t} are the counter-cyclical securities that are intermediated by banks, B^{1-w}_{t} are the counter-cyclical securities that are held by bankers and B^{g}_{t} are the counter-cyclical securities purchased by the central bank. In general, the interest rates in this model are nominal and not real interest rates, in contrast to Gertler and Karadi (2013).

Besides the definition of the market, the interest rates are defined. The central bank is able to purchase counter-cyclical securities on the market without limitations and is able to affect the interest rate via either the portfolio-balancing channel or the signalling channel. The nominal long-term interest rates on counter-cyclical securities are as follows:

$$r^{b}_{t} = r^{d}_{t} + \tau^{b}_{t,n}, \tag{41}$$

where the nominal long-term interest rate, r^{b}_{t}, contains the nominal short-term risk free interest rate, r^{d}_{t}, and the term-premium on the counter-cyclical securities, $\tau^{b}_{t,n}$. The nominal short-term interest rate depends on the policy interest rate of the central bank that is zero or negative in time of crisis and the expectations of the market, see equation (44). The policy rate is assumed stable so that only the expectations can alter this interest rate.

The prices of counter-cyclical securities are the inverse of the respective nominal long-term interest rate, hereby capturing the relationship between the return on bonds and the price of bonds:

$$p_{t} = \frac{1}{r^{b}_{t}} = \frac{1}{r^{d}_{t} + \tau^{b}_{t,n}}. \tag{42}$$

Transmission channel 1: Portfolio-balancing channel

The portfolio-balancing channel is adapted from Kimura and Small (2006) and is defined as follows:

$$\begin{aligned}\tau_{t,n}^{j} &\equiv E\left[r_t^j - r_t^d\right] = Cov\left[r_t^m, r_t^j\right]\frac{E\left[r_t^m - r_t^d\right]}{Var\left[r_t^m\right]} \\ &= \{\lambda_1^j\lambda_1^m Var[Z_t] \\ &\qquad + Y_t^j Var\left[\varepsilon_t^j\right]\}\frac{E\left[\sum_{j=1}^N Y_t^j\lambda_0^j + \sum_{j=1}^N Y_t^j\lambda_1^j Z_t + \sum_{j=1}^N Y_t^j\varepsilon_t^j - r_t^d\right]}{\left(\sum_{j=1}^N Y_t^j\lambda_1^j\right)^2 var[Z_t] + \sum_{j=1}^N \left(Y_t^j\right)^2 var\left[\varepsilon_t^j\right]}.\end{aligned} \tag{43}$$

Where $E\left[r_t^j - r_t^d\right]$ or $\tau_{t,n}^j$ is the expected term premium on asset j, r_t^d is the short-term interest rate and Y_t^j is the amount of asset j available in the market[13]. It is important to stress that the portfolio-balancing channel only affects the term-premium by changing the specific return on a counter-cyclical security and not the short-term safe interest rate.

The portfolio-balancing channel comes forward in this model as follows. The central bank purchases counter-cyclical securities, B_t^g. With this the total amount of these securities in the market decreases – see equation (40). In turn, this will alter the total market supply in equation (39), altering the term-premium through Y^j in equation (41). Remind that this channel is able to function because institution investors rebalance their portfolio when QE occurs. These institutional investors and their effect are assumed exogenous to the model.

Transmission channel 2: Signalling channel

The second central bank channel is the signalling channel. The central bank is credible and is able to affect the expectations of the short-run interest rate even though it is close to zero. The expectations are built as

$$r_t^d = \frac{1}{n}\left[r_t + E_t\sum_{j=1}^{n-1} r_{t+j}\right] = \frac{1}{n}[r_t + X_t], \tag{44}$$

where the nominal short-term risk free interest rate is a combination of the current nominal short-term interest rate and the cumulative expectations of the future nominal short-term interest rate, $E_t\sum_{j=1}^{n-1} r_{t+j}$. When the central bank engages in QE, market participants expect the nominal short-term risk free interest rates to stay low for a longer period of time, decreasing $E_t\sum_{j=1}^{n-1} r_{t+j}$ or X_t, and with it the current nominal short-term risk free interest rate.

[13] Asset j is either the counter-cyclical securities or the pro-cyclical securities.

6.2 The general solution

This subsection discusses the first-order conditions as a general solution of the model. The derivation of the first-order conditions and the bordered hessians can be found in appendix D and E. Next subsection formulates the propositions following these results.

To formulate the first-order conditions for households and banks, the Hamiltonian method is used. Though very similar to the Lagrangian method, this technique takes into account the time dimension (Chiang & Wainwright, 2005). Where the solving technique of the Hamiltonian differs from the Lagrangian is the needed transversality condition, equations (33), (34) and (38). To check whether the solutions are optimal, the corresponding bordered hessians are calculated[14] – see appendix D. The channels of QE are solved by means of the implicit function theorem, which is used to solve implicit functions as the two QE channels are. In appendix E, the details are shown.

The solution strategy for this and next subsection is as follows: the first-order conditions for the households, banks and the QE-channels are derived. The results of the QE-policy are with respect to the respective interest rates and are perturbed to see how QE affects the interest rates. Hereafter, the perturbations are connected with the first-order condition of households and banks. In other words, the perturbation changes the initial equilibrium of households and banks at time is t. At time $t+1$, households and banks re-optimize their portfolios based on the new QE-policy, this will lead to different investments in counter-cyclical assets and deposits. The answers to these questions are formulated into four propositions. This strategy of solution is chosen because it clearly and sufficiently illustrates the effects of QE.

6.2.1 Households

The households maximise their utility (equation (30)) substitute to the budget constraint (equation (31)) and the transversality conditions (equations (33) and (34)) with respect to $C_t, L_t, D_t^h, B_t^{1-w}$ and λ. Let $u'C_t$ and $u'C_{t+1}$ be the first derivative of the utility of consumption at the respective period and $\alpha' L_t$ the first derivative of the utility of labour, then the first-order conditions are:

$$u'C_t W_t = \alpha' L_t; \tag{45}$$

$$E_t \Lambda^i r_{t+1}^d = 1 \tag{46}$$

$$B_t^{1-w} = \overline{B^{1-w}} + \frac{E_t \Lambda^i r_{t+1}^b}{k p_t} - \frac{1}{k} = \overline{B^{1-w}} + \frac{E_t \Lambda^i r_{t+1}^b r_t^b - 1}{k}. \tag{47}$$

The first two equations are very similar to Gertler and Karadi (2013). Equation (45) states the relative preferences of households to consume and provide labour and equation (46) states the relative preference of holding deposits. If the wage rate in equation (45) increases, the relative utility of labour of households increase, meaning that households are inclined to provide more labour. If the expected short-term safe deposit rate in equation (46) decreases households are less inclined to put money into deposits.

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[14] to obtain an optimal solution the bordered hessians should be smaller than zero

Equation (48) is somewhat different form Gertler and Karadi (2013), stating that bankers increase their investment in counter-cyclical securities if the expected next-period return increases, relative to the marginal costs of investment. An increase in the current return on securities can also be seen as a decrease in the price of the securities, which leads to lower investment. The marginal costs are assumed stable overtime.

To check whether the results of the households are optimal and the bordered hessians are constructed, see Appendix D for more details. It was found that the utility function of consumption and labour are correctly specified and that the results are optimal.

6.2.2 Banks

The banks maximise their utility function (equation (35)) subject to the constraints given equations (36) and (37), and the transversality condition (equation (38)) and maximise with respect to B_t^p. Their first order condition is as follows:

$$\mathrm{E}_t \Lambda^{\mathrm{i}} \left(r_{t+1}^b - r_{t+1} \right) = 1 \tag{48}$$

Equation (48) states that if the excess return on any security increases, the banks' investments will increase accordingly. Hereafter the solution is check whether it is optimal. Checking the bordered hessian, it is found that the solution is optimal. See appendix D for more details.

6.2.3 Central bank

In the model QE is represented as the two channels discussed above. The first-order conditions, which are derived according to the implicit function theorem, are discussed separately. Because there are no endogenous variables, the results of the implicit function theorem are equal to a normal derivation. See appendix E for details about the derivations.

Transmission channel 1: The portfolio-balancing channel

In this channel, QE is seen as an increase in B_t^g in equation (40). Before maximising equation (43) with respect to B_t^g, equation (40) is substituted into equation (39), which is then substituted into equation (42). Notice that $Z_t < 0$, $\lambda_1^b < 0$, which means that the economy is in an economic downturn, and that the securities purchased by the central bank are counter-cyclical.

The first-order conditions for the purchase of counter-cyclical securities with respect to the long-term interest rate is:

$$\begin{aligned} \frac{dr_t^b}{dB_t^g} = \{ & -\left(\lambda_1^b \lambda_1^b + \lambda_1^m\right) Var[Z_t] \\ & - Var\left[\varepsilon_t^b\right] \} \frac{E_t\left[-(\lambda_0^b + \lambda_1^b Z_t + \varepsilon_t^b)\right]}{-2\left(\sum_{j=1}^N \lambda_1^j\right) \lambda_1^b var[Z_t] + 2(-B_t^g) var[\varepsilon_t^b]} \end{aligned} \tag{49}$$

In general, equation (49) shows that when the central bank engages in QE by purchasing counter-cyclical assets, the long-term interest rate decreases. To explain the effect of QE on the term-premium, equation (49) is split into three: the covariance, the variance and the excess return of the market portfolio.

The variance of the purchase of long-term government bonds is as follows:

$$\frac{\partial Var[r_t^m]}{\partial B_t^g} = -2\left(\sum_{j=1}^{N} \lambda_1^j\right)\lambda_1^b var[Z_t] + 2\left(-B_t^g\right)var\left[\varepsilon_t^b\right]. \tag{50}$$

The first term on the right-hand side of the equation is strictly positive, which means that the purchase of counter-cyclical securities in an economic downturn increases the variance of the market portfolio. The second term on the right-hand side is negative, which means that the variance of the counter-cyclical securities decreases and with it the variance of the market portfolio. In a situation where the first term on the right-hand side is more positive than the second term, than the variance of the market portfolio increases, leading to a decrease of the term-premium.

The first-order condition of the covariance is as follows:

$$\frac{\partial Cov\left[r_t^m, r_t^b\right]}{\partial B_t^g} = -\left(\lambda_1^b\lambda_1^b + \lambda_1^m\right)Var[Z_t] - Var\left[\varepsilon_t^b\right] < 0. \tag{51}$$

This first-order condition states that when the central bank purchases counter-cyclical securities, the covariance between the market return and the return on counter-cyclical securities decreases. This leads to a decrease in the long-term interest rates.

