



# International Portfolio Diversification and the Issue of Estimation Errors in Mean-Variance Efficient Portfolios

A German Investor Perspective

**Master Thesis**

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## ABSTRACT

The following study applies a popular asset allocation framework—the mean-variance optimization—to examine the potential international diversification benefits for the German investor, from an ex-post perspective, over a period of almost 15 years. The effect of a German home bias is explicitly considered in the analysis. Moreover, the effects of the recent period of severe stock market stress (2007–2008) are analyzed for further comparison of the benefits of international diversification.

However, practical implementation of the traditional mean-variance optimization is limited. The procedure suffers, almost always, from estimation errors. Estimation errors refer to the fact that the input parameters are estimated from past data, and as such, are likely to contain various errors, such as measurement errors, small sample biases, and noises. An application of a Monte Carlo simulation is used to illustrate this potential shortcoming. Furthermore, a resampling procedure known as the resampled efficient frontier is introduced to account for estimation errors in mean-variance optimized portfolios.

The analysis shows that, in almost all cases, internationally diversified portfolios are superior to regionally or purely domestically diversified portfolios for the German investor. An international portfolio, where estimation errors are explicitly considered, still performs better than purely domestically diversified indices, and the resampled efficient frontier, as a large-scale sensitivity analysis, is plotted to improve the quality of the mean-variance optimized international portfolio.



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## I. Introduction

Portfolio theory and the related literature on international portfolio diversification are still important subjects of discussion in economic theory. The rich literature on this particular topic suggests differing results. Many authors emphasize the importance of international diversification for an investor, while others do not see any benefits from international diversification or, at least, highlight various disadvantages of international diversification, such as exchange rate risks and information asymmetry. Additionally, the procedure of international integration of countries, termed globalization, results in the fact that markets around the world move closer together and become, therefore, much more correlated. This decreases the benefits of international portfolio diversification. The framework of the mean-variance optimization is used in the following study to examine those negative consequences. Especially interesting is how the framework of the mean-variance optimization has behaved during the recent global financial crisis, considered as one of the worst financial crises since the great depression of the 1930s.

Thus, the aim of the following analysis is to determine the optimal portfolio weights and to form mean-variance efficient portfolios in order to explore the effects of the full sample as well as the crisis period. However, the applied framework may result in optimal portfolio weights where the domestic market could be completely ignored. This is a highly doubtful assumption because investors typically hold a fair amount of assets from their domestic markets. Thus, the effects of a typical home bias for the German investor are especially considered in this study. The goal is to get a thorough understanding of the consequences of a relatively weak home bias, as well as higher degrees of bias, exclusively invested in the German market.

Therefore, the research questions can be summarized as follows: Are international portfolio diversification benefits still present nowadays during a lengthy and, thus, relatively stable period as well as during a period of severe stock market stress, and what are the optimal portfolio allocations for the German investor? And if they are present nowadays, how much of the international diversification benefits can be achieved by only investing regionally? Furthermore, what are the consequences of a German home bias in mean-variance efficient portfolios over the whole sample period?

Additionally, as one of the main aspects of the study, the weaknesses of traditional mean-variance optimization are discussed. As mentioned, the main weakness of the mean-variance optimization procedure is the aspect of estimation errors. A Monte Carlo simulation helps in evaluating the extension of estimation errors in mean-variance optimized portfolios. A statistical resampling procedure was applied to the international portfolio formed for this study. The goal was to receive more robust results for the highly sensitive input parameters.

This leads to two more research questions: Are the consequences of estimation errors in mean-variance efficient portfolios too strong to experience a positive impact of international diversification on those portfolios? Additionally, does the concept of the resampled efficient

frontier account for the uncertainty inherent in the input parameters and add value for practitioners when applying the framework of the mean-variance optimization?

The study contributes to the recent literature due to its relevance to portfolio theory and its unique perspective. The overwhelming majority of studies investigate the possible international diversification benefits from a US investor's perspective. In this study, we take the viewpoint of the local investor from Germany, another developed country. This is an important distinction because investors in Germany are operating in a more bank-based system and a comparatively underdeveloped stock market; therefore, they are more likely to benefit from investing internationally than US investors. The United States has one of the most diversified economies and one of the most developed stock markets in the world by any standard. Therefore, a perspective of US investors is unlikely to be representative for investors in most other countries. Moreover, no other papers evaluating international as well as regional diversification benefits from a German perspective in recent times were found, especially studies carried out on investment decisions during the global financial crisis.

The results of the mean-variance optimization show that international portfolio diversification is superior to regional as well as domestically diversified portfolios over the whole sample period. In fact, it would be optimal not to invest in the German market at all. The investigation of the crisis period gives different results. The German market becomes a relatively important component in the optimal portfolio on the efficient frontier. Therefore, the German market shows potential as a safer and more stable market during times of global financial crisis. The consequences of a German home bias, depending on the severity of the bias, decrease the efficiency of the mean-variance efficient frontier portfolios drastically. However, the results are still superior investment strategies compared to investing solely in the domestic market.

Most importantly, recognizing estimation errors in the construction of mean-variance efficient portfolios changes the benefits for the German investor considerably. However, these statistically equivalent resampled portfolios still perform better than a purely domestically diversified investment. The application of the resampled efficient frontier on the international portfolio results in the following implications. First, the resampled efficient frontier exhibits greater diversification compared to traditional mean-variance efficient portfolios. Another striking result is that the resampled optimal portfolios exhibit less sudden shifts and provide smooth transitions along the portfolio ranks (resampled frontier). Portfolios on the resampled frontier show, therefore, highly desirable characteristics for practitioners and investors.

Following the literature review in Chapter I, Chapter II continues with an elaboration on academic research in portfolio theory, mean-variance optimization, international diversification, and the aspect of the estimation error. This helps in constructing the methodological framework concerning this study. Hence, Chapter III and Chapter IV are concerned with the data gathering process and the methodological aspects of this study. Chapter V presents the empirical results. These results are discussed and interpreted in the following step. Chapter VI closes the paper with a detailed conclusion and recommendations for further research.

# **I. Literature Review**

## **1. Mean-Variance Optimization (Modern Portfolio Theory)**

More than 60 years have elapsed since Markowitz (1952) famously published his seminal article “Portfolio Selection.” His work was the starting point of the mathematical framework called modern portfolio theory (MPT). Modern portfolio theory relies on the mean-variance optimization and has been the standard for efficient portfolio construction ever since. Modern finance and, therefore, almost every portfolio program for asset allocation, is still based on some form of the theory introduced and extended by Markowitz (1959). Prior to the work of Markowitz, portfolio selection models were mainly concerned with the returns generated by investment opportunities. However, an investor behaving according to modern portfolio theory will choose a portfolio that is mean-variance optimal. Therefore, the investor cares about the expected return offered by the portfolio and the risk attributed to holding this portfolio. More specifically, when the assumptions surrounding modern portfolio theory hold, an investor would be interested in the highest expected return for a given amount of risk or, conversely, the lowest possible portfolio risk for a given return expectation. A situation, therefore, exists in which risk is traded off against returns. Moreover, Markowitz showed the impact of diversification in a portfolio. Variance can be decreased by selecting portfolios based upon their overall risk–reward characteristics instead of focusing on each security individually (Chen et al., 2010). To put it differently, the risk of investing can be reduced while maintaining the return by investing in a diversified portfolio rather than in individual assets.

One of the most important concepts of modern portfolio theory is Markowitz’s efficient frontier. Any possible asset combination in the risk–return space can be computed and plotted by applying the concept of the efficient frontier. The combination of assets offers the optimal return for a given amount of risk. Standard mean-variance optimization typically includes a higher amount of diversification of the available asset classes for the lower risk parts of the frontier, while the higher risk parts are progressively more concentrated in fewer asset classes. The portfolio with the lowest risk on the efficient frontier is called the minimum-variance portfolio (MV). Portfolios situated on the efficient frontier below the MV portfolio have an inferior risk–reward distribution and can, therefore, be rejected as inefficient. Efficient portfolios that lie above it, on the upward sloping portion, all have a specific trade-off between expected return and standard deviation. The best choice among those portfolios is not as obvious. In fact, the choice will depend on the individual risk attitudes of investors (Chen et al., 2010). The maximum return portfolio, as the highest point on the efficient frontier, allocates 100% to the highest return asset class, which is often, but not necessarily, the highest risk asset class (Idzorek, 2006).

## **2. Benefits of International Portfolio Diversification**

The first part of this section is concerned with early work on the benefits of international portfolio diversification. During these times, research was mainly conducted from the

perspective of the US investor. The following paragraphs are devoted to the analysis and examination of international diversification benefits from an international investor's perspective. At the end of this chapter, various critical positions on the advantages of international portfolio diversification are discussed.

The first paper conducting research on the benefits of international portfolio diversification by applying modern portfolio theory was published by Grubel (1968). The paper deals with potential gains for US investors if they pursue an internationally diversified portfolio strategy. For the first time in practice, Grubel's results point out that international portfolio diversification achieves higher returns compared to purely national diversified portfolios for the US investor over the sample period 1959–1966.

Two years later, Levy and Sarnat (1970) delivered another famous application of the mean-variance analysis to examine the benefits from international portfolio diversification for the US investor from an ex-post perspective. The study covers the period from 1951 to 1967. While Levy and Sarnat demonstrate that diversified domestic portfolios are dominated by internationally diversified portfolios, they also find other interesting implications. Diversification across 16 high-income countries results in only marginal improvements for the US investor. However, including investments not only from Western Europe and Canada, but rather from all over the world, gains a substantial improvement in investors' portfolio results.

Following these two papers, Solnik (1974) used a sample of eight different countries with stock returns over six years. In order to control for foreign exchange risk, the paper assumes that only US dollars were invested in stocks from every country. Solnik assessed different scenarios with random diversification and found that selection across countries was superior to domestic diversification, again with the perspective of a US investor. This can be seen in the reduction of the total portfolio risk by adding international assets.

Taken all together, early research on international diversification focused on the existence and degree of potential benefits through diversification by applying the mean-variance model of Markowitz. The papers are all in favor of diversification and have suggested several benefits that motivate investors to invest internationally, such as the lower return correlation between foreign and domestic securities.

A rather small part of the economic literature examined international portfolio diversification benefits from the point of view of countries other than the US. The perspective of the US investor is by far the most analyzed in studies in this particular domain. However, a few selected studies on international investors have been conducted.

Haavisto and Hanson (1992) examined the Nordic stock markets during the period 1970–1988 by analyzing each Nordic equity market as a single and separate market. The results for the period under consideration show that the Nordic optimal portfolios would mostly consist of only Finnish and Swedish assets and are, therefore, extremely concentrated. However, a long-term investor would have had diversification benefits by keeping an unhedged and diversified

Nordic portfolio. Liljeblom et al. (1997) extended the analysis about the Nordic equity markets by examining the magnitude of international diversification benefits from the Nordic point of view together with several international countries. Special attention is paid to the period of 1987–1993, where the Nordic capital markets were liberalized. The study reveals substantial benefits from international diversification for the Nordic countries.

Eun and Resnick (1994) examined the portfolio diversification benefits from the point of view of the Japanese investor. Moreover, the perspective of the US investor is considered again. The results include, again, gains from international investment portfolios as opposed to purely domestic ones. However, gains from diversification are much bigger for US investors than for Japanese investors. These gains from international diversification are, for the US investor, considered to be due to higher returns. A different finding stems from the perspective of the Japanese investor, where gains are mainly due to a lower level of risk.

Bugár and Maurer (1997) studied the benefits derived from international diversification of stock portfolios from two different viewpoints—Hungary with the Hungarian stock exchange as an emerging market and Germany as a developed market and one of the largest markets in the world. After considering the availability of data, the two authors used a time series of 17 national stock indices. The portfolios were mainly represented by Morgan Stanley Capital International (MSCI) as well diversified stock indices. The authors found evidence for international diversification benefits. However, as in the previous study by Eun and Resnick (1994), the countries had different sources for these gains in an ex-post perspective. While, for German investors, these gains accrue in terms of higher expected returns and risk reduction, the Hungarian investor has gains in terms of a reduction of risk only.