The final first order condition is that of the expected excess return on the market portfolio:

$$\frac{\partial E_t\left[r_t^m - r_t^d\right]}{\partial B_t^g} = -\left(\lambda_0^b + \lambda_1^b Z_t + \varepsilon_t^b\right). \tag{52}$$

Here is the value of $Z_t < 0$, which means that the excess return of the market portfolio only decreases when $\lambda_1^b Z_t < \lambda_0^b + \varepsilon_t^b$.

Transmission channel 2: Signalling Channel

Channel 2 of QE operates is the signalling channel. The first-order condition of this is:

$$\frac{dr_t^d}{dX_t} = \frac{1}{n}X_t' > 0. \tag{53}$$

Here is X_t' the first derivative of X_t. In essence, equation (53) says that if the expectations of the future short-term interest rate decrease, the current short-term interest rate decreases as well.

6.3 Perturbation results

The propositions are split into two parts, the first part discusses the effects of the portfolio-balancing channel, and the second part discusses the effects of the signalling channel. The two channels are not mutually exclusive, meaning that in practice both channels may be observed and work simultaneously.

Table 7

Type of Quantitative Easing		Trans-mission channel	Perturbation variables	Propositions
Policy 1:	Large-scale long-term asset purchases	Portfolio balancing channel	B_t^g	1. Long-term interest rate decreases at time t Income inequality reduces 2. Long-term interest rate slightly increases at time $t+1$ Income inequality deteriorates
		Signalling channel	X_t	3. Short-term and corresponding long-term interest rate decreases at time t Effect on income distribution is ambiguous 4. Long-term interest rate slightly increases at time $t+1$ Income inequality deteriorates

Table 7 provides the perturbation table of section 6 including the corresponding propositions. In the subsequent analysis both channels of QE have different perturbation variables. The perturbation of these variables lead to two propositions for each channel, the first proposition of each is at time is t and the second of each is at time $t+1$.

Table 5 describes which variables in the model are perturbed and lead to what propositions. In the portfolio-balancing channel the counter-cyclical securities the central bank purchases, B_t^g, is perturbed – an increases in B_t^g leads to a decrease in the long-term interest rate. In the signalling channel, the expectations, X_t, is perturbed – a decrease in the expectations leads to a decrease in the short-term and long-term interest rate. Both perturbations lead to two propositions each. The first and third proposition are at time t, see figure 3, and checks the perturbation of both variables on the existing equilibrium. The second and fourth proposition are at time $t+1$ when the households and banks are able to react on the perturbation of B_t^g and X_t.

6.3.1 Transmission channel 1: The portfolio-balancing channel

Proposition one expresses the effect of the portfolio-balancing channel on the next-period returns of households and banks.

From equation (49) a situation could arise in which $\frac{dr_t^b}{dB_t^g} < 0$, meaning that if the central bank increases its holdings in counter-cyclical securities, B_t^g, the long-term interest rate, r_t^d, decreases. The long-term interest rates come forward in two separate equations, namely equation (47) and (48). Equation (47) shows that the holdings of counter-cyclical securities by households depend on the long-term interest. Equation (48) shows a similar argument but for banks, here the investments depend on the excess returns, which is defined as the difference between the short-term and long-term interest rate. By both, when the long-term interest rate decreases, the excess returns decrease and with it the next-period return on the investments in counter-cyclical securities. This is due to the fact that the households and banks are unable to react on the QE policy.

The change in long-term interest rates lead to two changes in the distribution of income. Firstly, because only bankers receive money from the investments in counter-cyclical securities only their next-period income is affected and not the income of the workers – see equation (31). Secondly, banks see a decrease in their growth of net-equity – see equation (37). Because the additional income of bankers depends on the growth of the net-equity, the bankers will see a decrease in their additional income. Both changes lead to a decline in income inequality and possibly to a decrease if the long-term interest rates turn negative.

Proposition one:
When the portfolio-balancing channel is salient, the return on counter-cyclical securities and the additional income of the bankers decreases, leading to, at least, a slow-down in the growth of income inequality and possibly a decline in income inequality.

The second proposition of the portfolio-balancing channel comes forward when the households and banks are able to react on the QE-policy. As equations (47) and (48) show, when the long-term interest rate decreases, households and banks will decrease their position in long-term securities for period $t+1$. The central bank keeps committed to it policy, meaning that B_t^g does not change.

The re-optimisation of households and banks lead to the following effect. Bankers and banks sell part of their position at $t+1$ for an increased price and therefore capitalise on this price increase, increasing their overall wealth[15]. When doing so, demand on the long-term securities decreases, increasing the long-term interest rate, but not by as much as *ex ante* the QE-policy. This is because the market holds less counter-cyclical securities, which, as equation (49) describes, leads to a higher variation of the market, leading households and banks to seek counter-cyclical securities relative to their return than previously.

Even though the interest rates on long-term securities decrease, bankers and banks are, to some extent, able increase their wealth by capitalising on the increased prices. This means that only bankers and banks see this rise in wealth, whereas the workers do not, because the return on deposits and labour income remain unchanged, income inequality increases. This leads to the following proposition:

Proposition two:
When the portfolio-balancing channel is salient, bankers and banks will decrease their position in counter-cyclical securities ex post, with it capitalising, to some extent, on the increased prices of these long-term securities, leading to a slight decrease in the prices of the counter-cyclical securities and to an increase in the income inequality between workers and bankers.

[15] See that the price of a long-term security is the inverse of the interest, equations (41)

6.3.2 Transmission channel 2: The signalling channel

The first proposition of the signalling channel has three effects at time is t: i) it alters the next-period return on counter-cyclical securities, ii) it decreases the next-period return on deposits and iii) it changes the growth in net-equity of the bank.

The first effect is that the signalling channel changes the return on counter-cyclical securities. Equation (53) states that there could be a situation in which $\frac{dr_t^d}{dX_t} > 0$, meaning that if the short-term interest rate expectations decline, the short-term interest rate follows correspondingly. Combining equation (53) with (41), a decline in the short-term interest rate leads to a similar decrease in the long-term interest rate. This has the effect that the next-period return on counter-cyclical securities for bankers and banks decreases, decreasing the growth in income inequality.

The second effect of the signalling channel is that the return on deposits in $t+1$ decreases similarly as counter-cyclical securities – see equation (31). The decreased return on deposits has the effect that it can increase the growth of income inequality, because the short-term interest rates decrease. Since the decrease in long-term interest rates and short-term interest rates is equal, a change in both lead to no net effect change in income inequality – both workers and bankers have a lower next-period return.

The third effect is with respect to the income of the banks and bankers. The next-period return on counter-cyclical securities declines for banks, decreasing the profit of the bank. The costs of deposits, though, also decrease due to the lower short-term interest rate, increasing the profit of the bank. Depending on how much of the banks' investments at time is t are financed by equity and deposits (see equation (37)) the next-period profit of the bank can either increase or decrease, leading to either an increase or decrease of the income of the banker, Π_t^{1-w}.

Proposition three:

When the signalling channel is salient, the return on counter-cyclical securities and deposits, and the costs of deposits for banks decrease. The equal decrease in short-term and long-term interest rates and the uncertain effect of the banks' profitability on the income of bankers lead to an ambiguous effect on the growth of income inequality.

Proposition four is very similar to proposition two. The difference between proposition two and four is that the effect on income inequality in proposition four is stronger than in proposition two. Bankers and banks are able to capitalise on the increased prices in long-term securities, increasing the growth of income inequality and, possibly, changing income inequality altogether. Also applicable to this channel is that the decrease in demand for long-term securities by bankers and banks will lead to lower long-term security prices. Different in the signalling channel is that the short-term interest rate on deposits have decreased, lowering the total next-period return of workers either because they hold less deposits or receive less a return. In all this means that the while the bankers and banks are able to capitalise on the increased prices of long-term securities, the workers only see a decrease in there next-period return. This has the effect that the growth in income inequality will increase. The proposition is as follows:

Proposition four:

When the signalling channel is salient, bankers and banks will ex post sell part of their position in counter-cyclical securities, with it capitalising, to some extent, on the increased prices of these securities, leading to a slight decrease in the prices of the counter-cyclical securities. Thereby, the return of deposits of workers will decrease, increasing the income inequality between workers and bankers more so than in the portfolio-balancing channel.

7 Additions to the basic illustrative model

Previous section introduced the basic illustrative model in a situation of QE-policy 1 in which households could invest either in long-term counter-cyclical securities or in deposits. This section introduces three additions to the basic framework of the model. Firstly, it introduces the two other types of QE, namely ILFs and operation twist. Secondly, this section allows households and banks to invest into pro-cyclical assets in addition to the basic illustrative model. These three additions are separately added to the basic illustrative model of section 6 but are not mutually exclusive. Appendix F provides the mathematical details and appendix G provides the elaboration on the symbols.

Table 6 provides information about which variables are perturbed per addition to the basic illustrative model and introduces six new propositions. In the remainder of the section the perturbation of the other propositions are at time $t + 1$ when households and banks re-optimise based on the perturbed variables.

7.1 Policy 2: Improved central bank lending facilities

In the illustrative model, QE-policy 2 would directly facilitate the banks by increasing the amount that can be borrowed by banks at the central bank. This means that the banks optimisation problem has to be altered; households are indirectly affected by this policy. In section 6, the banks' balance sheet constraint (equation (36)) contains assets in which is invested, and as liabilities net-equity (equation (37)) and deposits. In order to add the improved central bank lending facilities, a third form of liabilities has to be added, namely borrowings from the central bank, see equation (54). Banks use these central bank borrowings to partly finance their investments in counter-cyclical securities, however, the central bank borrowings are constraint by the amount of assets banks have that function as collateral. This means that the more collateral the banks have, the more they can borrow from the central banks and the more they are able to invest in counter-cyclical securities. When the central bank improves the lending facility, it effectively increases the collateral pools of the banks and increases the total amount a bank can borrow

Table 8

Type of Quantitative Easing		**Trans-mission channels**	**Perturbation variables**	**Propositions**
Policy 2:	Improved central bank lending facilities		θ	5. Long-term interest rate decreases at time t Income inequality reduces 6. Long-term interest rate slightly increases at time $t+1$ Income inequality deteriorates
Policy 3:	Operation twist		B_t^g, D_t^g	7. Long-term interest rate decreases at time t Short-term interest rate increases at time t Income inequality reduces 8. Long-term interest rate slightly increases at $t+1$ Short-term interest rate slightly decreases at $t+1$ Income inequality deteriorates
Extension to policy 1:	Large-scale long-term asset purchases including pro-cyclical assets	Portfolio balancing channel	B_t^g	9. Long-term interest rate on pro-cyclical securities increases at time t Income inequality deteriorates 10. Long-term interest rate on pro-cyclical securities slightly decreases creases at time $t+1$ Income inequality still deteriorates though less quickly as before
		Signalling channel	X_t	No new propositions, same effects as with counter-cyclical securities