Driessen and Laeven (2007) did not focus on a specific country while examining international portfolio diversification benefits. Rather, the two authors presented cross-country evidence in their study. The authors used a sample of 52 countries, and they, indeed, found global as well as regional diversification benefits for domestic investors in developed or developing countries. Even the use of short-selling constraints as a more realistic assumption did not change the outcome. However, developing countries dominate developed countries due to substantially larger gains, a finding that can be explained with less integrated and, therefore, correlated world financial markets in developing countries. More interestingly, diversification benefits seem to have shrunk for most countries over the sample period.

In conclusion, all these papers seem to have found overwhelming evidence of the presence of benefits while diversifying internationally. However, there are existing studies which found evidence against international diversification benefits. While not every study is against international diversified portfolios in general, some studies, at least, have cast doubt on this matter.

First, the study by Hanna et al. (1999) found no evidence for international diversification benefits at all. The study is based on the perspective of the US investor and compiled data from the equity markets of Canada, UK, Japan, France, Germany, and Italy during the period from 1988 to 1997. The authors argue that a two-country portfolio, consisting of the S&P 500

and one of the major market indices of the G-7 countries, would have been dominated by a portfolio consisting solely of the S&P 500 index. The risk-adjusted returns during this period were much higher for the US than for other countries. Moreover, the correlation between these countries was too high to offset the lower foreign market returns (Hatemi & Roca, 2006). These findings were, however, put into perspective by acknowledging that their results might be sample specific. Nevertheless, they emphasized another perspective on diversifying internationally.

Second, Abidin et al. (2004) investigated, from a Malaysian perspective, whether international diversification benefits still exist. Index data were gathered for 20 different countries. Furthermore, they divided their analysis into periods and sub-periods over a range of 17 years from 1987 to 2003. The results did not show clear-cut international diversification benefits. In fact, they observed mixed outcomes while examining the efficient frontiers. Therefore, considering different crisis and non-crisis periods, the findings are not in full support of investing internationally, and they show benefits for purely domestic portfolios (e.g., the Post-Asian Financial Crisis period).

Third, the academic literature has, additionally, addressed the issue of the stability of benefits through diversification over time. With their study on market correlation over the period of 1960 to 1990, Longin and Solnik (1995) emphasized that correlation is unstable over time. More specifically, explicit modeling of conditional correlation shows an increase of the international correlation between markets over this particular period. Growing market correlation due to financial integration can have substantial disadvantages for international diversification benefits. Hence, a growing amount of studies in the literature have found that diversification benefits tend to be unstable over time and are even reduced by the globalization of financial markets (Hyde et al., 2007; Shawky et al., 1997; Quinn & Voth, 2008).

### **3. Estimation Errors in the Mean-Variance Optimization**

Other papers have addressed the issue of estimation errors in mean-variance efficient portfolios. The application of the mean-variance optimization in practice is not straightforward because the set of inputs, such as means and (co)variances, are obtained from historical data. However, these inputs are likely to be affected by sampling error and, therefore, have an estimation risk. In the mean-variance framework, a small change in the values of means or covariances often leads to large changes in the portfolio composition and, therefore, biased findings (Cuthbertson & Nitzsche, 2005). In the next step, the most important work on the existence of estimation errors in mean-variance efficient portfolios and the sensitivity of optimal portfolio weights is introduced. In addition, various attempts based on Monte Carlo simulations are discussed to create better and more stable optimal portfolios to overcome the shortcomings of the mean-variance optimization.

Best and Grauer (1991) investigated the sensitivity of optimal portfolio weights to changes in expected return estimates. They confirmed that portfolio weights can be extremely sensitive

to changes in the means of assets. Britten-Jones (1999) presented another statistical procedure to show the sampling error in estimates of mean-variance efficient portfolio weights. The regression approach, used on 20 years of data from 11 country stock indices, showed the importance and impact of the estimation errors in estimates of the weights of an international mean-variance efficient portfolio.

Chopra and Ziemba (1993) extended the debate on the effect of errors with an analysis of the relative impact of estimation errors in mean, variance, and covariances. They agreed with the general consensus of recent studies in the literature that investors should spend a lot of time and resources on finding the best parameters, especially the expected return, for a better portfolio allocation process. Otherwise, mean-variance optimization will not result in a satisfying performance. Furthermore, the authors came up with interesting implications. They argued that investors with a moderate to high tolerance for risk experience a higher cash-equivalent loss due to errors in estimates of expected returns than for errors in estimates of variances and covariances. This is due to the fact that errors in expected returns have a relatively strong influence on the mean-variance optimization process.

One early study of Jobson and Korkie (1980) already conducted a sampling experiment based on Monte Carlo simulations. The historical input parameters, mean return vector, and covariance matrix were used to generate sample observations of the parameters. The authors found that the inputs for the optimization process have, indeed, significant errors.

Jorion (1992) presented a simulation approach based on random sampling that explicitly measures and illustrates the effect of estimation errors in portfolios. The author applied his concept, from the viewpoint of a US investor, on efficient global bond portfolios. He stated that the optimization algorithm used for mean-variance optimization is biased toward the choice of assets that form the optimal portfolios. Assets with the highest return are commonly chosen as the assets that are weighted heavily in the portfolio optimization process. This results in a lack of diversification, and those assets are most likely to suffer from estimation errors. Ultimately, the optimal portfolio weights are unstable relative to small changes in the input parameters, and the statistically equivalent global bond portfolios show substantial estimation errors with and without short-selling constraints.

Moreover, Michaud (1989) indicated that the Markowitz mean-variance framework, as the most widely used method for portfolio construction, has deficiencies as a practical tool in the investment community and, therefore, fails to meet widespread acceptance. The unconstrained mean-variance optimization process magnifies the errors associated with the input estimates. Therefore, this optimization process does more harm than good.

A few years later, Michaud (1998) introduced the concept of portfolio resampling and the resampled efficient frontier, termed resampled efficiency. Current theory on traditional mean-variance optimization, in conjunction with portfolio resampling grew, in part, out of the above discussed work of Jobson and Korkie (1980), Jorion (1992), and Michaud (1989). The overarching goal of Michaud is to improve the traditional mean-variance optimization technique and to address the aspect of estimation risk in the input parameters. Additionally,

resampled mean-variance optimization addresses input sensitivity and diversification in those optimal portfolios. The procedure is an application of Monte Carlo simulations and defines a new set of optimal portfolios called the resampled efficient frontier. Scherer (2002) criticized the portfolio resampling framework by expressing his concern about the extent to which the results can be generalized. However, he indicated that resampling remains an interesting procedure for further discussion while dealing with the important problem of error maximization.

Not every author in the academic literature agrees with the generally held view on estimation errors in the mean-variance optimization framework. The overwhelming majority of papers published emphasize the negative effects of estimation errors on mean-variance optimization and portfolio selection. Kritzman's (2006) findings oppose the general consensus on this issue. His study acknowledges that the results of mean-variance optimizations are sometimes the subject to highly sensitive input errors. However, relatively small input errors are of little consequence to the estimates of an investor. Therefore, the concern for estimation errors is exaggerated and, in his opinion, just "hype."

In summary, the topics of mean-variance portfolio optimization, international portfolio diversification, and estimation errors in mean-variance efficient portfolios have been investigated over a period of more than 60 years now. The rich literature on this particular matter suggests different and opposing findings considering the benefits of international diversification as well as the aspect of estimation errors. Nevertheless, a lot of questions remain open, and further investigations need to be conducted. The next two sections will explain the chosen approach concerning this study, starting with a section on the data gathered.

## **II. Data**

The study uses weekly closing prices of domestic, regional, and international stock market indices. The data was obtained from the Thomson Reuters Eikon database, which is a set of software products to help monitor and analyze financial information (Thomson Reuters, 2017). The sample period of this study ranges from June 2002 to December 2016, yielding a total of 760 observations. The analysis covers the whole sample period as a comprehensive study of international portfolio diversification benefits under different market conditions. However, the sample is also divided into a sub-period to examine the possible effects during the global financial crisis. This period begins in January 2007 and ends in December 2008, yielding a total of 105 observations. The crisis period covers the climax of the world financial crisis.

The sample used in this study comprises indices computed by Morgan Stanley Capital Investment (MSCI). The MSCI indices have been the benchmark of index construction and maintenance for more than 40 years and are well known in the world of finance. In particular, portfolio managers rely on reasonable index data to appropriately reflect international

investable opportunity sets of equities. One of the advantages of using the MSCI index data for this study is the MSCI index policy. The company provides consistent treatment across all markets and ensures that only liquid stocks are used for its indices. Otherwise, a company will be deleted from the market indices due to low liquidity (MSCI, 2017). Moreover, using only one type of index data from the same source ensures standardization and, therefore, better comparison between the different countries selected.

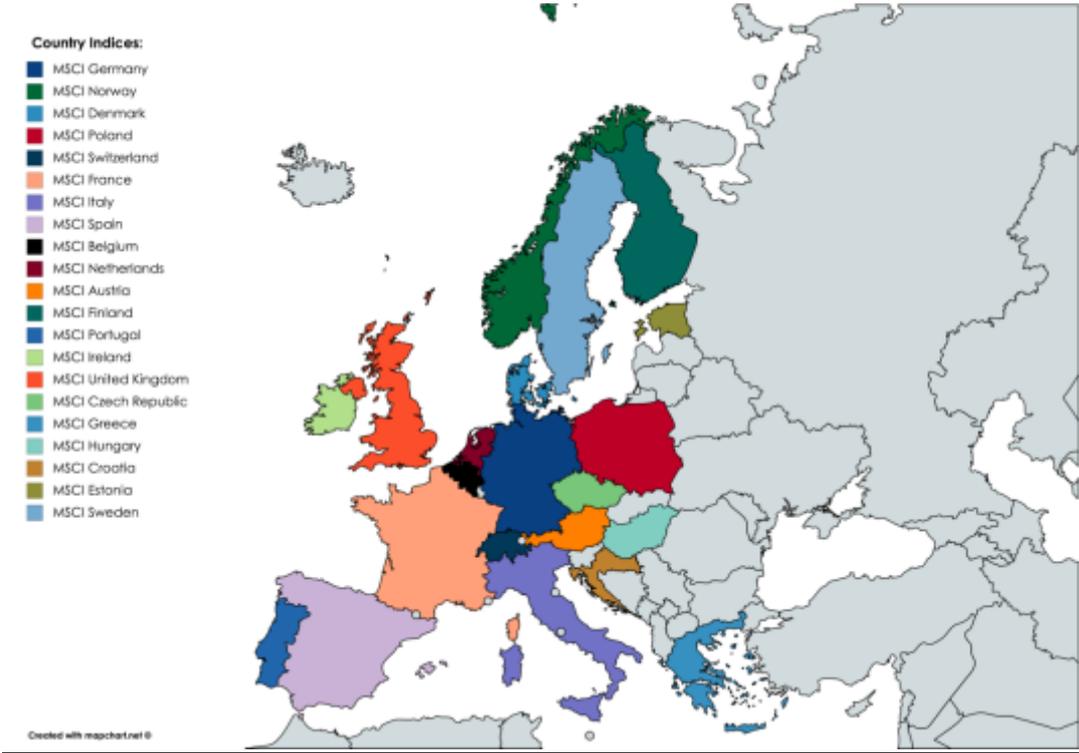
Additionally, it is important to note that currency exchange rate risk needs to be considered while examining international portfolio diversification benefits. Therefore, all MSCI indices are either already gathered in euros (€) or are gathered in different local currencies and subsequently denominated in euros (€) for this study. The countries' currencies denominated in euros (€) were computed with the appropriate exchange rate at every point in time of the whole sample period (2002–2016).

The study analyzes the performance of purely domestic portfolio diversification with regional and internationally diversified portfolios. The domestic portfolio consists of the MSCI Germany index, which reflects large and mid-cap segments of the German market. The index has 58 constituents and covers about 85% of the equity universe in Germany.