Table 8 provides the perturbation table of section 7 including the corresponding propositions. Each type of QE and the relevant channel has its own perturbation variables. These, as in section 6, lead to two propositions per type of QE. The first propositions are formulated at time is t and the second at time is $t+1$.

per unit of collateral. Because banks are only able to invest in one type security here it is modelled as an increase in θ, which is a measure of how much a bank can borrow based on their investment in counter-cyclical securities of the previous period. In the model, this means that equation (36) is altered and a new constraint is added. As in section 6, the propositions per addition are spread over two time-periods. Proposition five, seven and nine and formulated at time is t, before households and banks can react. The

$$p_t B_t^p = N_t + D_t^h + G_t \tag{54}$$

$$G_t = \theta(p_{t-1} B_{t-1}^p) \tag{55}$$

where G_t is the total amount banks can borrow from the central bank at time t and $0 < \theta < 1$, which is the maximum amount that can be borrowed on a single unit of collateral. Re-optimizing the banks' utility (equation (35) substitute to (55), (37) and (56)) with respect to B_t^p yields:

$$\mathrm{E_t}\Lambda_{t,t+1}\left(r_{t+1}^b - r_{t+1}\right) + \theta = 1, \tag{56}$$

which differs on one point from equation (48), in that banks now increase their holdings in long-term securities if θ increases. To see how this type of QE affects the long-term interest rates the implicit function theorem is used to find the perturbation results – see appendix F for the details:

$$-\frac{p_t}{\theta} = 0. \tag{57}$$

These results suggest that if the central bank increases the θ, prices of counter-cyclical securities increase, which corresponds to a decline in the long-term interest rate. From this, it can be inferred that the ILFs have the effect that the banks are able to borrow more from the central bank and are able to invest more in long-term securities, leading to an increased demand in these long-term securities, increasing the prices of these securities and decreasing the long-term interest rate. If households are able to capture the increase in prices, the income inequality could increase. The underlying mechanisms are therefore very similar to the portfolio-balancing channel in section, leading to similar propositions. In general, this leads to two new propositions:

Proposition five:

When the central bank improves its lending facilities, long-term interest rates will decrease due to the increased investment in long-term securities of banks, thereby decreasing the next-period return of the bankers and banks, leading to a decrease of income inequality.

Proposition six:

When the central bank improves its lending facilities, bankers and banks will decrease their position in counter-cyclical securities, with it capitalising, to some extent, on the increased prices of these long-term securities, leading to a slight decrease in the prices of the counter-cyclical securities and to an increase in the income inequality between workers and bankers.

7.2 Policy 3: Operation twist

Policy 3 has a different effect when added into the basic model. Effectively, a central bank purchases long-term bonds with short-term bonds in a situation of operation twist.

Operation twist can be modelled the following way: assume that the central banks purchases in long-term securities have to be equal to the sale of short-term government debt, D_t^g, which means that the central bank cannot finance itself indefinitely, though has enough reserves to pursue this policy. This can be captured in the following central bank balance sheet constraint:

$$B_t^g = D_t^g. \tag{58}$$

A second assumption is that all households, workers and bankers, hold deposits and short-term central bank debt instead of only deposits of the banks, D_t^p, which are both assumed to be safe one-period securities:

$$D_t^h = D_t^p + D_t^g. \tag{59}$$

Lastly, it is assumed that the short-term interest rate on short-term central bank debt is a function of the equilibrium between supply and demand of the short-term central bank debt market, here shown as $f(D_t^g)$:

$$r_t^g = f(D_t^g). \tag{60}$$

Meaning that if demand increases or supply decreases, the short-term interest rate increases and *vice versa*. The short-term interest of the total short-term government debt and deposits hold by households is then defined as:

$$r_t^h = \frac{D_t^p r_t^d + D_t^g r_t^g}{D_t^p + D_t^g}, \tag{61}$$

which is the average of both short-term interest rates on government debt and deposits. Notice here that the interest rate of deposits is assumed safe and depends on the policy interest rate, which is assumed constant. This means that only the interest rate on short-term central bank debt can change.

Equation (58) shows that when the central bank purchases long-term securities, it sells short-term securities, increasing the supply of them on the market. Assuming that the demand of the short-term central debt remains constant, an increase in supply of short-term government debt will result in a lower the market equilibrium, $f(D_t^h)$. A lower market equilibrium results in lower market prices of short-term government debt, but higher short-term interest rates, r_t^g.

The inclusion of operation twist also has some effect on the budget constraint of the households and the balance sheet constraint of banks. The new budget constraint and the balance sheet constraints are:

$$\begin{aligned} C_t + D_t^h + p_t\left[B_t^{1-w} + \frac{1}{2}k\left(B_t^{1-w} - \overline{B^{1-w}}\right)^2\right] \\ = W_t L_t + \Pi_t^{1-w} + r_t^h D_{t-1}^h + r_t^b B_{t-1}^{1-w}; \end{aligned} \tag{62}$$

$$p_t B_t^p + q_t D_t^g = N_t + D_t^p; \tag{63}$$

$$N_t = (r_t^b - r_t^d) p_{t-1} B_{t-1}^p + (r_t^g - r_t^d) q_{t-1} D_{t-1}^g + r_t N_{t-1}. \tag{64}$$

In the budget constraint of the households (equation (62)) the return on the total holdings of short-term assets by households, D_t^h, now depends on the average short-term interest rate of short-term government debt and deposits. The balance sheet constraint of the banks (equations (63) and (64)) have two new additions: i) is the possibility of the banks to invest in short-term central bank debt, D_t^g, and ii) the net-equity of the banks can now grow with an extra factor of $(r_t^g - r_t^d)D_t^g$, meaning that the excess return on short-term central bank debt augments the net-equity as well.

This change leads to different first-order conditions for households and banks with respect to D_t^h. For households the new first-order condition now becomes:

$$E_t \Lambda_{t,t+1} r_{t+1}^h = 1 \tag{65}$$

Now meaning that the choice of investing in short-term assets depends on the expected discounted future average short-term interest rate on short-term assets. For banks, the first-order condition becomes:

$$\mathrm{E}_t\Lambda_{t,t+1}\left(r_{t+1}^{g}-r_{t+1}^{d}\right)=1, \tag{66}$$

which shows that when the excess return on short-term government debt increases banks will invest more into these short-term securities.

The perturbation results with respect to the long-term interest rate is exactly equal to the portfolio-balancing channel in the previous section. The results for short-term interest rates, however is new:

$$f'\left(D_t^{g}\right)=0, \tag{67}$$

meaning that when the central bank sells short-term central bank debt, the equilibrium value of the short-term central bank debt decreases, increasing the short-term interest rate altogether.

Thus, in addition to the lower long-term interest rates in the portfolio-balancing channel, this type QE also increases the short-term interest rate. In effect, this means that if bankers are able to capitalise on the increased long-term security prices, income inequality increases. However, due to the increased return on short-term assets workers will also gain more wealth than in a situation absent of operation. The effects on income inequality are therefore less strong than in the portfolio-balancing channel with QE as a purchasing programme only. This leads to two new propositions:

Proposition seven:

When the central bank engages in operation twist, the long-term interest rate decreases similarly to the portfolio-balancing channel, however, the short-term interest rates increase, making the total effect on next-period return of households and banks uncertain, but decreasing income inequality.

Proposition eight:

When the central bank engages in operation twist, bankers and banks will decrease their position in long-term securities, possibly capturing the increased prices of long-term securities, and will invest more in short-term government debt; this has the effect that it pushes up the long-term interest rates slightly up and the short-term interest rates slightly down, leading to an increase in income inequality.

7.3 Extension to policy 1: Cyclical securities

The basic model only allowed for banks and households to invest in counter-cyclical long-term securities. This subsection allows households and banks to invest in pro-cyclical long-term securities, such as low-grade long-term private securities and equities, as argued by Kimura and Small (2006). In general, this only means that the household budget constraint and the balance sheet constraint of the banks are altered and in terms of the QE-policy, nothing changes. The new household budget constraint and the balance sheet constraint of the banks are:

$$\begin{aligned} C_t+D_t^h+P_t\left[A_t^{1-w}+\frac{1}{2}k\left(A_t^{1-w}-\overline{A^{1-w}}\right)^2\right]+p_t\left[B_t^{1-w}+\frac{1}{2}k\left(B_t^{1-w}-\overline{B^{1-w}}\right)^2\right] \\ =W_tL_t+\Pi_t^{1-w}+r_t^dD_{t-1}^h+r_t^aA_{t-1}^{1-w}+r_t^bB_{t-1}^{1-w}; \end{aligned} \tag{68}$$

$$P_t A_t^p + p_t B_t^p = N_t + D_t^p; \tag{69}$$

$$N_t = (r_t^a - r_t^d) P_{t-1} A_{t-1}^p + (r_t^b - r_t^d) p_{t-1} B_{t-1}^p + r_t N_{t-1}. \tag{70}$$

Here is A_t^{1-w} and A_t^p the total amount of pro-cyclical securities bankers and banks can invest in respectively, r_t^a is the rate of return on cyclical assets. This leads to new first-order conditions with respect to pro-cyclical securities:

$$A_t^h = \overline{A^h} + \frac{\mathrm{E_t}\Lambda_{t,t+1} r_{t+1}^a}{k P_t} - \frac{1}{k} = \overline{A^h} + \frac{\mathrm{E_t}\Lambda_{t,t+1} r_{t+1}^a r_t^a - 1}{k}; \tag{71}$$

$$\mathrm{E_t}\Lambda_{t,t+1}(r_{t+1}^a - r_{t+1}) = 1. \tag{72}$$

Here the interpretation is familiar. Households, equation (71) will increase their holdings in cyclical assets if the expected future long-term interest is high and the current price is low. Banks will increase their holdings in pro-cyclical assets when the excess return increases, equation (72)

The difference with the basic model occurs when the central bank engages in QE. The signalling channel will yield the same results as in section 6 and will not be further discussed. The effects of the portfolio-balancing channel are different. The perturbation result is as follows:

$$\frac{dr_t^a}{dB_t^g} = -\lambda_1^a \lambda_1^b Var[Z_t] \qquad \forall Y_t^T \neq B_t. \tag{73}$$

This result is different from section 6 in that when the central bank purchases counter-cyclical securities, the long-term interest rate on pro-cyclical securities increase due to the definition of $\lambda_1^b < 0$. This means that QE as a purchasing programme leads to higher long-term interest rates on pro-cyclical securities. This is similar to the argument made by Kimura and Small (2006) in section 4.

Referring to equation (49), QE as a purchasing programme increases the market variance and decreases the expected market return, however, QE increases the covariance between the market return and the pro-cyclical assets. This means that the term-premium on pro-cyclical assets decreases less than that on counter-cyclical assets and possibly even increase. As a result, the excess return on cyclical securities will increase. Bankers and banks will exhibit increased next-period returns and will increase their position in pro-cyclical securities. This will lead a new proposition:

Proposition nine:

When households and banks are able to invest in pro-cyclical assets, the purchases of long-term securities could lead to an increase in excess return of pro-cyclical assets compared to the decrease in excess return of counter-cyclical securities, increasing the income inequality

Proposition ten:

When households and banks are able to invest in pro-cyclical assets, the interest rate on these assets increases, leading to an increase of the position of bankers and banks into these securities and to a decrease in the position of counter-cyclical assets, with it, again, capitalising on the increased prices of the counter-cyclical assets of bankers and banks, increasing the income inequality.

8 *Discussion and conclusion*

The research question of this study is: *How does Quantitative Easing affect the distribution of income of private households?* As the propositions suggest, QE in a macroeconomic setting does not influence the income distribution in a uniform way. This is demonstrated by means of two time-periods: i) the initial shock of the QE-policy and ii) the reaction of economic agents on the QE-policy. During the initial shock, the effect of the three QE-policies, large-scale long-term asset purchases (LSAPs), improved central banks lending facilities (ILFs) and operation twist, lead to different results. The effects of LSAPs and operation twist during the initial shock are ambiguous. However, when economic agents invest in pro-cyclical assets during an LSAP, the income inequality deteriorates. If the LSAP affects the long-term interest rate, the central bank engages in operation twist or in ILFs, the income inequality reduces. During the second time-period, when economic agents are able to react, all three policies lead to an increase in income inequality. Thus, before households are able to adjust to QE, the effect on income inequality is ambiguous; when the households are able to adjust to QE the effect is increasing income inequality.

These results are supported by earlier evidence discussed in section 3 of this study. Firstly, the three types of QE as added in the model comply with the aims discussed in section 3, albeit the model does not allow for inflation. All three types of QE decrease in the long-term interest rates and, additionally, operation twist increases the short-term interest rate, for which the programmes were designed to do. The model presented in the current study, therefore, justifies the use of these policies by central banks. Secondly, the channels of QE as described in section 3 are supported by the empirical evidence presented in section 3 and 4. The portfolio-balancing channel decreases the long-term interest which was empirically shown by Christensen and Rudebusch (2012), Joyce et al. (2012), Ugai (2007), and Kimura and Small (2006). This suggests that the transmission channel 1 is not only theoretically interesting but also empirically supported. This also means that these findings are more likely to be strong and not by construct. The same holds for the signalling channel, Krishnamurthy and Vissing-Jorgensen (2011), and Ugai (2007) found that this channel decreases the short-term rate in the US and Japan, which is precisely the effect of the channel in the model and exactly the objective of the channel.

There are some limitations to the current study. An important point to stress is that this study is not empirical, therefore the propositions provided in sections 6 and 7 have to be confirmed by suitable empirical evidence. The model clearly illustrates the mechanisms of QE but nothing further. For future studies, researchers should disentangle all the mechanisms of QE in the model and empirically test whether they have an effect on income inequality. A second important limitation of this study and the subsequent model is the way the financial crisis of 2007-8 is involved. As said earlier, QE is a post-crisis measure to reinvigorate the economy, meaning that the economic situation of QE on itself is already unique. Now the economic situation is reflected in the *ex post* returns of securities in the portfolio-balancing channel. A third limitation is about the inclusion of the type of financial system. The model presented in the current study is not a one-size-fits-all solution for every central bank contemplating QE. Primarily, each central bank should choose the types of QE most suitable for the financial system in which the central bank operates. The effect of different system, however, are not shown in the model. Because bank-based and market-based systems are differently structured, the reaction of both may differ from each other.

Future research can improve the current study in a few ways. Future studies should investigate how financial crises affect income inequality and how QE has an effect on this unique situation. This can be done with formal models. These models, however, should be supported by empirical evidence. Additionally, future studies may want to investigate the differences in reaction of the financial systems to conventional monetary policy, unconventional monetary policy and QE. This can then be added to the model in order to provide a tailor-made solution for central banks. From a policy perspective, this has the benefit that central banks are better able to choose suitable policies for their economies and hereby preventing wasting of central bank resources. Besides the financial systems, the model can be improved in a couple of ways by future research. Firstly, now the channels are derived separately and the effects of the channels are thereafter plugged into the model with the households and banks. To improve the model, future studies can combine the channels and the households and banks into one comprehensive model. One might even go as far as creating a general equilibrium model to improve the model. A general equilibrium model could show more clearly show the effects on the macro-economy compared to the model presented in this study, but even then, concessions have to be made. Secondly, the inflation channel can be added to the model. This has two advantages, now the changes in real interest rates can be seen, which is more relevant for investors, and the inflation aim of QE can now be tested. Thirdly, besides the inflation channel, some other channels may be added to the model to create a more elaborate model of the effects of QE. Krishnamurthy and Vissing-Jorgensen (2011) provided extra channels that can be relevant for QE. Future studies have to assess these and then, possibly, add them to the model. Fourthly, the model now assumes that institutional investors rebalance their portfolios and thereby drive the portfolio-balancing channel. To improve the model, the institutional investors can be added to the model, in order to makes this relationship explicit.

In conclusion, the current study shows that *ex post* QE-policies can increase the income inequality in an economy, adding a new effect of QE to the existing effects on interest rates and inflation.

9 *Bibliography*

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Appendix A Policy 1: LSAPs of different central banks

The current appendix provides more in depth information about the different LSAPs of the Fed, BoE, BoJ and the ECB.

The Federal Reserve

The Fed's LSAP programme is often split into three periods (Hausken & Ncube, 2013; Krishnamurthy & Vissing-Jorgensen, 2011). During the first period (QE1; 2008-09), the Fed bought MBS and treasury bonds at a value of 500 and 100 billion dollars respectively. The second QE-programme of the Fed (QE2) was introduced in 2010 after QE1 was phased down and consisted of only government bonds with a value of 600 billion dollars. The last programme (QE3) followed QE2 in 2012 and consisted of the purchase of MBS and government bonds with a monthly value of 40 billion and 45 billion dollars respectively with no definite ending. Among the three periods of QE are two differences, namely the volume of the purchases and the type of securities purchased.

The Bank of England

The programme in the UK was introduced in the beginning of 2009 with the purchase of governmental and private securities with a value of 50 billion pounds. The BoE increased the purchases several times and halfway through 2012 it was again increased to an amount of 375 billion pounds (Fawley & Neely, 2013). By the end of March 2013 the BoE had purchased securities worth 330 billion pounds (Steeley, 2015). As of the date of the current study, the BoE still pursues the LSAP.

The Bank of Japan

In the beginning of the 2000s the BoJ combined IFLs and an LSAP (Ugai, 2007). At the start of this program, the BoJ would buy five trillion yen. After several increases, the BoJ promised to purchase 1.200 trillion yen at the beginning of 2002. On March 9 2006 the BoJ decided to stop the programme, because the bank expected the year-on-year inflation growth to remain positive (Ugai, 2007). Nevertheless, in 2010 the BoJ reintroduced QE in the form of asset purchases (government bonds as well as corporate bonds) with an amount of 35 trillion yen (BoJ, 2010a), this was combined with ILFs as in the earlier period (BoJ, 2010b). In 2016 the programme was expanded to 80 trillion yen on an annual basis (BoJ, 2016b).

The European Central Bank

In addition, the ECB has engaged in an asset-purchasing QE-programme. One of the measures was the securities markets programme[16] (SMP), which the ECB used to purchase sovereign debt on the secondary market on an as-needed basis – this means that the purchases were not preannounced. The operation was also sterilised, which means that the money supply did not change (Fawley & Neely,

[16] Was announced on May 10th 2010

2013). The SMP is not a typical form QE and nudges more towards a CE programme due to the focus on the functioning of the European sovereign debt market and the sterilisation of the purchases. It can, however, be seen as the forerunner of the QE-programme introduced in 2015 (ECB, 2015a) and was due to last to the end of September 2016 (ECB, 2015b). The QE-programme was introduced due to the low economic growth and low inflation in the eurozone. With this programme, the ECB monthly purchased 60 billion euros in investment-grade bonds issued by governments, national agencies and EU-institutions (Annunziata, 2015/2016). The purchase of the members' securities is proportional to the GDP of the members, which prevents the impression that the programme is introduced to alleviate funding for specific countries. In January 2016 the programmes was extended to the end of March 2017 (ECB, 2016). Three months later in April 2016 the ECB announced to extend the programme to €80 billion a month (ECB, 2016).

Appendix B Transmission channel 1: Re-writing the CAPM

This appendix shows the steps of rewriting the CAPM as argued in section 4. The standard CAPM is as follows:

$$E\left[r_t^j - r_t^f\right] = Cov\left[r_t^m, r_t^j\right]\frac{E\left[r_t^m - r_t^f\right]}{Var[r_t^m]}. \tag{A.1}$$

First the market price of the risk, $\frac{E\left[r_t^m - r_t^f\right]}{Var[r_t^m]}$, is held constant:

$$E\left[r_t^j - r_t^f\right] = kCov\left[r_t^m, r_t^j\right] \tag{A.2}$$

Then the covariance, $Cov\left[r_t^m, r_t^j\right]$, is re-written as a function of the correlation coefficient and the variances of the return on asset j and the return on the market:

$$E\left[r_t^j - r_t^f\right] = k\rho\left[r_t^m, r_t^j\right]\sqrt{Var[r_t^j]}\sqrt{Var[r_t^m]}. \tag{A.3}$$

Appendix C Transmission channel 1.1: Derivation of the market variance

The variance of the market as argued in section 4 is derived as follows. The definition of the return on market portfolio is known:

$$r_{t,m} = \sum_{j=1}^{N} w_j\lambda_o^j + \sum_{j=1}^{N} w_j\lambda_1^j Z_t + \sum_{j=1}^{N} w_j\varepsilon_t^j. \tag{A.4}$$

Estimating the variance of r_m is as follows:

$$var[r_t^m] = var\left[\sum_{j=1}^{N} w_j\lambda_o^j\right] + var\left[\sum_{j=1}^{N} w_j\lambda_1^j Z_t\right] + var\left[\sum_{j=1}^{N} w_j\varepsilon_t^j\right] \tag{A.5}$$