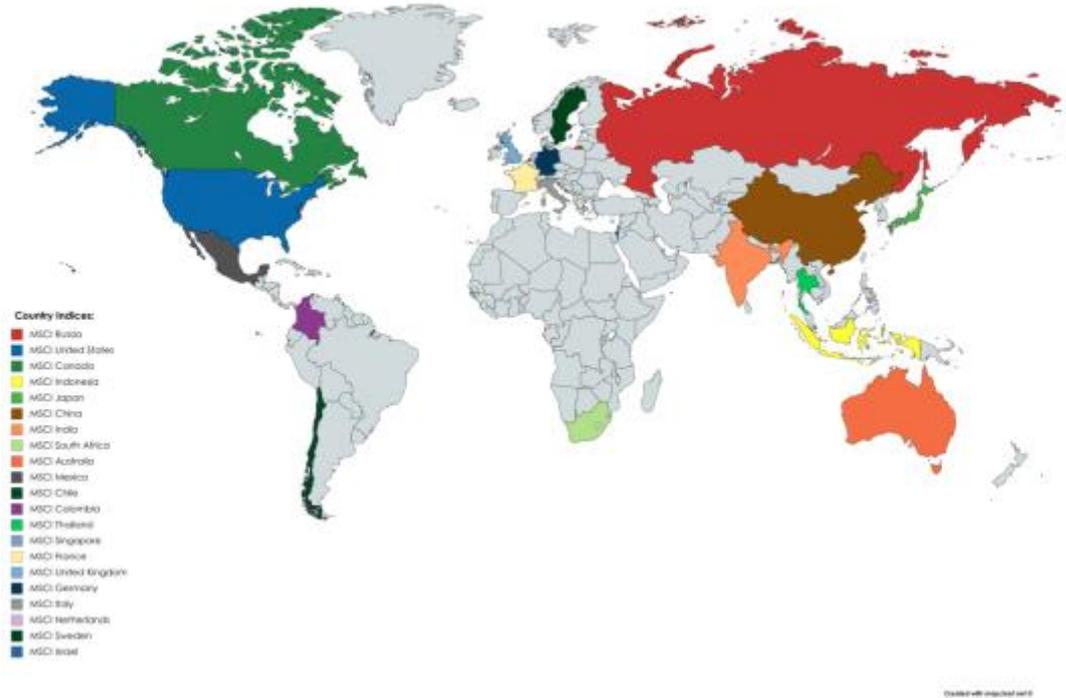
Regional diversification helps to allow investors to invest in any regional equity index for the region in which the country is located. As previously mentioned, the perspective of this study is German. Therefore, the regional portfolio referred to is the European portfolio. The European portfolio's main purpose is to examine how much of the international diversification benefits can be achieved by solely regional investments. Furthermore, public policy and, thus, policy makers have a strong interest in the topic because many countries have been considering the setup of regional stock exchanges in recent years. On the contrary, international portfolios may include investments anywhere in the world. Thus, the international or world portfolio is the main subject of this study to examine international portfolio diversification benefits and, subsequently, the aspect of the estimation errors in mean-variance efficient portfolios.

The countries and, therefore, the markets were selected according to their market capitalization and liquidity. Nevertheless, the availability of data was also considered for the selection of the portfolios. The European and international portfolios consist of the following 21 country markets.

**Figure 1:** Composition of the European Portfolio



**Figure 2:** Composition of the International Portfolio



The European and international portfolios include the MSCI Germany index as a proxy for a domestic portfolio. The reason for this is that an international diversified portfolio, in practice, should include a mixture of domestic as well as European or international assets to capture the possible benefits of international investments. Hence, the analysis was conducted in a more

realistic way. Moreover, the international portfolio consists of a mixture of countries from all relevant continents, including country indices from Europe.

As a final point, the calculation of the Sharpe ratio in order to maximize the risk–return ratio in a portfolio optimization process requires a risk-free interest rate. The Sharpe ratio analysis is discussed explicitly in the methodology section. This study uses the German government benchmark bond of 10 years as the risk-free interest rate. The whole sample, as well as the crisis period, has different risk-free rates calculated specifically for those periods. The risk-free rate is crucial in determining the optimal risky portfolio and can, therefore, affect the results considerably.

### III. Methodology

First, the traditional Markowitz mean-variance portfolio optimization model, which is used to examine possible preferences for international diversification versus domestic diversification, is analyzed. Furthermore, various extensions are discussed. Most importantly, the full sample period imposes various guidelines on the optimized portfolios. This results in a detailed examination of those portfolios. Finally, this section is concerned with the methodology to illustrate and account for estimation errors in mean-variance optimized portfolios. The chosen approach is based on portfolio resampling.

#### 1. Mean-Variance Diagnosis and Extensions

In the first step, the closing prices need to be converted into historical returns in order to perform the portfolio optimization. Therefore, after retrieving the closing prices of the underlying assets, it is important to note the chosen calculation of the asset returns. The study uses logarithmic or, in other words, continuously compounded returns. This is due to the fact that logarithms are commonly used in finance and are often a more useful and insightful way to look at economic data, despite their particular shortcomings (Hudson & Gregoriou, 2015). Furthermore, continuously compounded returns are time additive, and the study uses weekly closing price data (see Chapter II); therefore, the natural logarithm of returns has advantages in dealing with price fluctuations. The underlying formula can be expressed in the following way:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

where

$R_t$  = rate of return

$P_t$  = actual period price

$P_{t-1}$  = price, one period before  $P_t$

After obtaining all the rate of returns over the period, the mean returns of the individual assets are computed and annualized in the next step. The annualized rate has practical implications because it allows for a better comparison over a commonly used period over one year. Therefore, standard deviation and the variance–covariance matrix are also annualized.

The second step is concerned with the mathematical framework underlying the mean-variance optimization.

First, the return can be described as the reward in any investment process, and the expected return is the estimated return which may or may not occur. The expected return of the portfolio is obtained by finding the weighted average return of the assets included in the portfolio.

$$E(r_p) = \sum_{i=1}^n w_i E(r_i) \quad (2)$$

where

$E(r_p)$  = expected return of the portfolio

$\sum_{i=1}^n w_i$  = number of assets to evaluate and the weighting of asset i in the portfolio

$E(r_i)$  = expected return of asset i

Second, the concept of risk defines the possibility that the actual return from an investment may deviate from the expected outcome due to the unpredictability of future returns. Therefore, risk is considered to be a chance of variation in returns, where greater chances of variation are perceived as riskier investments than those with smaller chances of variation (Omisore et al., 2011). The portfolio variance can be expressed as the weighted average covariance of its individual asset returns:

$$Var(r_p) = \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j Cov(r_i r_j) \quad (3)$$

In other words, the covariance can be written in terms of the correlation coefficient.

$$Cov(r_i r_j) = \rho_{ij} \sigma_i \sigma_j = \sigma_{ij} \quad (4)$$

Therefore, the equation expressed in terms of the correlation coefficient will look as follows:

$$Var(r_p) = \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \rho_{ij} \sigma_i \sigma_j \quad (5)$$

where

$Var(r_p)$  = variance of the portfolio

$\rho_{ij}$  = correlation coefficient between the returns on assets i and j

$\sigma_i\sigma_j$  = standard deviations of  $r_i$  and  $r_j$ , respectively

Third, the portfolio volatility (standard deviation), as a measure of risk, is obtained by taking the square root of the portfolio variance:

$$\sigma_p = \sqrt{\sigma_p^2} \quad (6)$$

The actual portfolio optimization process was conducted by applying Excel's Solver. Portfolios with different target returns are constructed to find optimal portfolio weights, such that the standard deviation of the portfolios is minimized. Therefore, the optimization algorithm solves for the set of portfolios in the optimization process, which can be considered as the efficient parts in the risk–return spectrum. A portfolio is called efficient when there is no other combination possible with a better expected level of return compared to its level of risk. The return is plotted on the vertical axis, and the risk or standard deviation is plotted on the horizontal axis. The resulting curve is called the efficient frontier (Markowitz, 1952, 1959).

The advantage of an unconstrained mean-variance optimized portfolio is that it produces mathematically optimized solutions. However, it is not realistic to assume these portfolios in practice. The two following constraints are, therefore, always imposed on the portfolios:

$$\sum w_i = 1 \quad (7)$$

$$w_i \geq 0 \quad (8)$$

The sum of the weights of the portfolio sum up to 1, and the portfolio is, thus, fully invested. Moreover, the assumption of unlimited short-selling for an investor results in portfolios with many large long and short positions. However, most of the countries in the international portfolio as well as the European portfolio do not have the resources or know-how, have self-imposed constraints, or are forbidden by law to engage in unlimited short selling of equity indices. Therefore, the remainder of the study has imposed a non-negativity constraint on the portfolios. As a side note, the effect of unlimited short-selling is shown on the European portfolio, as an additional robustness check, in the appendix. The reason is that it is more realistic to assume unlimited short-selling on this portfolio than on the international portfolio with its various developing and emerging countries and different political systems.

Next, the optimization process results in an efficient frontier, spanning from the minimum-variance to the maximum return portfolio. The results section highlights the minimum-variance portfolio (MV) as well as the maximum Sharpe ratio portfolio (max Sharpe) (Sharpe, 1994). The maximum Sharpe ratio portfolio accounts for the optimal portfolio weights of risky assets in the presence of a risk-free asset (Eptas & Leger, 2010). The tangent line that

connects the risk-free rate to the feasible region for the risky assets on the efficient frontier is known as the capital market line (CML).

The tangency, or market portfolio, is the point where the Sharpe ratio is maximized and is, therefore, the optimal risky or most efficient portfolio from a risk–reward standpoint. The Sharpe ratio is commonly used to compare the performance of benchmarks or other indices. A higher ratio indicates a better risk-adjusted performance. The calculation is conducted, with the help of Excel’s Solver, in the following way:

$$\text{Max } SR = \frac{E(r_p) - r_f}{\sigma} \text{ subject to the constraints (7), (8)} \quad (9)$$

where

SR = Sharpe ratio

$E(r_p)$  = expected return of the portfolio

$r_f$  = risk-free rate

$\sigma$  = standard deviation of the portfolio

Instead of maximizing the Sharpe ratio, the crisis period is examined by a quadrant analysis, similar to Eptas and Leger (2010). The reasons for this are explained in the next section based on the results of this study. Additionally, correlation coefficients between the German equity index and all the other international indices are calculated. An investor can reduce portfolio risk by holding combinations of assets (e.g., i and j) with a correlation coefficient between -1 and 1 ( $-1 \leq \rho_{ij} < 1$ ). The more uncorrelated those assets are, the better the diversification benefits are. However, there are no diversification benefits and, therefore, there is perfect diversification when those two assets are perfectly correlated ( $\rho_{ij} = 1$  or  $-1$ ).

In addition to the mean-variance efficient frontier portfolios from the European and international portfolios, several investment rule sets of the international portfolio are determined. This allows the study to examine the benefits arising from internationally diversified portfolios from another more insightful perspective. The traditional efficient frontier of the international portfolio is, from now on, referred to as the EFI.

The first guideline represents a totally naïve investment strategy. This investor weights every national index equally ( $w_i = \frac{1}{n}$ ), also referred to as the equally weighted portfolio (EWP). The strategy can be described best as a hope of the investor that the variance of the expected return on the portfolio is lowered without sophisticated optimization models.

The second investment rule constrains the maximum weight of each individual country index to be at 20% ( $0 \leq w_i \leq 0.2$ ) and is called the limited-weighted portfolio (LWP). Hence, the optimization process cannot allocate a huge amount of the portfolio weights on one probably

superior national index, which commonly happens in the mean-variance optimization framework. Therefore, increasing diversification between the various indices is expected.