Then re-writing the equation to take the summations out of the variance:

$$var[r_t^m] = \left(\sum_{j=1}^{N} w_j \lambda_o^j\right)^2 Var[1] + \left(\sum_{j=1}^{N} w_j \lambda_1^j Z_t\right)^2 var[Z_t] + \sum_{j=1}^{N} w_j^2\, var\left[\varepsilon_t^j\right] \tag{A.6}$$

$$\text{where } \left(\sum_{j=1}^{N} w_j \lambda_o^j\right)^2 Var[1] = 0 \tag{A.7}$$

Therefore, the variance of $r_m$is:

$$var[r_t^m] = \left(\sum_{j=1}^{N} w_j \lambda_1^j\right)^2 var[Z_t] + \sum_{j=1}^{N} w_j^2 var\left[\varepsilon_t^j\right] \tag{A.8}$$

Appendix D Policy 1: Optimisation of households and banks in the basic illustrative model

This appendix shows the optimization problem of the households and the banks. For both the solving technique is the same. Firstly, the Hamiltonian is given. As argued before, the Hamiltonian has the ability to allow a time dimension to enter the equation. When the transversality condition is added to the Hamiltonian, the technique is very similar to the Lagrangian method. Secondly, the first-order conditions are given. The first-order conditions are used as a solution in section 6. Thirdly, the bordered hessian is calculated to check if the solution is optimal. For the results of the households and the banks, the second-order conditions should be positive and, if they are not, the bordered hessian should be negative. Notice that only the optimisation of households and banks in section 6 are given in this appendix.

Households

The Hamiltonian and the transversality conditions of the households are as follows:

$$\begin{aligned}\mathcal{H}_h = E_t \sum_{i=0}^{\infty} \beta^i [uC_{t+i} - \alpha L_{t+i}] \\ + \lambda \Bigg[E_t \sum_{i=0}^{\infty} \Lambda^i \Big\{ W_{t+i} L_{t+i} + \Pi_{t+i}^{1-w} + r_{t+i}^d D_{t+i-1}^h \\ + r_{t+i}^b B_{t+i-1}^{1-w} - D_{t+i}^h - p_{t+i} \left[B_{t+i}^{1-w} + \frac{1}{2} k \left(B_{t+i}^{1-w} - \overline{B^{1-w}} \right)^2 \right] \\ - C_{t+i} \Big\} \Bigg].\end{aligned} \tag{A.9}$$

$$\lim_{I\to\infty} \Lambda^I D_{t+I} = 0, \tag{A.10}$$

$$\lim_{I\to\infty} \Lambda^I B_{t+I} = 0. \tag{A.11}$$

Notice here that in equation (A.9), the left part of the right-hand side is the households' utility function and the right part is the budget constraint. Here λ is the co-state variable that is very similar to the

Lagrange-multiplier (Chiang & Wainwright, 2005). The budget constraint differs from equation (32), for here it is the intertemporal household budget constraint. The problem is optimised with respect to C_t, L_t, D_t^h, B_t^h and λ, then the following are the first-order conditions:

$$\frac{\partial \mathcal{H}_h}{\partial C_t} = u'C_t - \lambda = 0; \tag{A.12}$$

$$\frac{\partial \mathcal{H}_h}{\partial L_t} = \alpha' L_t + \lambda(W_t) = 0; \tag{A.13}$$

$$\frac{\partial \mathcal{H}_h}{\partial D_t^h} = \lambda\left(E_t \Lambda^i \{r_{t+1}^d\} - 1\right) = 0; \tag{A.14}$$

$$\frac{\partial \mathcal{H}_h}{\partial B_t^{1-w}} = \lambda\left(E_t \Lambda^i \{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right]\right) = 0; \tag{A.15}$$

$$\begin{aligned} \frac{\partial \mathcal{H}_h}{\partial \lambda} = E_t \sum_{i=0}^{\infty} \Lambda^i \Big\{ & W_{t+i} L_{t+i} + \Pi_{t+i}^{1-w} + r_{t+i}^d D_{t+i-1}^h + r_{t+i}^b B_{t+i-1}^{1-w} - D_{t+i}^h \\ & - p_{t+i}\left[B_{t+i}^{1-w} + \frac{1}{2} k\left(B_{t+i}^{1-w} - \overline{B^{1-w}}\right)^2\right] - C_{t+i} \Big\} = 0. \end{aligned} \tag{A.16}$$

All the first-order conditions are correctly specified and equal to zero. These first-order conditions are rewritten and stated in section 6.2.1. in which equation (46) is $\frac{\partial \mathcal{H}_h}{\partial C_t} = \frac{\partial \mathcal{H}_h}{\partial L_t}$ and (47) and (48) are re-written from (A.14) and (A.15).

To check whether the first-order conditions are optimal the second-order conditions and the bordered hessian are given. The bordered Hessians are:

$$H_h f(\lambda) = \left| \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} \right| = 0; \tag{A.17}$$

$$H_h f(C_t, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ -1 & u''C_t \end{vmatrix} = -1 < 0; \tag{A.18}$$

$$\begin{aligned} H_h f(C_t, L_t, \lambda) &= \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} \end{vmatrix} = \begin{vmatrix} 0 & -1 & W_t \\ -1 & u''C_t & 0 \\ W_t & 0 & \alpha'' L_t \end{vmatrix} \\ &= -u''C_t W_t^2 - \alpha'' L_t < 0; \end{aligned} \tag{A.19}$$

$$H_h f\left(C_t, L_t, D_t^h, \lambda\right) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial l_t \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \left(D_t^h\right)^2} \end{vmatrix} \tag{A.20}$$

$$= \begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 \\ -1 & u'' C_t & 0 & 0 \\ W_t & 0 & \alpha'' L_t & 0 \\ \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & 0 & 0 & 0 \end{vmatrix} = 0;$$

$$H_h f(C_t, L_t, D_t^h, B_t^{1-w}, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial (D_t^h)^2} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial B_t^{1-w}} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial (B_t^{1-w})^2} \end{vmatrix} = \tag{A.21}$$

$$\begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & E_t\Lambda^i\{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] \\ -1 & u'' C_t & 0 & 0 & 0 \\ W_t & 0 & \alpha'' L_t & 0 & 0 \\ \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & 0 & 0 & 0 & 0 \\ E_t\Lambda^i\{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] & 0 & 0 & 0 & -p_t k \end{vmatrix}$$

$$= 0;$$

Equations (A.17), (A.20) and (A.21) are equal to zero, and equations (A.18) and (A.19) are smaller than zero. This means that the system is negative semi-definite, suggesting a correct specification of the model.

Banks

For the banks the same process holds as for households. The Hamiltonian and the transversality condition of the bank is as follows:

$$\mathcal{H}_b = E_t \sum_{i=0}^{\infty} \Lambda^i N_{t+i} + \lambda \left[E_t \sum_{i=0}^{\infty} \Lambda^i \left\{ (r_{t+i}^b - r_{t+i}^d) p_{t+i-1} B_{t+i-1}^p + r_{t+i}^d N_{t+i-1} + D_{t+i}^h - P_{t+i} S_{t+i}^p - p_{t+i} B_{t+i}^p \right\} \right]. \tag{A.22}$$

$$\lim_{I\to\infty} \Lambda^{I} B^{p}_{t+I+1} = 0. \tag{A.23}$$

Here is the left part of the right-hand side of equation (A.22) the utility function of the bank and the right part the intertemporal balance sheet constraint of the bank. The problem is maximised with respect to B_t^p and λ, making the first-order conditions:

$$\frac{\partial \mathcal{H}_b}{\partial B_t^p} = \lambda\left(E_t \Lambda^i\left\{\left(r_{t+1}^b - r_{t+1}^d\right)p_t\right\} - p_t\right) = 0; \tag{A.24}$$

$$\begin{aligned}\frac{\partial \mathcal{H}_b}{\partial \lambda} = E_t \sum_{i=0}^{\infty} \Lambda^i \{(r_{t+i}^b - r_{t+i}^d) p_{t+i-1} B_{t+i-1}^p + r_{t+i}^d N_{t+i-1} + D_{t+i}^h - P_{t+i} S_{t+i}^p \\ - p_{t+i} B_{t+i}^p\} = 0.\end{aligned} \tag{A.25}$$

To check whether the optimisation is optimal the bordered hessian in constructed:

$$H_b f(\lambda) = \left|\frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2}\right| = 0 \tag{A.26}$$

$$\begin{aligned} H_b f\left(B_t^p, \lambda\right) &= \begin{vmatrix} \dfrac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} & \dfrac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t^p} \\ \dfrac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t^p} & \dfrac{\partial^2 \mathcal{H}_b}{\partial \left(B_t^p\right)^2} \end{vmatrix} \\ &= \begin{vmatrix} 0 & E_t \Lambda^i\left\{\left(r_t^b - r_t\right)p_{t-1}\right\} + p_t \\ E_t \Lambda^i\left\{\left(r_t^b - r_t\right)p_{t-1}\right\} + p_t & 0 \end{vmatrix} \\ &= -\left[E_t \Lambda^i\left\{\left(r_t^b - r_t\right)p_{t-1}\right\} + p_t\right]^2 < 0 \end{aligned} \tag{A.27}$$

The bordered hessian of the banks' optimisation problem is equal to zero (A.26) and smaller than zero (A.27), meaning that the optimisation problem is negative semi-definite and correctly specified.

Appendix E Policy 1: Solution of the central bank channels

The solution of the portfolio-balancing channel and the signalling channel in section 6 is done with the implicit function theorem. Because most variables are assumed exogenous, the results are not different from normal derivation. The equation (43) is re-written; equation (39) is substituted in equation (40) and set equal to zero, and equation (43) is set to zero. Firstly, the results of the portfolio-balancing channel are discussed, hereafter the results of the signalling channel are given.

Transmission channel 1: Portfolio-balancing channel

The implicit function of the portfolio-balancing channel with respect to the long-term interest rate is as follows:

$$r_t^b - r_t^d - \{\lambda_1^j \lambda_1^m Var[Z_t] + Y_t^j Var[\varepsilon_t^j]\} \frac{E[\sum_{j=1}^N Y_t^j \lambda_0^j + \sum_{j=1}^N Y_t^j \lambda_1^j Z_t + \sum_{j=1}^N Y_t^j \varepsilon_t^j - r_t^d]}{\left(\sum_{j=1}^N Y_t^j \lambda_1^j\right)^2 var[Z_t] + \sum_{j=1}^N \left(Y_t^j\right)^2 var[\varepsilon_t^j]} = 0. \tag{A.28}$$

Notice that the equation above is equal to equation (44) substituted in equation (42). If an equation is implicitly specified, then a normal derivation cannot be done, therefore the implicit function theorem has to be used. To find the derivative of r_t^b with respect to B_t^g, the following has to be done:

$$\frac{dr_t^b}{dB_t^g} = -\frac{F_{B_t^g}}{F_{r_t^b}}. \tag{A.29}$$

Therefore, the First-order derivative is:

$$\frac{dr_t^b}{dB_t^g} = -\frac{-\left(\{-(\lambda_1^b \lambda_1^b + \lambda_1^m) Var[Z_t] - Var[\varepsilon_t^b]\} \frac{E_t[-(\lambda_0^b + \lambda_1^b Z_t + \varepsilon_t^b)]}{-2\left(\sum_{j=1}^N \lambda_1^j\right) \lambda_1^b var[Z_t] + 2(-B_t^g) var[\varepsilon_t^b]}\right)}{1} = 0 \tag{A.30}$$

Re-written:

$$\frac{dr_t^b}{dB_t^g} = \{-(\lambda_1^b \lambda_1^b + \lambda_1^m) Var[Z_t] - Var[\varepsilon_t^b]\} \frac{E_t[-(\lambda_0^b + \lambda_1^b Z_t + \varepsilon_t^b)]}{-2\left(\sum_{j=1}^N \lambda_1^j\right) \lambda_1^b var[Z_t] + 2(-B_t^g) var[\varepsilon_t^b]} = 0 \tag{A.31}$$

Transmission channel 2: Signalling channel

The same technique is used to derive the signalling channel. Firstly, the implicit function is given:

$$r_t^d - \frac{1}{n}[r_t + X_t] = 0. \tag{A.32}$$

To obtain the derivative of r_t^d with respect to X_t the following has to be done:

$$\frac{dr_t^d}{dX_t} = -\frac{F_{B_t^g}}{F_{r_t^b}}. \tag{A.33}$$

Meaning:

$$\frac{dr_t^d}{dX_t} = -\frac{-\frac{1}{n} X_t{}'}{1} = \frac{1}{n} X_t', \tag{A.34}$$

Where X_t' is the first derivative of X_t and is assumed to be positive.

Appendix F Solution of the additions to the model

This appendix provides the solutions of the additions to the model as provided in section 7. Only the solutions that differ from section 6 are given in this appendix, if the solutions are the same it will be stated.

Policy 2: Improved central bank lending facilities

As argued in section 7, only the optimisation problem of the banks changes with respect to the addition of the improved central bank lending facilities. The solution of this new optimisation problem is as follows, where the Hamiltonian and the transversality conditions are:

$$\mathcal{H}_b = E_t \sum_{i=0}^{\infty} \Lambda^i N_{t+i} + \lambda \left[E_t \sum_{i=0}^{\infty} \Lambda^i \left\{ \left(r_{t+i}^b - r_{t+i}^d\right) p_{t+i-1} B_{t+i-1}^p + r_{t+i}^d N_{t+i-1} + D_{t+i}^h + \theta\left(p_{t+i-1} B_{t+i-1}^p\right) - p_{t+i} B_{t+i}^p \right\} \right]; \tag{A.35}$$

$$\lim_{I \to \infty} \Lambda^I B_{t+I+1}^p = 0. \tag{A.36}$$

The banks optimise their portfolios with respect to the counter-cyclical securities, B_t^p, giving the following first-order:

$$\frac{\partial \mathcal{H}_b}{\partial B_t^p} = \lambda\left[E_t \Lambda^i \left\{ \left(r_t^b - r_t\right) p_t + \theta(p_t) \right\} - p_t\right] = 0 \tag{A.37}$$

To check whether the solutions is optimal, the bordered hessian is constructed:

$$H_b f(\lambda) = \left| \frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} \right| = 0 \tag{A.38}$$

$$\begin{aligned} H_b f\left(B_t^p, \lambda\right) &= \begin{vmatrix} \dfrac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} & \dfrac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t^p} \\ \dfrac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t^p} & \dfrac{\partial^2 \mathcal{H}_b}{\partial \left(B_t^p\right)^2} \end{vmatrix} \\ &= \begin{vmatrix} 0 & E_t \Lambda^i \left\{ \left(r_t^b - r_t\right) p_t + \theta(p_t) \right\} - p_t \\ E_t \Lambda^i \left\{ \left(r_t^b - r_t\right) p_t + \theta(p_t) \right\} - p_t & 0 \end{vmatrix} \\ &= -\left[E_t \Lambda^i \left(r_t^b - r_t\right) p_t + \theta(p_t) - p_t\right]^2 < 0. \end{aligned} \tag{A.39}$$

The bordered hessians above are equal to zero (A.38) and smaller than zero (A.39), meaning that the curve is negative semi-definite and the outcomes are optimally derived.

Besides the new banks' optimisation problem, the results of the QE-policy have to be derived with the help of the implicit function theorem. Argued in section 7, the amount of central bank borrowings are equal to a fraction of the counter-cyclical securities banks hold. Furthermore, the central bank can augment the central bank borrowings by increasing the θ. The one-period implicit function of this relationship is:

$$G_t - \theta\left(\frac{1}{r_{t-1}^b} B_{t-1}^p\right) = 0. \tag{A.40}$$

Using the implicit function theorem, the results can be derived:

$$\frac{dr_t^b}{d\theta} = -\frac{F_\theta}{F_{r_t^b}}; \tag{A.41}$$

$$\frac{dr_t^b}{d\theta} = -\frac{-E_t\Lambda^i \frac{1}{r_t^b} B_t^p}{-E_t\Lambda^i \theta B_t^b} = -\frac{\frac{1}{r_t^b}}{\theta} = -\frac{p_t}{\theta} = 0. \tag{A.42}$$

Equation A.45 shows that if the central bank improves the lending facilities – a increase in θ – the long-term interest rate decreases.

Policy 3: Operation twist

Operation twist alters the basic illustrative model differently. Here only the changes in the solutions of the model are given, meaning that the effect on the long-term interest rates are not given for it is the same as the portfolio-balancing channel. With operation twist assumed is that the definition short-term interest rate changes, which changes the optimisation problem of households and banks.

Households

The new Hamiltonian of the households is:

$$\begin{aligned}\mathcal{H}_h = E_t \sum_{i=0}^{\infty} \beta^i [uC_{t+i} - \alpha L_{t+i}] \\ + \lambda\Bigg[E_t \sum_{i=0}^{\infty} \Lambda^i \Big\{ W_{t+i}L_{t+i} + \Pi_{t+i}^{1-w} + r_{t+i}^h D_{t+i-1}^h \\ + r_{t+i}^b B_{t+i-1}^{1-w} - D_{t+i}^h - p_{t+i}\left[B_{t+i}^{1-w} + \frac{1}{2}k\left(B_{t+i}^{1-w} - \overline{B^{1-w}}\right)^2\right] \\ - C_{t+i}\Big\}\Bigg].\end{aligned} \tag{A.43}$$

$$\lim_{I\to\infty} \Lambda^I D_{t+I+1}^h = 0, \tag{A.44}$$

$$\lim_{I\to\infty} \Lambda^I B_{t+I+1}^{1-w} = 0. \tag{A.45}$$

Because only the definition of the short-term interest rate changes, only the results of this is given. The rest of the results are the same as in appendix D. The new first order condition is:

$$\frac{\partial \mathcal{H}_h}{\partial D_t^h} = \lambda\left(E_t \Lambda^i \{r_{t+1}^h\} - 1\right) = 0. \tag{A.46}$$

To check whether the optimisation is optimal the bordered hessians are given:

$$H_h f(\lambda) = \left|\frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2}\right| = 0; \tag{A.47}$$

$$H_h f(C_t,\lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ -1 & u''C_t \end{vmatrix} = -1 < 0; \tag{A.48}$$

$$H_h f(C_t, L_t, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} \end{vmatrix} = \begin{vmatrix} 0 & -1 & W_t \\ -1 & u''C_t & 0 \\ W_t & 0 & \alpha''L_t \end{vmatrix} \tag{A.49}$$

$$= -u''C_t W_t^2 - \alpha'' L_t < 0;$$

$$H_h f\left(C_t, L_t, D_t^h, \lambda\right) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial l_t \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \left(D_t^h\right)^2} \end{vmatrix} \tag{A.50}$$

$$= \begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t}\Lambda^i r_{t+1}^h - 1 \\ -1 & u''C_t & 0 & 0 \\ W_t & 0 & \alpha''L_t & 0 \\ \mathrm{E_t}\Lambda^i r_{t+1}^h - 1 & 0 & 0 & 0 \end{vmatrix} = 0;$$

$$H_h f(C_t, L_t, D_t^h, B_t^{1-w}, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial (D_t^h)^2} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial B_t^{1-w}} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial (B_t^{1-w})^2} \end{vmatrix} = \tag{A.51}$$

$$\begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & E_t \Lambda^i \{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] \\ -1 & u''C_t & 0 & 0 & 0 \\ W_t & 0 & \alpha''L_t & 0 & 0 \\ \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & 0 & 0 & 0 & 0 \\ E_t \Lambda^i \{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] & 0 & 0 & 0 & -p_t k \end{vmatrix}$$

$$= 0;$$

Both bordered hessians (A.47), (A.50) and (A.51) are equal to zero, and equations (A.48) and (A.49) are smaller than zero, suggesting that the optimisation is optimal.