As a final and most important investment rule set, the home bias effect of German investors is analyzed. The proportion of portfolio assets that investors invest in their home markets is, in many countries, higher than what standard mean-variance analysis predicts. The purpose of this extension is to analyze the performance of a German investor subject to various degrees of home bias. The study imposes three different minimum weightings for the German portfolio on the optimization process. The first guideline imposes a minimum German weight of 25% ( $w_i = 0.25$ ) in the portfolio (HB25). Thereafter, the next minimum German weight was determined to be 50% ( $w_i = 0.5$ ) in the portfolio (HB50). Ultimately, the portfolio (HB75) had an efficient frontier with a minimum German weight of 75% ( $w_i = 0.75$ ) and, therefore, severe home bias is observed.

## **2. Portfolio Resampling and Resampled Efficiency**

As a final point, the results from the ex-post mean-variance analysis are put into perspective with a discussion of the estimation errors, input sensitivity, and the lack of diversification in mean-variance efficient portfolios. The chosen approach, concerning this study, is to (a) apply a Monte Carlo procedure known as portfolio resampling. Portfolio resampling is a technique which draws repeatedly from the return distribution to create statistically equivalent portfolios to visualize and measure the effect of estimation errors on the international portfolio over the whole period (see e.g., the simulation approach of Jorion (1992)). Moreover, (b) the resampled efficient frontier is drawn (resampled efficiency).

The methodology of the applied simulation approach can be summarized in the following way:

The mean return vector and the variance–covariance matrix of the international portfolio were calculated from the actual sample of historical returns, as the first realization of the point estimates. The number of assets in the international portfolio comprises 21 country indices, and the sample size is 14.5 years. Subsequently, the mean-variance optimization technique was applied, and the efficient frontier of the original sample was drawn. The optimization was conducted with short-selling constraints, and the estimates were assumed to be the true values, as performed in Jorion’s study (1992). Thereafter, the statistical software program STATA was used to draw a multivariate normal distribution with the estimated mean return and variance–covariance as the input parameters to generate a random sample of returns. This parametric method is called a Monte Carlo simulation. The random sample of returns was generated for all the assets of the portfolio and spans the whole sample period. In the next step, the new mean return vector and the new variance–covariance matrix were calculated to perform an optimization with these particular inputs. The optimal risky portfolio from a risk–reward standpoint was selected from the simulated optimal portfolios on the efficient frontier. Most importantly, the corresponding allocation vectors (efficient portfolio weights) of the

simulated optimal risky portfolio were saved. This procedure was repeated many times, and, each time, a new set of optimization inputs was estimated.

Ultimately, the allocation vectors, each representing one simulated optimal portfolio, were assessed in conjunction with the original mean return vector and the original variance–covariance matrix of the assumed true portfolio. This forced all estimates to plot below the original efficient frontier due to the fact that no allocation vector optimal for the simulated portfolios would be optimal for the original optimal portfolio. This leads to an illustration of the extension of estimation errors in the international portfolio and may help portfolio managers in practice to decide whether to pursue an original optimal investment or not.

Subsequently, a large-scale sensitivity analysis, also known as Michaud’s resampled efficiency, was applied in a similar fashion to the international portfolio. The implementation of resampled efficiency accounts for the uncertainty inherent in input parameters and, therefore, seeks to avoid the pitfalls of the mean-variance optimization procedure (Michaud, 1998).

First, a standard mean-variance optimization, with the mean return vector and the variance–covariance matrix, was run for the international portfolio. The minimum-variance portfolio and the maximum return portfolio, as the lowest and the highest points of the efficient frontier, respectively, were calculated. Subsequently, the difference between the minimum-variance and maximum return portfolio was divided into  $m$  ranks. In this study, nine different ranks were chosen for the international portfolio. This helps in analyzing the efficient frontier by looking at a series of points rather than every point on the frontier.

Second, a Monte Carlo simulation was run, assuming a multivariate normal distribution, based upon these parameters. Hence, the generated returns are statistically equivalent. Moreover, the simulated mean vector and simulated variance–covariance matrix were calculated. Up to that point, there seemed to be no big difference to the simulation approach. However, the difference is that the simulated optimized portfolios were analyzed in a different way. Instead of focusing only on the optimal risky portfolio, the whole simulated efficient frontier was divided into nine ranks to determine the size of the expected return increments. The efficient portfolio weights were then calculated and saved for every return increment of the international portfolio. This computational intensive procedure was repeated for 100 efficient frontiers, and each frontier represents one set of inputs. Ultimately, this resulted in 100 portfolios for each rank.

In the next step, the simulated portfolio weights for each rank were averaged with the following formula:

$$\bar{w}_m^{resampled} = \frac{1}{n} \sum_{i=1}^n w_{im} \quad (10)$$

Finally, it is possible to redraw the efficient frontier by evaluating the simulated averaged portfolio weights for each rank with the original mean vector and variance–covariance matrix.

This should, by definition, force the resampled efficient frontier to plot below the original efficient frontier.

### IV. Results

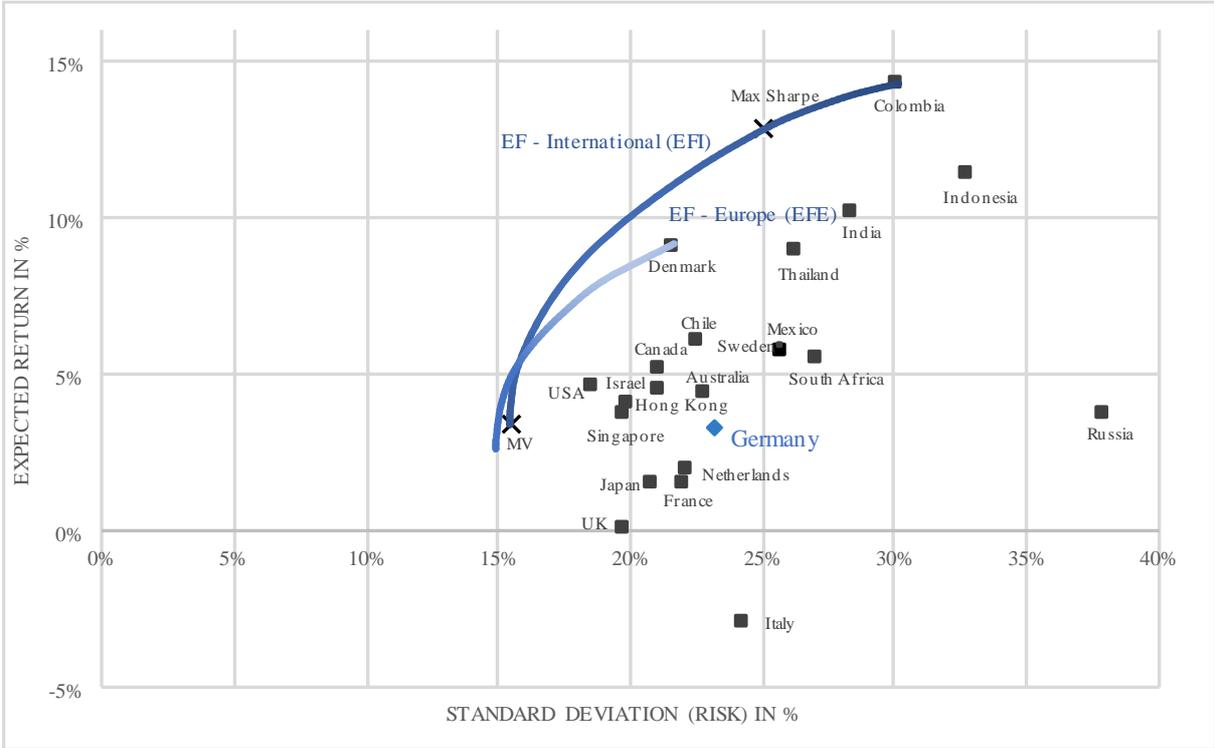
In this chapter, the results from the analysis are discussed. The chapter opens with the full sample period and takes, subsequently, a closer look on the crisis period (2007–2008). A discussion of estimation errors in the mean-variance optimization closes the chapter.

#### 1. Full Sample Period (June 2002–December 2016)

##### a) Mean-Variance Diagnosis and Extensions (Full Sample Period)

The figure plots two mean-variance efficient frontiers (EFI and EFE) with short selling constraints for the European and the international portfolio. Both efficient frontiers start at the minimum-variance portfolio and end at the maximum return portfolio. Moreover, the full sample of the domestically diversified national indices of the world portfolio and only partially for the regional portfolio are illustrated. This is due to reasons of clarity and comprehensibility and the focus on the world portfolio as the main subject of this study.

**Figure 3:** Mean-Variance Efficient Frontiers and National Indices (Full Sample Period)



Developed countries are superior in terms of the ranking of risk. Their corresponding standard deviation is far lower than that for developing countries in the sample. The US exhibits the lowest ranking of risk (18.52%), and Russia is ranked by far the worst (37.90%) in the risk ranking compared to the other countries in the world portfolio.

The mean-variance efficient international and European portfolios dominate all domestically diversified indices visibly. The exceptions are Denmark for the European portfolio and Colombia for the international portfolio. Both countries are the highest corner portfolios on the efficient frontier and offer the maximum returns. However, the risk involved is also maximized.

An efficient frontier “northwest” to another efficient frontier has advantages considering the risk–reward trade-off of the portfolios. It is interesting to observe that a mean-variance efficient international portfolio is, besides a small part in the beginning, superior to the European portfolio over the whole sample period.

Ultimately, by only applying visual evaluation, a German investor would benefit from diversifying in Europe as well as worldwide significantly. However, a rational investor would choose to hold a portfolio located on the international efficient frontier. In the next step, we extend the visual evaluation of this paragraph.

The same results become evident by taking a closer look on the weights, values, and Sharpe ratios (Table 1).

**Table 1:** Comparison of Mean-Variance Efficient International and European Portfolios (Full Sample Period)

Portfolio	EF - International (EFI)		EF - Europe (EFE)		Portfolio
	MV	Max Sharpe	MV	Max Sharpe	
Optimal Set on Efficient Frontier					Optimal Set on Efficient Frontier
United States	21.30%	0.00%	0.00%	0.00%	Norway
Japan	20.07%	0.00%	0.00%	98.91%	Denmark
India	0.00%	12.63%	0.00%	0.00%	Poland
Canada	0.00%	0.00%	0.00%	0.00%	Sweden
South Africa	0.00%	0.00%	48.45%	0.00%	Switzerland
Australia	0.00%	0.00%	0.00%	0.00%	France
Mexico	0.00%	0.00%	0.00%	0.00%	Italy
Indonesia	0.00%	13.46%	0.00%	0.00%	Spain
Chile	9.96%	0.00%	0.00%	0.00%	Belgium
Colombia	0.30%	63.86%	0.00%	0.00%	Netherlands
Hong Kong	0.00%	0.00%	0.00%	0.00%	Austria
Russia	0.00%	0.00%	0.00%	0.00%	Finland
Israel	23.94%	0.00%	11.73%	0.00%	Portugal
Thailand	0.00%	10.05%	0.00%	0.00%	Ireland
Singapore	16.97%	0.00%	0.00%	0.00%	United Kingdom
France	0.00%	0.00%	2.54%	0.00%	Czech Republic
United Kingdom	1.77%	0.00%	0.00%	0.00%	Greece
Italy	5.69%	0.00%	22.17%	0.00%	Croatia
Netherlands	0.00%	0.00%	15.11%	1.09%	Estonia
Sweden	0.00%	0.00%	0.00%	0.00%	Hungary
Germany	0.00%	0.00%	0.00%	0.00%	Germany

# Countries in Portfolio	8	4	5	2	# Countries in Portfolio
Return	3.45%	12.93%	2.64%	9.11%	Return
Standard Deviation	15.49%	25.10%	14.83%	21.51%	Standard Deviation
Sharpe Ratio	-	40.60%	-	29.64%	Sharpe Ratio

First, the optimal portfolio on the efficient frontier with the least risk (MV) will be analyzed. The regional portfolio exhibits slightly less risk (14.83%) compared to the world portfolio (15.49%). However, the return is also lower. The optimal risky portfolio, measured with max Sharpe, confirms the visual inspection of the previous paragraphs. The higher a portfolio's Sharpe ratio, the better its returns have been relative to the amount of investment risk taken. Therefore, the optimal portfolio, located on the international efficient frontier with a maximized Sharpe ratio of 40.6%, is superior to the regional portfolios' Sharpe ratio of 29.64%. As a side note, the Sharpe ratio of the domestically diversified index of Germany is 2.86% and, thus, substantially inferior to the optimized portfolios on the efficient frontiers.