Banks

The new Hamiltonian for banks is:

$$\mathcal{H}_b = E_t \sum_{i=0}^{\infty} \Lambda^i N_{t+i} + \lambda \Bigg[E_{t+i} \sum_{i=0}^{\infty} \Lambda^i \{ (r^b_{t+i} - r^d_{t+i}) p_{t+i-1} B^p_{t+i-1} + (r^g_{t+i} - r^d_{t+i}) q_{t+i-1} D^g_{t+i-1} + r^d_{t+i} N_{t+i-1} + D^p_{t+i} - P_{t+i} S^p_{t+i} - p_{t+i} B^p_{t+i} - q_{t+i} D^g_{t+i} \} \Bigg]; \tag{A.52}$$

$$\lim_{I\to\infty} \Lambda^I \, B^p_{t+I+1} = 0 \tag{A.53}$$

As with the households, only the solution with respect the short-term debt changes, therefore only these solutions are given – for the rest of the solutions see appendix D:

$$\frac{\partial \mathcal{H}_b}{\partial D^g_t} = \lambda \big(E_t \Lambda^i \{ (r^g_{t+1} - r^d_{t+1}) p_t \} - 1 \big) = 0; \tag{A.54}$$

To check whether the optimisation is optimal, the bordered hessians are given:

$$H_b f(\lambda) = \left| \frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} \right| = 0 \tag{A.55}$$

$$H_b f(B^p_t, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B^p_t} \\ \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B^p_t} & \frac{\partial^2 \mathcal{H}_b}{\partial (B^p_t)^2} \end{vmatrix} = \begin{vmatrix} 0 & E_t \Lambda^i \{ (r^b_{t+1} - r^d_{t+1}) p_t \} - p_t \\ E_t \Lambda^i \{ (r^b_{t+1} - r^d_{t+1}) p_t \} - p_t & 0 \end{vmatrix} = -\big[E_t \Lambda^i \{ (r^b_{t+1} - r^d_{t+1}) p_t \} - p_t \big]^2 < 0 \tag{A.56}$$

$$H_b f(B^p_t, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B^p_t} & \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial D^g_t} \\ \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B^p_t} & \frac{\partial^2 \mathcal{H}_b}{\partial (B^p_t)^2} & \frac{\partial^2 \mathcal{H}_b}{\partial B^p_t \partial D^g_t} \\ \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial D^g_t} & \frac{\partial^2 \mathcal{H}_b}{\partial D^g_t \partial B^p_t} & \frac{\partial^2 \mathcal{H}_b}{\partial (D^g_t)^2} \end{vmatrix} \tag{A.57}$$

$$= \begin{vmatrix} 0 & E_t \Lambda^i \{ (r^b_{t+1} - r^d_{t+1}) p_t \} - p_t & E_t \Lambda^i \{ (r^g_{t+1} - r^d_{t+1}) p_t \} - 1 \\ E_t \Lambda^i \{ (r^b_{t+1} - r^d_{t+1}) p_t \} - p_t & 0 & 0 \\ E_t \Lambda^i \{ (r^g_{t+1} - r^d_{t+1}) p_t \} - 1 & 0 & 0 \end{vmatrix}$$

$$= 0$$

Equations (A.55) and (A.57) are equal to zero and (A.56) is smaller than zero, suggesting that the system is negative semi-definite and correctly specified.

Quantitative Easing programme

As argued in section 3, operation twist only differs with respect to the large-scale asset purchasing programmes that it finances itself with short-term government debt. Therefore, the portfolio-balancing channel remains the same; the signalling channel disappears completely. Then the changes in short-term interest rates is as follows:

$$r_t^h - \frac{D_t^p r_t^d + D_t^g f(D_t^g)}{D_t^p + D_t^g} = 0 \tag{A.58}$$

$$\frac{dr_t^h}{dD_t^g} = -\frac{F_{D_t^g}}{F_{r_t^h}}. \tag{A.59}$$

$$\frac{dr_t^h}{dD_t^g} = -\frac{-\frac{f'(D_t^g)}{1}}{1} = f'(D_t^g) = 0. \tag{A.60}$$

Equation (A.61) states that if the supply of short-term government debt increases, the market equilibrium decreases, decreasing the short-term interest rate.

Allowing Pro-cyclical assets

When households and banks are allowed to purchase pro-cyclical securities, a new variable to maximise occurs for both players. Only the results for this variable are given, the rest is equal to appendix C.

Households

For households the new Hamiltonian is:

$$\begin{aligned}\mathcal{H}_h = E_t \sum_{i=0}^{\infty} \beta^i [uC_{t+i} - \alpha L_{t+i}] \\ + \lambda \Bigg[E_t \sum_{i=0}^{\infty} \Lambda^i \Big\{ W_{t+i} L_{t+i} + \Pi_{t+i}^{1-w} + r_{t+i}^d D_{t+i-1}^h \\ + r_{t+i}^b B_{t+i-1}^{1-w} + r_t^s A_{t-1}^{1-w} - D_{t+i}^h \\ - p_{t+i} \left[B_{t+i}^{1-w} + \frac{1}{2} k \left(B_{t+i}^{1-w} - \overline{B^{1-w}} \right)^2 \right] \\ - P_t \left[A_t^{1-w} + \frac{1}{2} k \left(A_t^{1-w} - \overline{A^{1-w}} \right)^2 \right] - C_{t+i} \Big\} \Bigg].\end{aligned} \tag{A.61}$$

$$\lim_{I \to \infty} \Lambda^I D_{t+I} = 0, \tag{A.62}$$

$$\lim_{I \to \infty} \Lambda^I B_{t+I} = 0. \tag{A.63}$$

The new first-order is:

$$\frac{\partial \mathcal{H}_h}{\partial A_t^{1-w}} = \lambda\left(E_t \Lambda^i \{r_{t+1}^s\} - P_t\left[1 + k\left(A_t^{1-w} - \overline{A^{1-w}}\right)\right]\right) = 0; \tag{A.64}$$

The bordered hessians remain the same as in appendix D, suggesting that the solution is optimal.

$$H_h f(\lambda) = \left|\frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2}\right| = 0; \tag{A.65}$$

$$H_h f(C_t, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ -1 & u''C_t \end{vmatrix} = -1 < 0; \tag{A.66}$$

$$H_h f(C_t, L_t, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} \end{vmatrix} = \begin{vmatrix} 0 & -1 & W_t \\ -1 & u''C_t & 0 \\ W_t & 0 & \alpha'' L_t \end{vmatrix} \tag{A.67}$$

$$= -u''C_t W_t^2 - \alpha'' L_t < 0;$$

$$H_h f\left(C_t, L_t, D_t^h, \lambda\right) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial l_t \partial D_t^h} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \left(D_t^h\right)^2} \end{vmatrix} \tag{A.68}$$

$$= \begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t} \Lambda^i r_{t+1}^d - 1 \\ -1 & u''C_t & 0 & 0 \\ W_t & 0 & \alpha'' L_t & 0 \\ \mathrm{E_t} \Lambda^i r_{t+1}^d - 1 & 0 & 0 & 0 \end{vmatrix} = 0;$$

$$H_h f(C_t, L_t, D_t^h, B_t^{1-w}, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_h}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial C_t \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial L_t \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial (D_t^h)^2} & \frac{\partial^2 \mathcal{H}_h}{\partial D_t^h \partial B_t^{1-w}} \\ \frac{\partial^2 \mathcal{H}_h}{\partial \lambda \partial B_t^{1-w}} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial C_t} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial L_t} & \frac{\partial^2 \mathcal{H}_h}{\partial B_t^{1-w} \partial D_t^h} & \frac{\partial^2 \mathcal{H}_h}{\partial (B_t^{1-w})^2} \end{vmatrix} = \tag{A.69}$$

$$\begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & E_t\Lambda^i\{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] \\ -1 & u''C_t & 0 & 0 & 0 \\ W_t & 0 & \alpha''L_t & 0 & 0 \\ \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & 0 & 0 & 0 & 0 \\ E_t\Lambda^i\{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] & 0 & 0 & 0 & -p_t k \end{vmatrix} = 0;$$

$$H_h f\left(C_t, L_t, D_t^h, B_t^{1-w}, A_t^{1-w}, \lambda\right) \tag{A.70}$$

$$= \begin{vmatrix} \frac{\partial^2\mathcal{H}_h}{\partial\lambda^2} & \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial C_t} & \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial L_t} & \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial D_t^h} & \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial B_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial A_t^{1-w}} \\ \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial C_t} & \frac{\partial^2\mathcal{H}_h}{\partial C_t^2} & \frac{\partial^2\mathcal{H}_h}{\partial C_t\partial L_t} & \frac{\partial^2\mathcal{H}_h}{\partial C_t\partial D_t^h} & \frac{\partial^2\mathcal{H}_h}{\partial C_t\partial B_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial C_t\partial A_t^{1-w}} \\ \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial L_t} & \frac{\partial^2\mathcal{H}_h}{\partial L_t\partial C_t} & \frac{\partial^2\mathcal{H}_h}{\partial L_t^2} & \frac{\partial^2\mathcal{H}_h}{\partial L_t\partial D_t^h} & \frac{\partial^2\mathcal{H}_h}{\partial L_t\partial B_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial L_t\partial A_t^{1-w}} \\ \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial D_t^h} & \frac{\partial^2\mathcal{H}_h}{\partial D_t^h\partial C_t} & \frac{\partial^2\mathcal{H}_h}{\partial D_t^h\partial L_t} & \frac{\partial^2\mathcal{H}_h}{\partial (D_t^h)^2} & \frac{\partial^2\mathcal{H}_h}{\partial D_t^h\partial B_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial D_t^h\partial A_t^{1-w}} \\ \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial B_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial B_t^{1-w}\partial C_t} & \frac{\partial^2\mathcal{H}_h}{\partial B_t^{1-w}\partial L_t} & \frac{\partial^2\mathcal{H}_h}{\partial B_t^{1-w}\partial D_t^h} & \frac{\partial^2\mathcal{H}_h}{\partial (B_t^{1-w})^2} & \frac{\partial^2\mathcal{H}_h}{\partial B_t^{1-w}\partial A_t^{1-w}} \\ \frac{\partial^2\mathcal{H}_h}{\partial\lambda\partial A_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial A_t^{1-w}\partial C_t} & \frac{\partial^2\mathcal{H}_h}{\partial A_t^{1-w}\partial L_t} & \frac{\partial^2\mathcal{H}_h}{\partial A_t^{1-w}\partial D_t^h} & \frac{\partial^2\mathcal{H}_h}{\partial A_t^{1-w}\partial B_t^{1-w}} & \frac{\partial^2\mathcal{H}_h}{\partial (A_t^{1-w})^2} \end{vmatrix} =$$

$$\begin{vmatrix} 0 & -1 & W_t & \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & E_t\Lambda^i\{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] & E_t\Lambda^i\{r_{t+1}^s\} - P_t\left[1 + k\left(A_t^{1-w} - \overline{A^{1-w}}\right)\right] \\ -1 & u''C_t & 0 & 0 & 0 & 0 \\ W_t & 0 & \alpha''L_t & 0 & 0 & 0 \\ \mathrm{E_t}\Lambda^i r_{t+1}^d - 1 & 0 & 0 & 0 & 0 & 0 \\ E_t\Lambda^i\{r_{t+1}^b\} - p_t\left[1 + k\left(B_t^{1-w} - \overline{B^{1-w}}\right)\right] & 0 & 0 & 0 & -p_t k & 0 \\ E_t\Lambda^i\{r_{t+1}^s\} - P_t\left[1 + k\left(A_t^{1-w} - \overline{A^{1-w}}\right)\right] & 0 & 0 & 0 & 0 & -P_t k \end{vmatrix}$$

$$= 0;$$

The first five bordered hessians are equal to appendix D, the sixth is negative, suggesting negative semi-definiteness.

Banks

The new Hamiltonian for banks is:

$$\mathcal{H}_b = E_t \sum_{i=0}^{\infty} \Lambda^i N_{t+i} + \lambda \left[E_t \sum_{i=0}^{\infty} \Lambda^i \left\{ \left(r_{t+i}^b - r_{t+i}^d\right) p_{t+i-1} B_{t+i-1}^p + (r_{t+i}^a - r_{t+i}^d) P_{t+i-1} A_{t+i-1}^p + r_{t+i}^d N_{t+i-1} + D_{t+i}^p - P_{t+i} S_{t+i}^p - p_{t+i} B_{t+i}^p - P_{t+i} A_{t+i}^p \right\} \right]; \tag{A.71}$$

$$\lim_{I\to\infty} \Lambda^I \, B_{t+I+1}^p = 0 \tag{A.72}$$

Then the new first-order is:

$$\frac{\partial\mathcal{H}_b}{\partial A_t^p} = \lambda\left(E_t\Lambda^i\{(r_{t+1}^a - r_{t+1})p_t\} - P_t\right) = 0; \tag{A.73}$$

The new bordered hessians are:

$$H_b f(\lambda) = \left|\frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2}\right| = 0 \tag{A.74}$$

$$\begin{aligned} H_b f(B_t^p, \lambda) &= \begin{vmatrix} \frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t^p} \\ \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t^p} & \frac{\partial^2 \mathcal{H}_b}{\partial (B_t^p)^2} \end{vmatrix} \\ &= \begin{vmatrix} 0 & E_t \Lambda^i \{(r_{t+1}^b - r_{t+1}^d) p_t\} + p_t \\ E_t \Lambda^i \{(r_{t+1}^b - r_{t+1}^d) p_t\} + p_t & 0 \end{vmatrix} \\ &= -\left[E_t \Lambda^i \{(r_{t+1}^b - r_{t+1}^d) p_t\} + p_t\right]^2 < 0 \end{aligned} \tag{A.75}$$

$$H_b f(B_t^p, A_t^p, \lambda) = \begin{vmatrix} \frac{\partial^2 \mathcal{H}_b}{\partial \lambda^2} & \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t} & \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial A_t^p} \\ \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial B_t} & \frac{\partial^2 \mathcal{H}_b}{\partial (B_t^p)^2} & \frac{\partial^2 \mathcal{H}_b}{\partial B_t^p \partial A_t^p} \\ \frac{\partial^2 \mathcal{H}_b}{\partial \lambda \partial A_t^p} & \frac{\partial^2 \mathcal{H}_b}{\partial A_t^p \partial B_t^p} & \frac{\partial^2 \mathcal{H}_b}{\partial (A_t^p)^2} \end{vmatrix} \tag{A.76}$$

$$= \begin{vmatrix} 0 & E_t \Lambda^i \{(r_{t+1}^b - r_{t+1}^d) p_t\} + p_t & E_t \Lambda^i \{(r_{t+1}^a - r_{t+1}^d) p_t\} - P_t \\ E_t \Lambda^i \{(r_{t+1}^b - r_{t+1}^d) p_t\} + p_t & 0 & 0 \\ E_t \Lambda^i \{(r_{t+1}^a - r_{t+1}^d) p_t\} - P_t & 0 & 0 \end{vmatrix} = 0$$

The three bordered hessians suggest that the system is negative semi-definite.

QE

The portfolio-balancing channel has a different effect on pro-cyclical assets than on counter-cyclical assets. The difference is given below using the implicit function theorem. For the signalling channel nothing changes – see appendix D for the details.

$$\begin{aligned} & r_t^j - r_t^d - \{\lambda_1^j \lambda_1^m Var[Z_t] \\ & \qquad + Y_t^j Var[\varepsilon_t^j]\} \frac{E\left[\sum_{j=1}^N Y_t^j \lambda_0^j + \sum_{j=1}^N Y_t^j \lambda_1^j Z_t + \sum_{j=1}^N Y_t^j \varepsilon_t^j - r_t\right]}{\left(\sum_{j=1}^N Y_t^j \lambda_1^j\right)^2 var[Z_t] + \sum_{j=1}^N \left(Y_t^j\right)^2 var[\varepsilon_t^j]} \\ & \qquad = 0. \end{aligned} \tag{A.77}$$

$$\frac{dr_t^a}{dB_t^g} = -\frac{F_{B_t^g}}{F_{r_t^b}}. \tag{A.78}$$

$$\frac{dr_t^a}{dB_t^g} = -\frac{\lambda_1^T \lambda_1^b Var[Z_t]}{1} = -\lambda_1^a \lambda_1^b Var[Z_t] \qquad \forall Y_t^T \neq B_t. \tag{A.79}$$

Equation (A.75) suggest that if the central bank starts purchasing counter-cyclical securities, the long-term interest rate on pro-cyclical securities increases, because the $-\lambda_1^a \lambda_1^b Var[Z_t] > 0$.

Appendix G List of Symbols used

Symbol	Explanation
Section 2	
r_t	Real interest rate at time t
ρ	Addition factor to the inflation target
π^*	Inflation target
π_t	Actual inflation at time t
K_π	Weight factor of the inflation gap
$K_\pi(\pi_t - \pi^*)$	Inflation gap
y_t	Real output at time t
y^*	Hypothetical potential output
K_y	Weight factor of the output gap
$K_y(y_t - y^*)$	Output gap
Section 3	
R_{t+i}^B	Expected real interest rate at time t on a long-term bond
r_{t+i}^D	Average expected overnight (short-term) nominal interest rate over the following n-years at time t
$\tau_{t+i,n}$	Term-premium on a bond of n-years at time $t+i$
$E_{t+i}\pi_n^L$	Expected long-term inflation over the following n-years at time $t+i$
$E_{t+i}\sum_{j=1}^{n-1} r_{t+i+j}^D$	Expectations of all nominal interest rates in the future over n – 1 periods
π_{t+i}	Inflation at period $t+i$
$E_{t+i}\sum_{j=1}^{n-1} \pi_{t+i+j}$	Inflation expectations over a period of n years
Section 4	
r_t^j	Return on asset j
r_t^f	Return on the market portfolio
r_t^m	Risk free rate of return
$E[r_t^j - r_t^f]$	Risk-premium on asset j
$Cov[r_t^m, r_t^j]$	Covariance of the return on the market portfolio and the risk free rate of return
$E[r_t^m - r_t^f]$	Risk-premium on the market portfolio
$Var[r_t^m]$	Variance of the return on the market portfolio
$\frac{E[r_t^m - r_t^f]}{Var[r_t^m]}$ or k	Market price of risk, is assumed constant
$\rho[r_t^m, r_t^j]$	Correlation coefficient of the market portfolio and the risk free rate of return
Z_t	Variable for business cycles
ε_t^j	Captures the specific risk of asset j
λ_1^j	Measure of cyclicality of asset j
$\lambda_{1,m}$	Measure of cyclicality of the market portfolio
w_j	Share of asset j in the market portfolio

Section 5

S_t	Total amount of private securities in the market
S_{pt}, S_{ht}, S_{gt}	Total amount of private securities intermediated by banks, households and the central bank respectively
B_t	Total amount of government bonds in the market
B_{pt}, B_{ht}, B_{gt}	Total amount of government bonds intermediated by banks, households and the central bank respectively
u_t	Utility of the household
C_{t+i}	Current consumption at $t+i$
hC_{t+i-1}	Discounted consumption of the previous period
$\frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi}$	Labour time weighted by a factor of relative preference to work, where χ the relative risk aversion of households and φ the inverse elasticity of work effort which is the sum of the substitution elasticity and a measure of people's willingness to trade work for consumption over time
E_t	Expectancy operator
β^i	Subjective discount rate
$W_t L_t$	Labour income at $t+i$
Π_t	Transfers from banks to households at $t+i$
X	Total transfer for household members to become banker
T_t	Lump-sum tax from the government to the household
R_t	Real rate of return of deposits and safe government debt
R_{kt}	Real rate of return of private securities
R_{bt}	Real rate of return of government bonds
D_{ht}	Deposits held by households at time t
k	Marginal costs of investment of households
Q_t	Market price of private securities
q_t	Market price of government bonds
$\overline{S_h}$	Frictionless amount of investment in private securities
$\overline{B_h}$	Frictionless amount of investment in government bonds
V_t	Utility of banks
σ	Probability that the banker will stay banker next period
$\Lambda_{t,t+i}$	Marginal rate of substitution of the households
n_{t+i}	Total amount of equity of the bank at time $t+i$
s_t	Private securities held by a single bank
b_t	Government bonds held by a single bank
d_t	Households deposits held by a single bank
θ	Amount of funds the bank can divert of the private securities
$\Delta\theta$	Amount of funds the bank can divert from the government bonds
ϕ_t	Maximum ratio of the adjusted measure of assets to net worth that the bank may hold without violating the incentive constraint
$\widetilde{\Lambda}_{t,t+1}$	Augmented stochastic discount factor

Section 6 (only changes with respect to previous sections)

Y_t^T	Total assets in the market
O_t	Total long-term low-grade private securities in the market
A_t	Total equities in the market

B_t^p, B_t^{1-w}, B_t^g	Total amount of high-grade long-term private securities intermediated by banks, households and the central bank respectively
U_t	Utility of the households
uC_{t+i}	Utility with respect to current consumption at $t+i$
αL_{t+i}	Labour time weighted by a factor of relative preference to work
D_t^h	Total amount of deposits held by households
p_t	Market price of long-term government bonds
$\overline{B^{1-w}}$	Frictionless amount of investment in counter-cyclical securities
r_t^d	Nominal rate of return of deposits
r_t^b	Nominal rate of return of counter-cyclical securities
N_{t+i}	Total amount of equity of banks at time $t+i$
$\tau_{t,n}$	Term-premium on counter-cyclical securities

Section 7 (only changes with respect to previous sections)

G_t	Total amount banks can borrow from the central bank at time $t=1$
θ	Amount borrowed on a single unit of collateral increases counter-cyclical securities
D_t^g	Short-term government debt issued by the central bank
D_t^p	Deposits of households held by banks
r_t^g	Nominal rate of return on short-term government debt
$f\left(D_t^g\right)$	Function of the equilibrium on the market for short-term government debt
A_t^p, A_t^{1-w}, A_t^g	Total amount of cyclical assets intermediated by banks, households and the central bank respectively
r_t^a	Nominal rate of return on cyclical assets