Moreover, the number of countries forming both optimal portfolios on the efficient frontier, the MV and the max Sharpe portfolio, is relatively low compared to the 21 countries representing each portfolio in the beginning. The international portfolio is represented by eight countries in the MV portfolio, where Israel, the United States, and Japan are assigned the biggest weights in the portfolio (>20%). Colombia, only marginally represented in the MV portfolio, increases its portfolio weight substantially (63.86%) in the max Sharpe portfolio. The European portfolio offers an even lower number of countries in the two portfolios; only five countries account for the MV and two countries for the max Sharpe portfolios.

This may seem a small number at first sight. However, it is not an unusual finding when short-selling is restricted. Only a few dominant country indices were assigned with positive weights for the efficient frontiers. The lack of diversification is one of the reasons for the high estimation risk in the international portfolio and is treated in the third section of the results.

Unrestricted short-selling would assign a different positive or negative weighting in the optimal portfolios for every country. However, this would considerably weaken the practical relevance since it is not possible to achieve for most of the countries in the sample.

Most importantly, both efficient portfolios are better off without any German investment in the portfolios. Therefore, German investors could have optimized their portfolio performance by completely ignoring the German market. Instead, investors should have focused on investing in the markets, with the assigned portfolio weights, shown in Table 1. In fact, every other portfolio combination on the efficient frontier, besides the investigated MV and max Sharpe portfolios, point out a zero percent weighting for the German market, too (see Figure 8). Thus, regional or international portfolio diversification is highly recommended for the German investor.

However, it is not realistic to assume that an investor allocates funds solely on an international level. Some degree of home bias is evident for most of the national investors. Therefore, the following guidelines are imposed on the international portfolio to examine the performance of a German investor who actively participates in the German market.

**Figure 4:** Visualization of Efficient Frontiers Subject to Various Investment Rule Sets

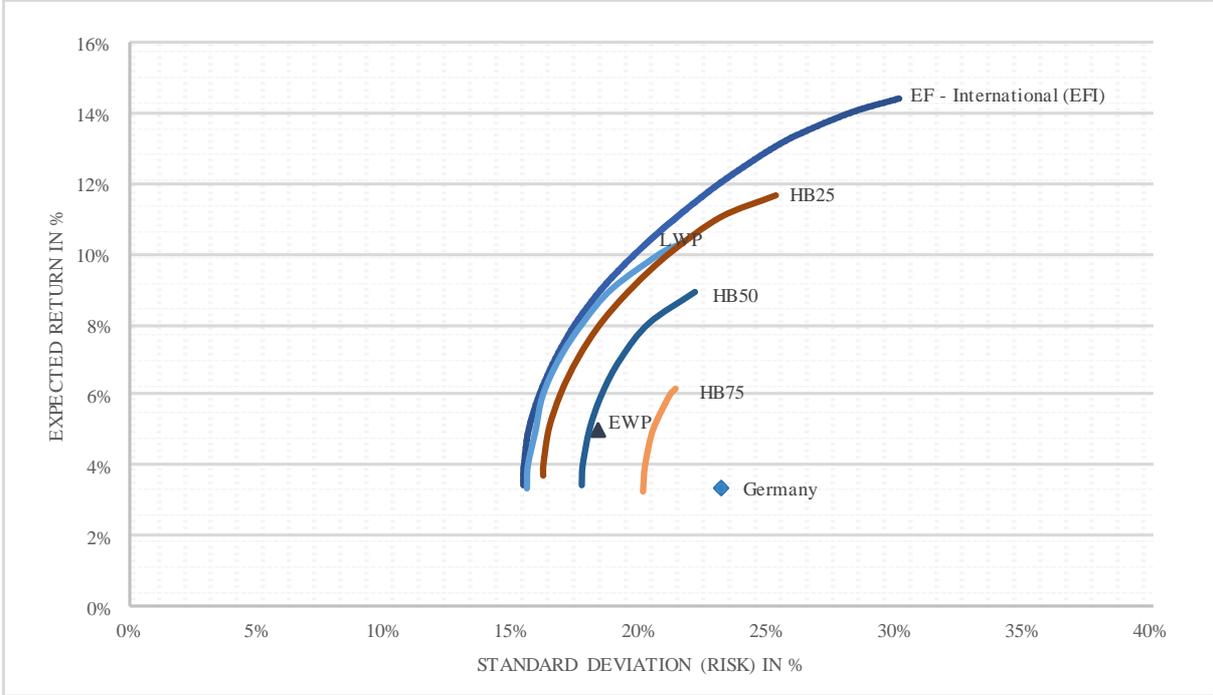


Figure 4 and Table 2 provide a summary of those results. The results of the visual inspection can be summarized in the following way:

The LWP runs almost in accordance with the EFI and drops only towards the highest corner portfolio. This shows that the EFI consists of a mixture of country markets, where one market is not explicitly dominant in the weighting of the portfolio. However, it is not possible to achieve the length of the original efficient frontier due to the limited weight constraint of 20%. Furthermore, the number of countries responsible for positive portfolio weights in the LWP has increased. The LWP shows, therefore, a higher rate of diversification. The German domestic market is still not represented in the efficient portfolio set.

In the next step, the effects of a German home bias are considered. These three efficient frontiers, to the right of the EFI and LWP, capture three minimum weightings in the German market. The intervals are chosen from a relatively moderate home bias with 25%, up to a severe home bias, with 75% of the assets in the set of optimal portfolios invested in the German market. The higher the German home bias in the efficient portfolio, the lower the performance of the efficient frontiers is.

**Table 2:** Comparison of Optimal Portfolios Subject to Various Investment Rule Sets

Portfolio	EF - International (EFI)		LWP		HB25		HB50		HB75		EWP
	MV	Max Sharpe	MV	Max Sharpe	MV	Max Sharpe	MV	Max Sharpe	MV	Max Sharpe	
Optimal Set on EF											
United States	21.30%	0.00%	20.00%	11.55%	8.87%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Japan	20.07%	0.00%	20.00%	0.00%	19.55%	0.00%	17.34%	0.00%	13.30%	0.00%	4.76%
India	0.00%	12.63%	0.00%	20.00%	0.00%	1.56%	0.00%	0.00%	0.00%	0.00%	4.76%
Canada	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
South Africa	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Australia	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Mexico	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Indonesia	0.00%	13.46%	0.00%	20.00%	0.00%	13.75%	0.00%	0.11%	0.10%	0.00%	4.76%
Chile	9.96%	0.00%	11.11%	8.45%	9.41%	0.00%	6.04%	0.00%	0.00%	0.00%	4.76%
Colombia	0.30%	63.86%	0.28%	20.00%	0.37%	59.68%	0.00%	49.89%	0.00%	25.00%	4.76%
Hong Kong	0.00%	0.00%	0.00%	0.00%	23.02%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Russia	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Israel	23.94%	0.00%	20.00%	0.00%	13.79%	0.00%	19.11%	0.00%	11.59%	0.00%	4.76%
Thailand	0.00%	10.05%	0.02%	20.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Singapore	16.97%	0.00%	18.56%	0.00%	0.00%	0.00%	7.50%	0.00%	0.00%	0.00%	4.76%
France	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
United Kingdom	1.77%	0.00%	4.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Italy	5.69%	0.00%	5.59%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Netherlands	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Sweden	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.76%
Germany	0.00%	0.00%	0.00%	0.00%	25.00%	25.00%	50.00%	50.00%	75.00%	75.00%	4.76%
# Countries in Portf.	8	4	9	6	7	4	5	3	4	2	21
Return	3.45%	12.93%	3.36%	10.10%	3.69%	11.16%	3.42%	8.88%	3.25%	6.14%	5.01%
Standard Deviation	15.49%	25.10%	15.51%	21.07%	16.21%	23.61%	17.75%	22.18%	20.13%	21.41%	18.35%
Sharpe Ratio	-	40.60%	-	34.97%	-	35.70%	-	27.69%	-	15.91%	12.39%

Furthermore, the composition of the countries representing each efficient frontier drops considerably compared to the EFI. Initially, eight countries form the MV portfolio, which decreases to seven countries for the HB25 portfolio, five for the HB50 portfolio, and only four countries represent the HB75 portfolio. The same effect is apparent for the max Sharpe portfolios. The length of the efficient frontiers also declines. All three efficient frontiers are inferior to the EFI and LWP.

The MV and max Sharpe values of Table 2 validate the visual inspection. For the MV portfolios on the efficient frontiers, the return of the LWP is only 0.09% lower and 0.02% riskier than for the EFI portfolios. Therefore, the two portfolios are almost similar. However, the optimal portfolio on the efficient frontier where the Sharpe ratio is maximized shows substantial differences. The Max Sharpe portfolio on the optimal set of the EFI has a superior Sharpe ratio of 40.6% compared to the 34.97% for the LWP.

More interestingly, an investor who had chosen to invest 25% in the German market (HB25) would have earned, referring to the MV optimal portfolio, 3.69%, slightly more compared to the EFI return of 3.45%. However, the standard deviation also rises. Moreover, the Sharpe ratio, referring to the max Sharpe column, is slightly higher for the HB25 (35.7%) than for the LWP (34.97%). Therefore, the optimal risky portfolio is in favor of the HB25. This measure deteriorates rapidly for HB50 (27.69%) and HB75 (15.91%).

The totally naïve investment strategy (EWP) is only slightly inferior to the HB50 portfolio. Furthermore, the naïve investment strategy is compared to the strategy of investing 75% of the assets in the German market (HB75) located in a superior position, in terms of the risk–return trade-off. Furthermore, an equally weighted portfolio is still advantageous considering return, standard deviation, and Sharpe ratio compared to the domestically diversified index of Germany.

In conclusion, the LWP effect on the EFI is rather weak. A German home bias, depending on the severity of the bias, decreases the efficiency of the EFI drastically. However, the results are still superior investment strategies compared to solely domestic investments. These observations further confirm the importance of international diversification for the German investor.

## 2. Crisis Period (January 2007–December 2008)

### a) Mean-Variance Diagnosis and Extensions (Crisis Period)

The crisis period opens with a section on the correlation coefficients. It is not possible to report full sample correlation coefficients for every country due to reasons of clarity and comprehensibility. Thus, Table 3 displays the correlation coefficients between Germany and the countries of the international portfolio. However, the same effects are apparent for the full sample.

**Table 3:** Correlation Coefficients Between Germany and Each of the 20 Remaining Countries of the International Portfolio

Correlation - International	MSCI Germany - Full Period	MSCI Germany - Crisis Period
MSCI United States	0.7134	0.6945
MSCI Japan	0.4493	0.4105
MSCI India	0.4866	0.6481
MSCI Canada	0.6817	0.7035
MSCI South Africa	0.6224	0.7294
MSCI Australia	0.6207	0.6625
MSCI Mexico	0.6261	0.7120
MSCI Indonesia	0.3610	0.5087
MSCI Chile	0.4967	0.5237
MSCI Colombia	0.3741	0.5658
MSCI Hong Kong	0.5317	0.6023
MSCI Russia	0.5249	0.6571
MSCI Israel	0.4958	0.5072
MSCI Thailand	0.4078	0.4365
MSCI Singapore	0.5480	0.6244
MSCI France	0.9228	0.8627
MSCI United Kingdom	0.8123	0.7801
MSCI Italy	0.8098	0.7615
MSCI Netherlands	0.8796	0.7948
MSCI Sweden	0.8417	0.7744
MSCI Germany	1.0000	1.0000
<b>Mean</b>	<b>0.6289</b>	<b>0.6648</b>

The average correlation of Germany with the countries of the international portfolio rises from 62.89% to 66.48%. This proves that diversification with international stocks is not as effective during the global financial crisis for the German investor. Appendix B reports the

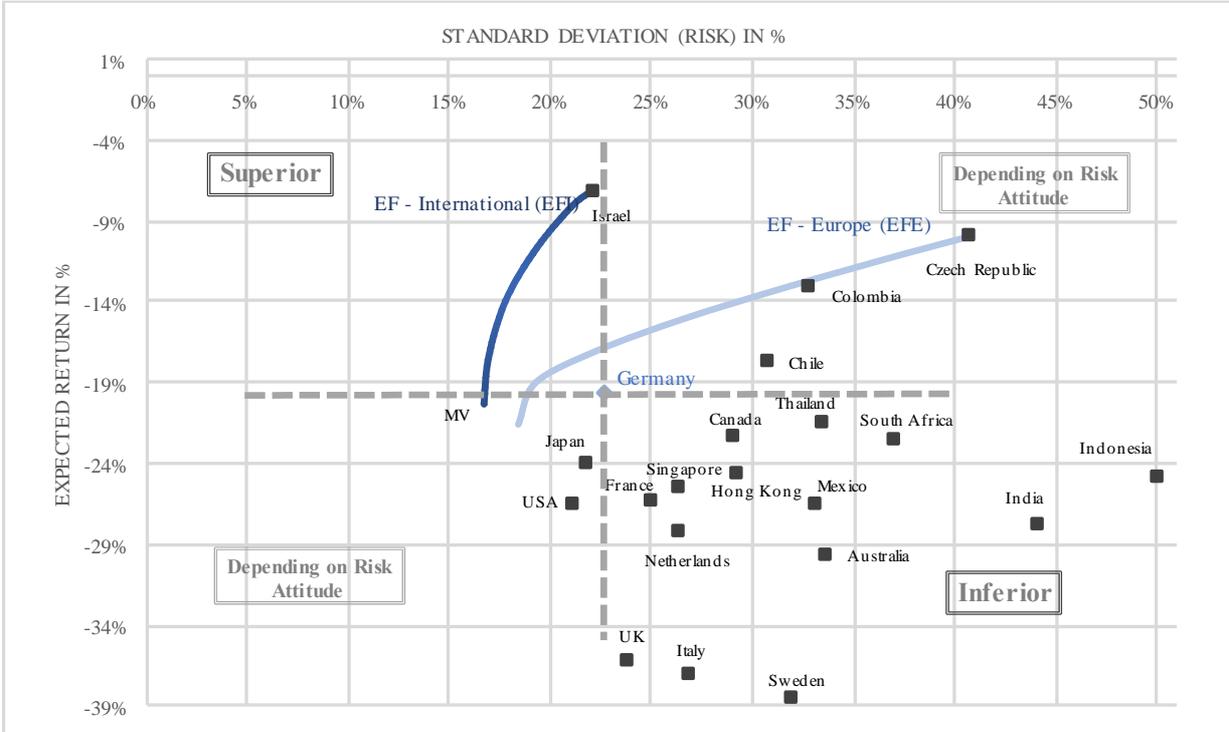
same effect for the European portfolio. Correlation rises during times of crisis from 69% to 70.94%. The European markets show, already, a relatively higher correlation with Germany compared to the Asian and South American markets. This observation is reasonable since the countries are in close geographical vicinity, and the markets are politically and economically connected through the European Union. It can be observed that countries located outside of the European continent indicate lower values in general. Asian and South American countries have a relatively weaker correlation with Germany. Indonesia and Colombia offer the best possibility to diversify internationally for the German investor due to the fact that they have the weakest correlation of all the countries analyzed.

Investors with different time horizons and strategies all suddenly focus primarily on the present. Hence, risky assets tend to all behave in a similar fashion, and the benefits of international diversification decrease.

In summary, we can observe increasing correlations. How will the mean-variance framework and the optimized portfolios on the efficient frontier behave during the global financial crisis?

Figure 5, again, excludes most of the domestically diversified national indices of the European portfolio and Russia as part of the international portfolio due to a large negative return and standard deviation. This happens for reasons of clarity and comprehensibility. The countries are, however, considered in the optimization process.

**Figure 5:** Mean-Variance Efficient Frontiers and National Indices (Crisis Period)



All the returns of the domestically diversified indices are negative, and the Sharpe ratio analysis would also be negative. Therefore, quadrant analysis is used for the crisis period. Every set of internationally diversified optimal portfolios would have been, during the world

financial crisis, clearly superior to the regionally diversified optimal portfolios on the efficient frontier. Almost the entire frontier lies in the superior quadrant of the figure, whereas the European portfolio would be only partially better off. For the most part, the investment decision depends on the risk attitudes of German investors. A better return would result in a higher standard deviation. The better result for the international portfolio, with its many developing countries, is also an indication that developed countries were hit the hardest during the world financial crisis.

As for the national indices, the German index already has a lower standard deviation and higher returns compared to most other countries in the sample. Only the US, Japan, and Israel are superior to Germany in terms of the standard deviation. An investment in the domestically diversified index of the United States and Japan would result in an even lower standard deviation. However, the return would also be limited. Conversely, Chile and Colombia offer a higher return but also an increasing level of risk over the sampled period. Israel is the only domestic index in the superior quadrant of the sample and emerges as a country with safe haven qualities for the German investor. Therefore, the domestic market of Israel offers a certain degree of protection for the German investor during times of crisis. The reasons for that are pure speculation and may be linked to Israel's unique economical activities and political arrangements (Eptas & Leger, 2010).

**Table 4:** Comparison of Mean-Variance Efficient International and European Portfolio (Crisis Period)

Portfolio	EF - International (EFI)	EF-Europe (EFE)	Portfolio
Optimal Set on Efficient Frontier	MV	MV	Optimal Set on Efficient Frontier
United States	28.84%	0.00%	Norway
Japan	38.37%	0.00%	Denmark
India	0.00%	0.00%	Poland
Canada	0.00%	0.00%	Sweden
South Africa	0.00%	73.00%	Switzerland
Australia	0.00%	0.00%	France
Mexico	0.00%	0.00%	Italy
Indonesia	0.00%	0.00%	Spain
Chile	0.00%	0.00%	Belgium
Colombia	0.00%	0.00%	Netherlands
Hong Kong	0.00%	0.00%	Austria
Russia	0.00%	0.00%	Finland
Israel	24.15%	0.00%	Portugal
Thailand	0.00%	0.00%	Ireland
Singapore	0.00%	0.00%	United Kingdom
France	0.00%	0.00%	Czech Republic
United Kingdom	0.00%	0.00%	Greece
Italy	0.00%	12.12%	Croatia
Netherlands	0.00%	0.00%	Estonia
Sweden	0.00%	0.00%	Hungary
Germany	8.64%	14.88%	Germany
# Countries in Portfolio	4	3	# Countries in Portfolio
Return	-20.26%	-21.59%	Return
Standard Deviation	16.67%	18.44%	Standard Deviation

The MV efficient portfolio is situated closely in the bottom left quadrant. As already

mentioned, correlations between countries have increased during the global financial crisis. This has an impact on the optimal portfolio weights, and the optimal international portfolio with the least risk (MV) relies, during times of crisis, even more on a few dominant countries: the United States, Japan, Israel, and, quite interestingly, Germany. The weight of the German market even increases for the optimal portfolios along the efficient frontier, up until column 4 (see Appendix C). Chile and Colombia are also part of the efficient frontier in higher return regions. However, this means Singapore, the United Kingdom, and Italy lose their place in the set of optimal portfolios, whereas Germany, not represented at all in the full sample period, emerges as the only new index in the optimal portfolios. Therefore, German investors should have invested 8.64% of their funds domestically for a better risk–reward position. More specifically, investors should do this for the optimal investment with the least risk (MV). The same effect can be observed in the European portfolio. Germany, not represented in the full sample, accounts for 14.88% of the optimal portfolio with the least risk during times of crisis.

In conclusion, international diversification remains an important benefit during times of severe stock market stress. Furthermore, the real advantages of international diversification appear for the German investor because the mean-variance optimization shows that a partial investment in the domestic market of Germany is optimal. Therefore, the portfolios consist of a mixture of international and domestic assets.

Nevertheless, a profitable German investor would need to invest in risk-free assets rather than risky national indices.

### **3. Portfolio Resampling and Resampled Efficiency**

The major drawback of Markowitz's methodology in the portfolio optimization process is, as mentioned in this study, the uncertainty inherent in the input parameters. Slight changes in expected returns or covariances can lead to significantly different optimal portfolio weights. Moreover, the optimization algorithm used for the mean-variance analysis tends to pick those assets with the best characteristics and disregard those assets with the worst characteristics; the algorithm treats them as if they were known with certainty. However, the impact of the estimation errors on those assets is, in almost all cases, exceptionally high (Scherer, 2002). This gives rise to a lack of diversification and a poor out-of-sample performance of mean-variance optimized portfolios. Therefore, do the international diversification benefits hold for the German investor, while considering the issue of estimation error, in mean-variance efficient portfolios?

**Figure 6:** Portfolio Resampling and Visualization of Estimation Errors

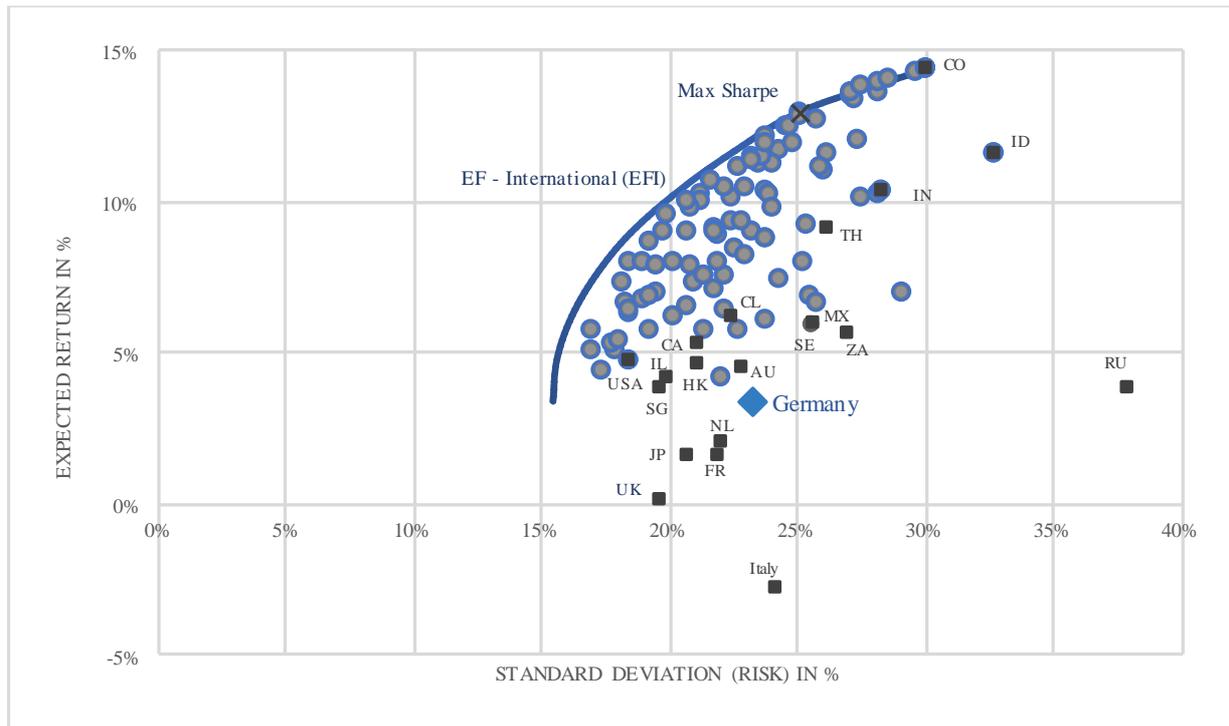


Figure 6 depicts the original mean-variance efficient frontier of the international portfolio with short-selling constraints, as determined in the previous sections. The minimum-variance portfolio (MV) is situated in the lower corner of the efficient frontier, and the optimal risky portfolio (max Sharpe) is labeled accordingly. Each additional blue dot represents the performance of a statistically equivalent portfolio that maximizes the risk–reward trade-off. These dispersed estimation error results highlight the sensitivity of the portfolio to changes in input parameters. The squares are the domestically diversified national indices of the international portfolio used in this study.

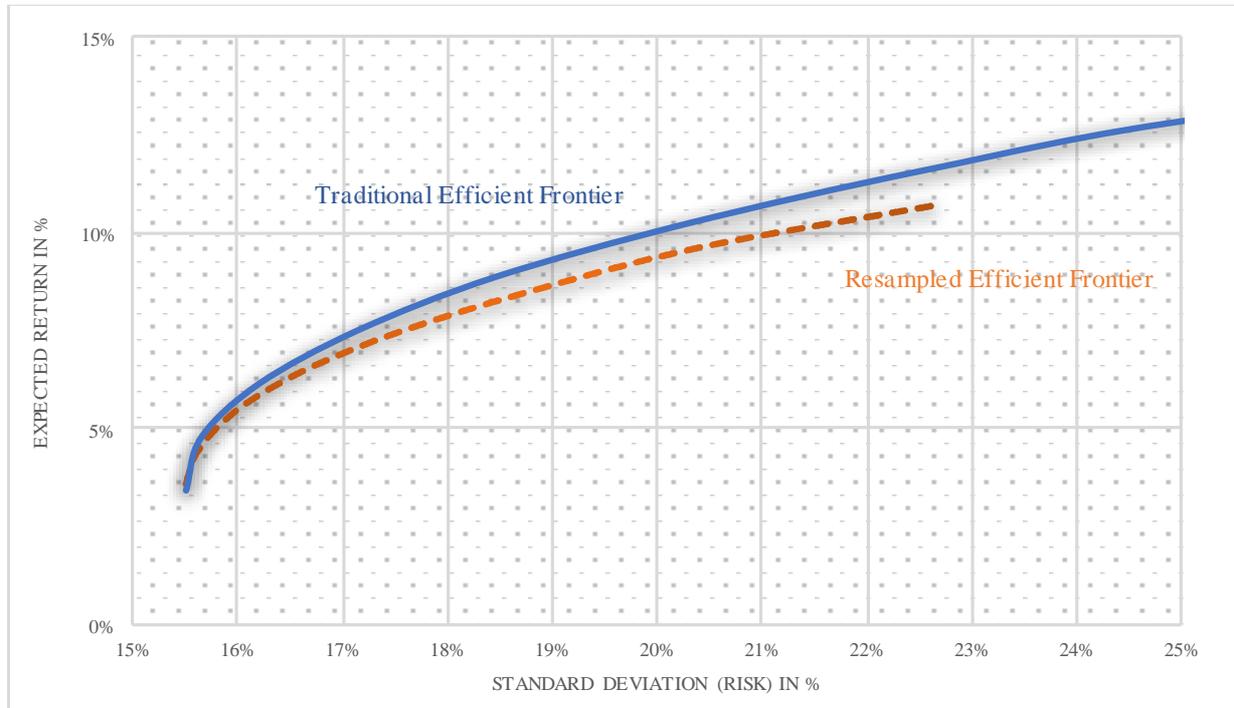
Firstly, the observation concerning the measurement of estimation errors is that the underlying world portfolio, indeed, suffers from substantial estimation errors. Furthermore, there are cases for the statistically equivalent portfolios apparent, where the domestic indices are suddenly superior to the world portfolios. The optimization algorithm has picked different optimal portfolio allocation weights. This demonstrates that even small changes in the sample data can cause significant changes in the mean-variance efficient frontier. These findings are, however, vanishingly low and can be neglected as outliers.

More importantly, it is still better to have an internationally diversified portfolio that suffers from estimation errors than a purely domestically diversified portfolio. To be more precise, the German investor has clear-cut international diversification benefits over the whole sample period since the domestically diversified index is clearly inferior to the original efficient frontier as well as the statistically equivalent regions.

Standard mean-variance optimization leads to highly concentrated and undiversified asset allocations. Moreover, the uncertainty inherent in the input parameters leads to an estimation

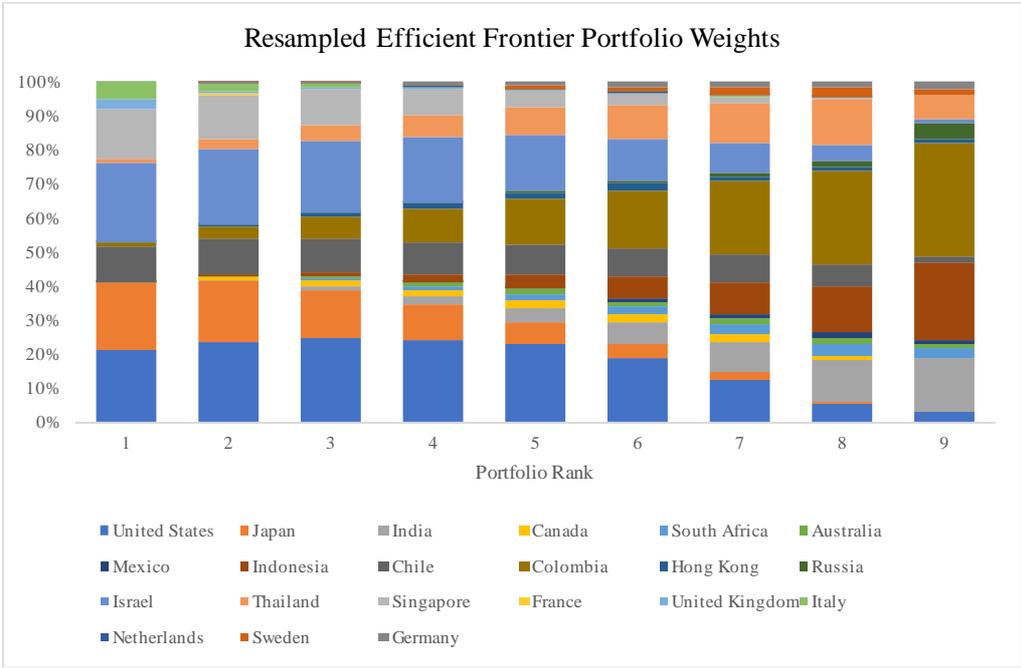
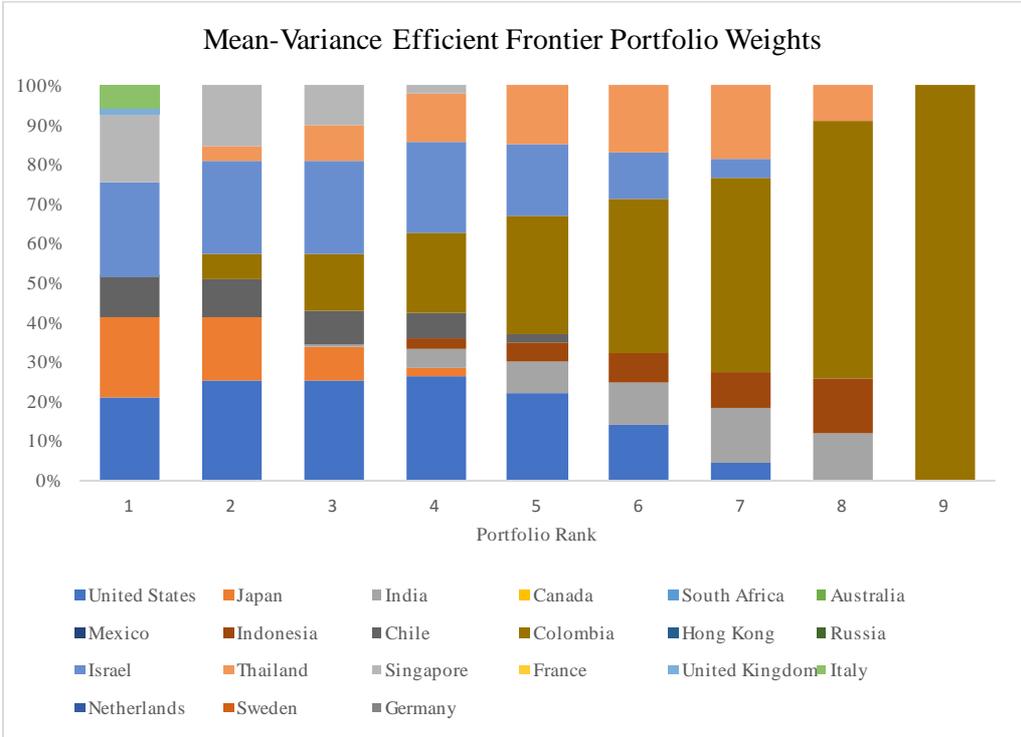
risk of the portfolio. This was confirmed for the international portfolio in the previous sections. Figure 7 displays, again, the traditional mean-variance efficient international portfolio over the full sample period.

**Figure 7:** Resampled Efficient Frontier



Furthermore, the dashed curve, known as the resampled efficient frontier, is the result of the approach explained in detail in the methodology section of this study. In short, sampling from the multivariate normal distribution and the subsequent optimization procedure result in 100 different simulated efficient frontiers along the portfolio ranks, where the portfolio weights are saved and averaged. Subsequently, these weights were linked back to the original efficient frontier. As a result, the resampled efficient frontier plots below the traditional efficient frontier. Thus, the risk–reward ratio is slightly inferior to the original efficient frontier. The resampled efficient frontier is also shorter than the traditional frontier. That is due to two features of the resampled portfolios. Figure 8 examines the traditional efficient frontier allocation together with the resampled efficient frontier allocation and is used to visualize these features.

**Figure 8:** Mean-Variance Efficient Frontier Portfolio Weights and Resampled Efficient Frontier Portfolio Weights



The first feature of the resampled portfolios, along the resampled frontier, is that the allocation vectors are significantly more diversified. The comparison above shows that every portfolio rank of the resampled efficient frontier consists of a higher number of countries in the optimal portfolio compared to the traditional efficient frontier. For instance, portfolio rank 1 includes eight country indices according to the traditional procedure, while 15 country

indices are included in the resampling procedure. Therefore, the approach helps to overcome the problem of highly concentrated asset allocations in the standard mean-variance efficient framework.

Second, the resampled portfolios show smooth transitions and, therefore, exhibit less sudden shifts in allocation along the resampled frontier. The traditional mean-variance procedure shows different characteristics. For instance, as we move from portfolios of rank 8 to portfolios of rank 9, the traditional mean-variance procedure provides a portfolio fully invested in Colombia. This is due to the fact that Colombia is the country with the highest return of all the indices during the sample period. These extreme portfolio characteristics are prone to likely maximize estimation errors and exhibit poor out-of-sample performance. In contrast, the resampled efficient frontier provides smooth transitions in allocation weights along the efficient frontier. Hence, the averaged simulations lead to more stable results and are not dependent on only one set of input parameters, which were probably measured with substantial estimation errors.

These features make the concept of resampled efficiency highly attractive for the German investor. However, the study acknowledges recent criticism of the procedure. Strictly speaking, the resampled efficient frontier is not mean-variance efficient anymore, and it depends heavily on no short-selling constraints. Otherwise, the procedure provides no improvement over the traditional mean-variance optimization. The reason is that, as can be seen in Figure 8, the resampled portfolio has a substantial increase in diversification. Unconstrained optimal portfolios already consist of optimal portfolios with large long or short positions, and this would not change during the resampling procedure. As mentioned, this study is, however, conducted with these non-negativity constraints to impose more practicability.

## V. Conclusion

The results of the mean-variance optimization show that international portfolio diversification is superior to European as well as domestically diversified portfolios over the whole sample period. One measure specifically considered for this study is the optimal portfolio on the efficient frontiers, where the Sharpe ratio is maximized. The maximized Sharpe ratio confirms the visual inspection. The optimal portfolios for the full sample period do not support German investment. In fact, it would be optimal to not invest in the German market at all. The portfolios along the efficient frontier do not show any weighting for the German market. However, international diversification is generally defined as the interplay and mixture between domestic and international markets.

Therefore, various investment guidelines have been imposed on the portfolios. A German home bias, depending on the severity of the bias, decreases the efficiency of the EFI drastically. However, the results are still superior investment strategies compared to domestic investments alone. The limited-weight portfolio plots in almost the same area as the

traditional efficient frontier, but the length of the optimal portfolios, forming the efficient frontier, is constrained. An equally weighted portfolio (EWP), as well, outperforms the purely domestically diversified investment. Thus, the EWP is a valuable tool for less informed or time-restricted investors. Ultimately, every considered portfolio performs better than the domestic portfolio when measured by the maximized Sharpe ratio. These observations further confirm the importance of international diversification for the German investor.

After the full sample, the study investigated the crisis period, known as the global financial crisis. Correlations increase for the international and European markets, and diversification in the optimal portfolios is, thus, restricted. Furthermore, visual inspection shows that the international efficient frontier has clear-cut benefits over the European efficient frontier. However, the investigation of the optimal portfolio weights of the crisis period compared to the full sample yields different results. The German market becomes, suddenly, responsible for a relatively high percentage of the optimal portfolio weights on the efficient frontier. Quadrant analysis shows that the German market has a potential as a safer and more stable market during the global financial crisis. The international efficient frontier performs the best, as its dimensions are almost entirely in the superior quadrant. Most of the optimal portfolios on the European efficient frontier are dependent on the risk attitudes of German investors.

Most importantly, portfolio resampling points out that the aspect of estimation errors in mean-variance efficient portfolios constrains the benefits of international diversification for the German investor considerably. Therefore, Kritzman's (2006) findings that the concern for estimation errors might be exaggerated cannot be confirmed for this study. However, the statistically equivalent portfolios (Figure 6) remain, for the most part, superior to the purely domestically diversified country indices. This is another finding in favor of international diversification, even though estimation errors are explicitly considered.

The concept of the resampled efficient frontier accounts for the estimation risk apparent in the international portfolio. Two implications become evident for the international portfolio. First, the resampled international portfolios exhibit greater diversification compared to the traditional mean-variance efficient portfolios. Second, the resampled international portfolios exhibit less sudden shifts and provide, therefore, smooth transitions along the portfolio ranks (resampled frontier). Portfolios on the resampled frontier show, therefore, highly desirable characteristics for practitioners and investors. At first sight, these practical implications offer enormous advantages for the investor. However, these results should be treated with caution. The assumption of non-negativity constraints is essential for resampled frontiers.

Nevertheless, the resampling procedure is compared to the traditional mean-variance optimization an attempt to create better optimization solutions. Thus, the aim of the combination of Monte Carlo simulations and the traditional mean-variance optimization is to develop robust, more forward-looking asset allocations.

Additionally, the results of the study might be sample specific. It may be difficult to extrapolate the findings of the crisis period of this study to any other crisis period in the

future. Furthermore, a continuing increase in correlations among the markets in the future is expected to happen.

Suggestions for further research include a stochastic dominance (SD) approach while considering international diversification benefits. Instead of applying the classical mean-variance framework to the portfolio selection problem, recent literature has emphasized the use of SD approaches. For instance, Abid et al. (2014) compared the classical mean-variance framework with the SD framework. The idea of SD is closely related to models of risk-averse preferences. The SD framework puts another perspective on the vast field of portfolio theory and asset allocation. Thus, the approach might overcome specific shortcomings of the mean-variance optimization model.

Another topic of discussion considering the aspect of estimation errors in mean-variance efficient portfolios is a procedure called robust optimization. The approach incorporates the issue of uncertainty in unknown parameters directly and explicitly in the optimization problem. Robust optimization was introduced by Ben-Tal and Nemirovski (1997) but applied for the first time by Lobo et al. (1998) in a portfolio optimization problem. Ceria and Stubbs (2006) proposed robust optimization to explicitly consider estimation errors in mean-variance optimal portfolios. However, the methodology is a more complex optimization problem and has yet to find significant relevance in academics as well as in practice.

Finally, the limitations of modern portfolio theory have led to recent advances in the field, termed as post-modern portfolio theory (PMPT). PMPT attempts to expand the risk–return paradigm, and further research will offer important insights on international diversification and portfolio theory in general.

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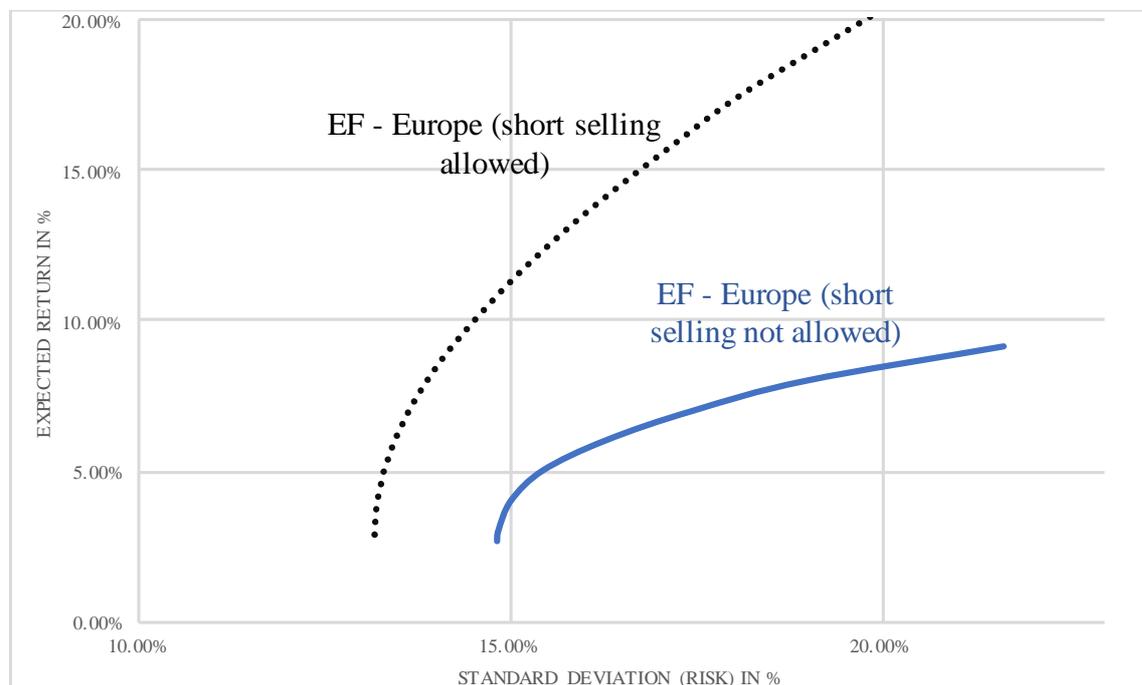
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## Appendix

### Appendix A: Effect of Short-Selling on the European Portfolio



**Note:** An efficient frontier with unlimited short-selling is clearly superior to an efficient frontier where short-selling is restricted in the risk–reward space. However, an efficient frontier where short selling is permitted has no finite upper bound and is hardly relevant in practice.

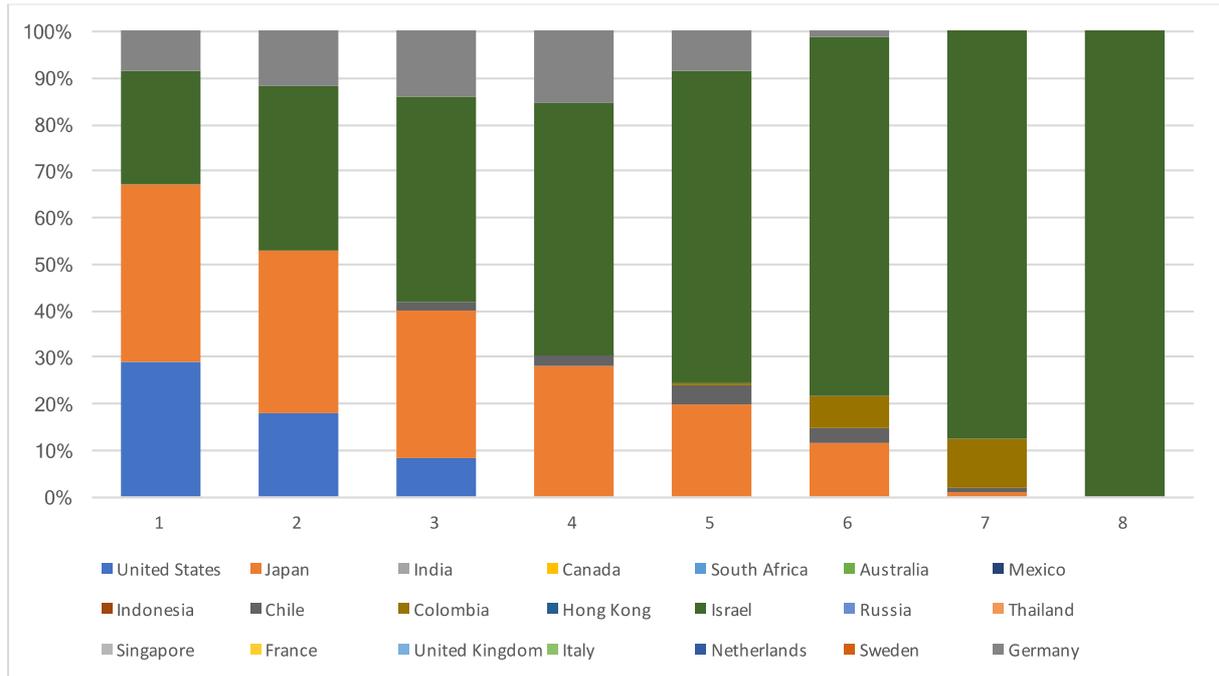
### Appendix B: Correlation Coefficients Between Germany and Each of the Remaining 20 Countries for the European Portfolio

Correlation - Europe	MSCI Germany - Full Period	MSCI Germany -Crisis Period
MSCI Norway	0.6961	0.7617
MSCI Denmark	0.6585	0.6937
MSCI Poland	0.6079	0.7545
MSCI Sweden	0.8417	0.7744
MSCI Switzerland	0.8101	0.7307
MSCI France	0.9228	0.8627
MSCI Italy	0.8098	0.7615
MSCI Spain	0.7585	0.6482
MSCI Belgium	0.7753	0.6770
MSCI Netherlands	0.8796	0.7948
MSCI Austria	0.6792	0.7299
MSCI Finland	0.7827	0.7371
MSCI Portugal	0.6166	0.6921
MSCI Ireland	0.6448	0.6079
MSCI United Kingdom	0.8123	0.7801
MSCI Czech Republic	0.4801	0.6215
MSCI Greece	0.4766	0.7133
MSCI Croatia	0.3504	0.4789
MSCI Estonia	0.3475	0.4336

MSCI Hungary	0.5400	0.6429
MSCI Germany	1.0000	1.0000
Mean	0.6900	0.7094

**Note:** Correlation coefficients for the full sample are not reported due to reasons of clarity and comprehensibility. The German correlation coefficients give, however, an indication of the effects for the full and crisis period.

**Appendix C: Crisis Period Distribution of Optimal Portfolio Weights for the International Portfolio.**



**Note:** Column 1 represents the minimum-variance portfolio, and column 8 shows the maximum return portfolio. As already mentioned, correlations between countries increase during the global financial crisis. This has an impact on the optimal portfolio weights. The optimal portfolio weights along the efficient frontier are less diversified during the global financial crisis and rely even more on a few dominant indices.